SEMI-LAGRANGIAN ADVECTION SCHEME WITH CONTROLLED DAMPING - - AN ALTERNATIVE WAY TO NONLINEAR HORIZONTAL DIFFUSION IN A NUMERICAL WEATHER PREDICTION MODEL

FILIP VÁŇA
CHMI

Introduction
SLHD scheme definition
SLHD scheme properties
Demonstration of SLHD scheme skills
Conclusion

Horizontal diffusion in atmospheric models

- Formal mathematical reason

  avoid the hyperbolic kind of model equations

  $\Rightarrow$ **with current models no need to take care about...**

- Parameterization of the physical processes

  horizontal turbulence and the molecular exchange

  $\Rightarrow$ **non-linear operator using flow field characteristics**

- Numerical filter

  removing the accumulated energy from the end of a model resolved spectrum and filtration of the numerical noise

  $\Rightarrow$ **linear operator of the kind of $K\nabla^r$ is sufficient**

SLHD scheme definition

Semi-Lagrangian advection

$$\frac{d\Psi}{dt} = \mathcal{R} + \mathcal{F}$$

Discretised in 3TL:

$$\frac{\Psi(\vec{x}, t + \Delta t) - \Psi(\vec{x} - 2\vec{\alpha}, t - \Delta t)}{2\Delta t} =$$

$$\frac{1}{2}\left[\mathcal{R}(\vec{x} - 2\vec{\alpha}, t) + \mathcal{R}(\vec{x}, t)\right] + \mathcal{F}(\vec{x} - 2\vec{\alpha}, t - \Delta t)$$

$$\Psi(\vec{x}, t + \Delta t) = \Delta t \mathcal{R}(\vec{x}, t) +$$

$$\left[\underbrace{\Psi(\vec{x} - 2\vec{\alpha}, t - \Delta t) + 2\Delta t \mathcal{F}(\vec{x} - 2\vec{\alpha}, t - \Delta t) + \Delta t \mathcal{R}(\vec{x} - 2\vec{\alpha}, t)}_{I}\right]$$

SLHD scheme definition II.

$$I = (1-\kappa)I_A + \kappa I_D = I_A + \kappa(I_D - I_A)$$

$$\kappa \propto d = \sqrt{\left(\underbrace{\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}}_{d_T}\right)^2 + \left(\underbrace{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}_{d_S}\right)^2}$$

$$\kappa = \frac{F(d)\Delta t}{1 + F(d)\Delta t}$$

$$F(d) = \left(\frac{\Delta h_{ref}}{\Delta h}\right)^P \cdot a \left[\mathrm{max}(1, \frac{d}{d_0})\right]^b d$$

SLHD properties

Comparison of the SLHD scheme characteristics with the ALADIN spectral linear diffusion parameters $\mathcal{H}$ and r:

General form of linear diffusion:

$$\left.\frac{\partial \Psi}{\partial t}\right|_{\text{diff}} = (-1)^{(1+r/2)} K \nabla^r \Psi$$

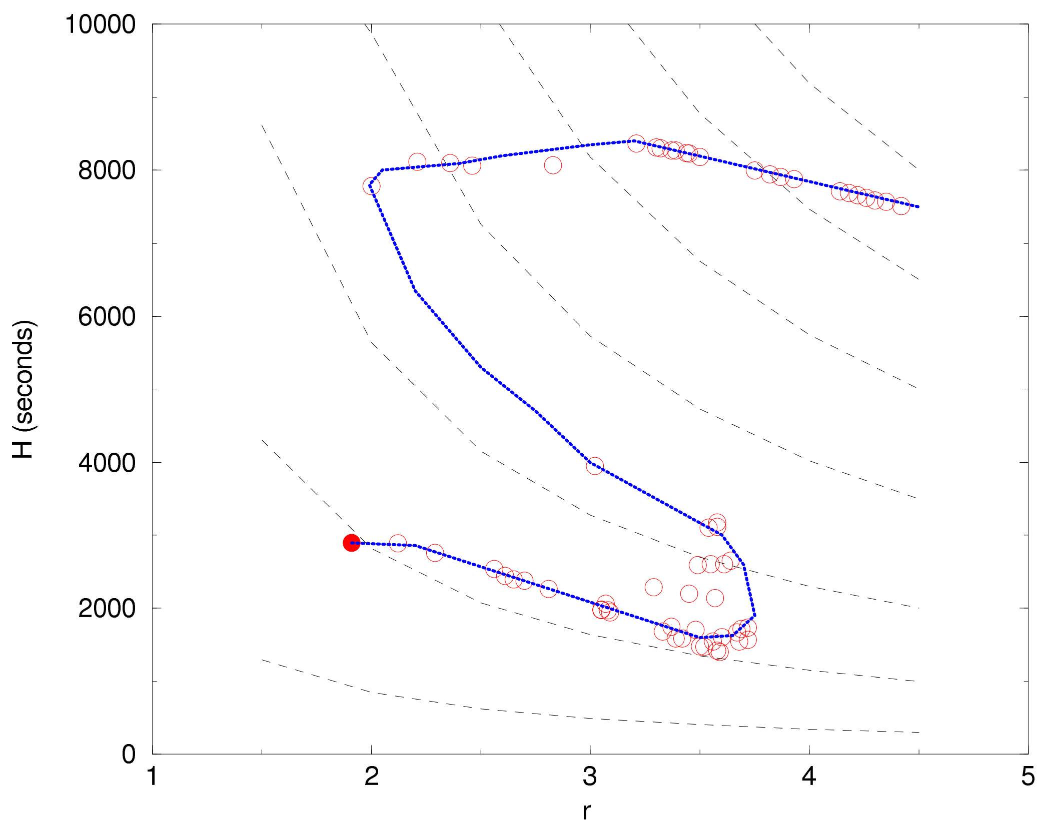
For the model ALADIN it stands:

$$\left.\frac{\partial \Psi}{\partial t}\right|_{\text{diff}} = -(2\pi)^{-r/2} \left[\frac{L_x^2}{\mathcal{M}} + \frac{L_y^2}{\mathcal{N}}\right]^{r/4} \exp\left(-\frac{i\pi r}{2}\right) \Omega h_\Psi G(l) \frac{1}{\mu} \nabla^r \Psi$$

$$\mathcal{H}_\Psi = \frac{1}{\Omega h_\Psi}$$

SLHD properties II.

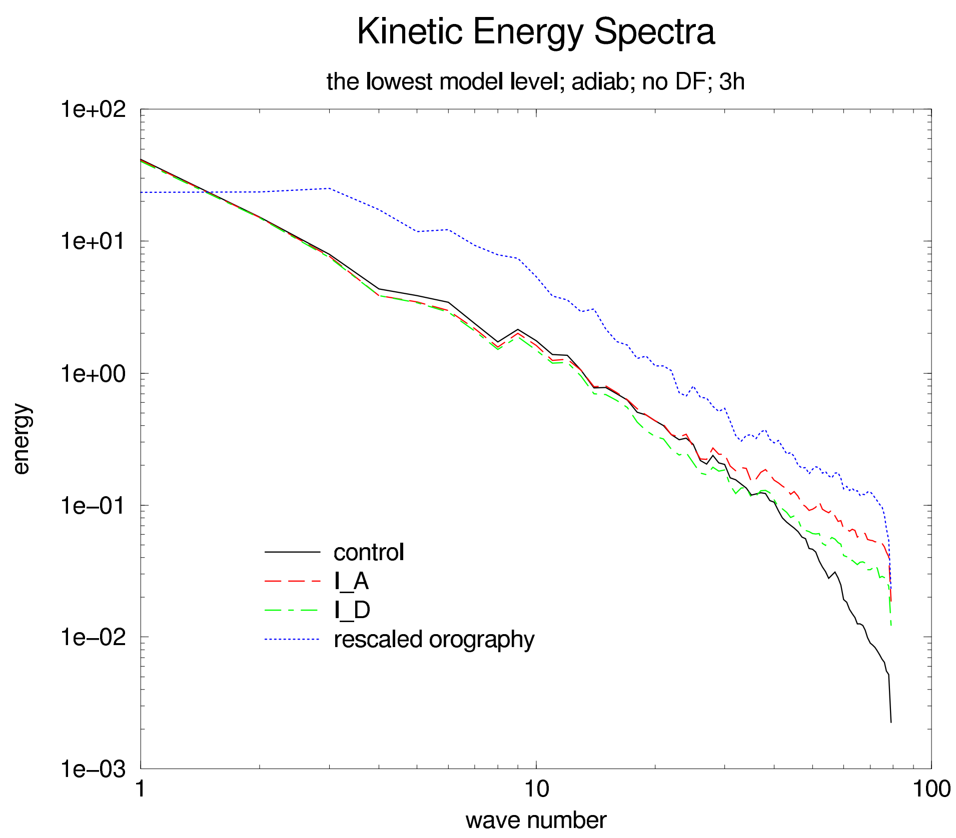
Characteristic curve of the SLHD expressed in equivalent $\mathcal{H}$ and r parameters (obtained from the idealised experiment).



SLHD properties III.

Real case

$$\left.\frac{\partial \Psi}{\partial t}\right|_{diff} = \mathcal{D}_{SLHD}\left[\Psi(\vec{x}, t + \Delta t) - \Delta t \mathcal{R}(\vec{x}, t)\right]$$



SLHD potential problems

- Can the SLHD scheme influence the distribution of the origin points of S-L trajectories? Is the scheme performance dependent to this distribution?

- Can the non-uniform smoothing of the SLHD diffusive interpolator depending to the position of the S-L origin point be responsible for malfunctioning or small scale noise generation?

Distribution of the S-L origin points distances from the closest grid point (2D)

With SLHD

Number of diagnosed items : 14 315 239

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
6.077e-05	2.040e-01	3.331e-01	3.289e-01	4.519e-01	7.071e-01

Quantile of 1% cases: 0.03697184

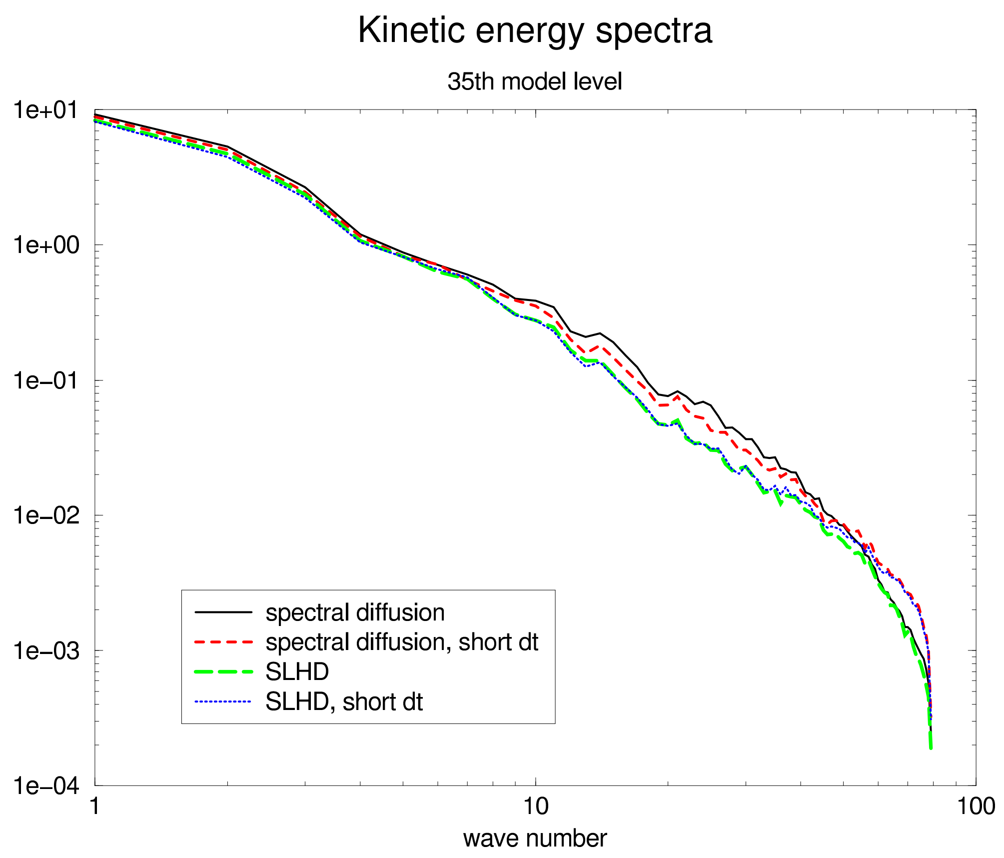
Reference

Number of diagnosed items: 14 315 288

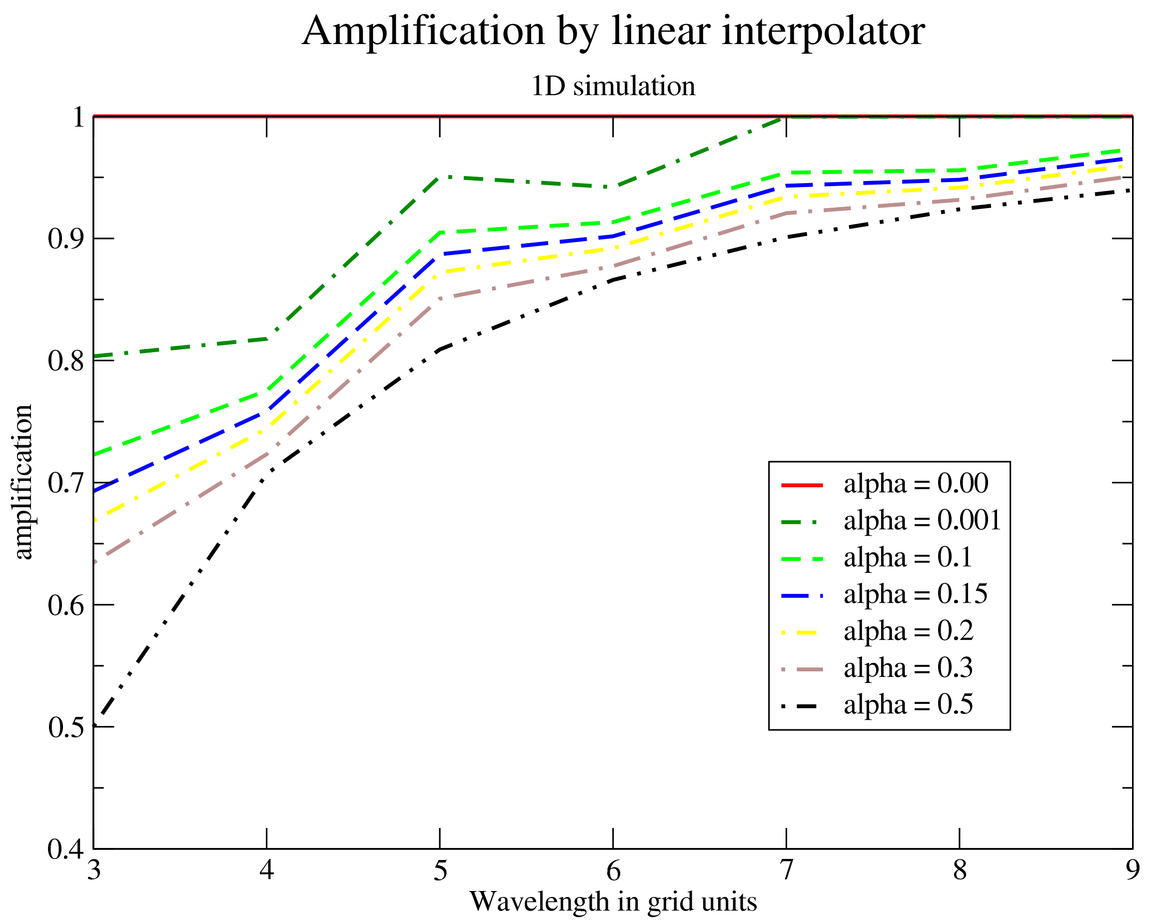
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
9.021e-05	2.041e-01	3.332e-01	3.290e-01	4.520e-01	7.069e-01

Quantile of 1% cases: 0.03700731

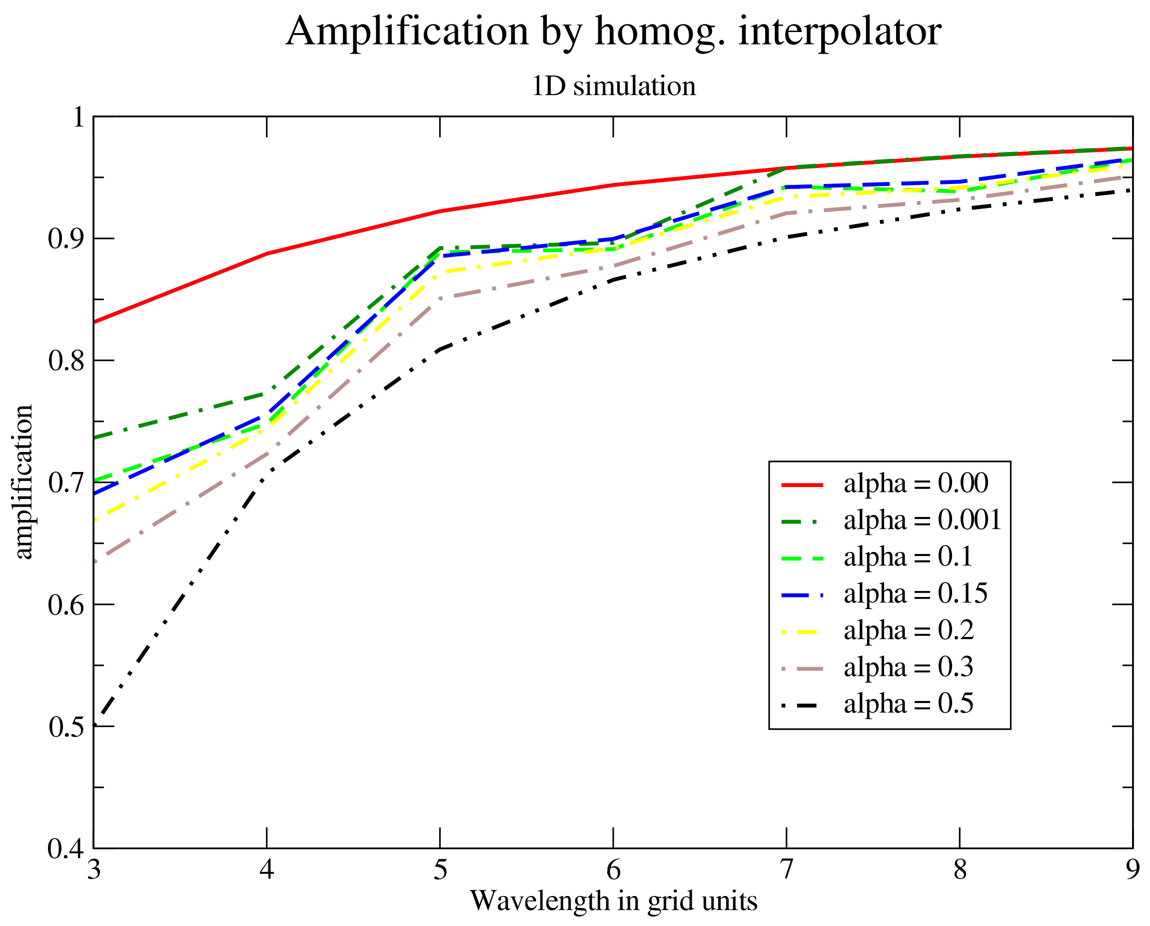
Influence of the S-L origin points distribution to the SLHD performance



Amplification factor of the diffusive interpolator with respect to the S-L origin point distance from the grid point

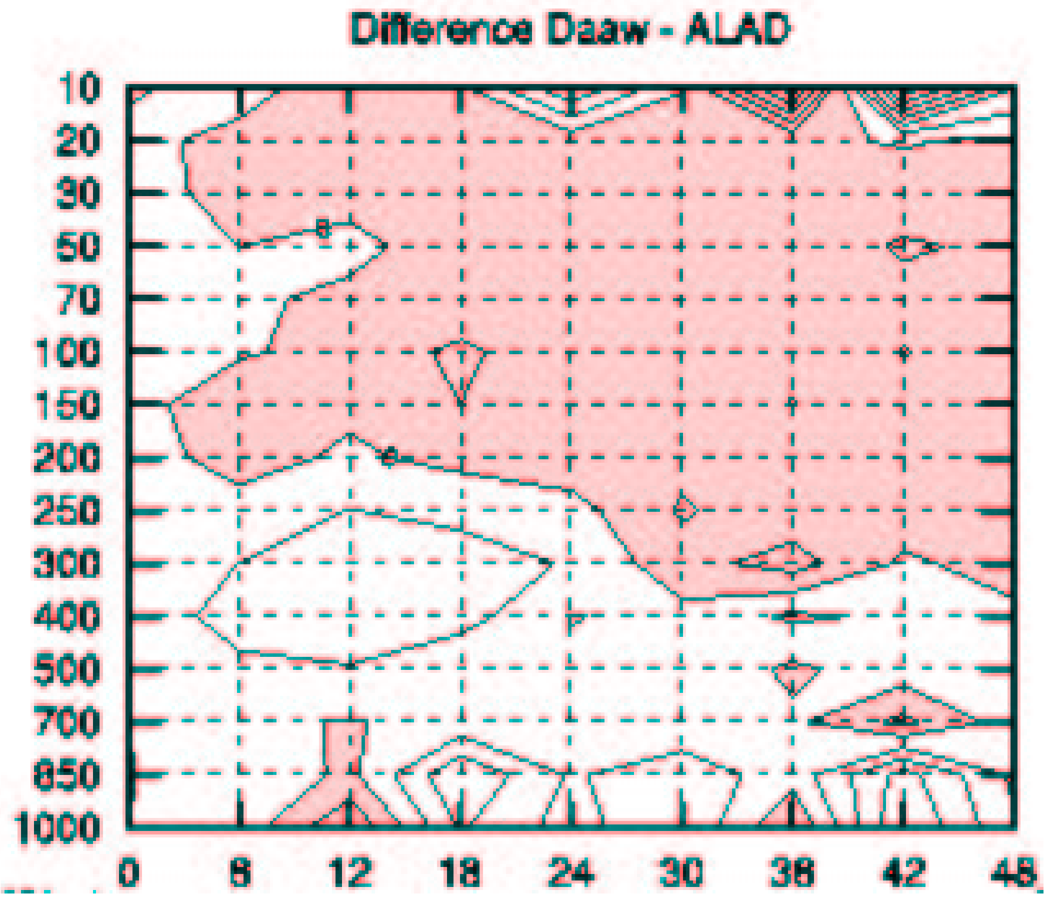


Amplification factor of the diffusive interpolator with respect to the S-L origin point distance from the grid point

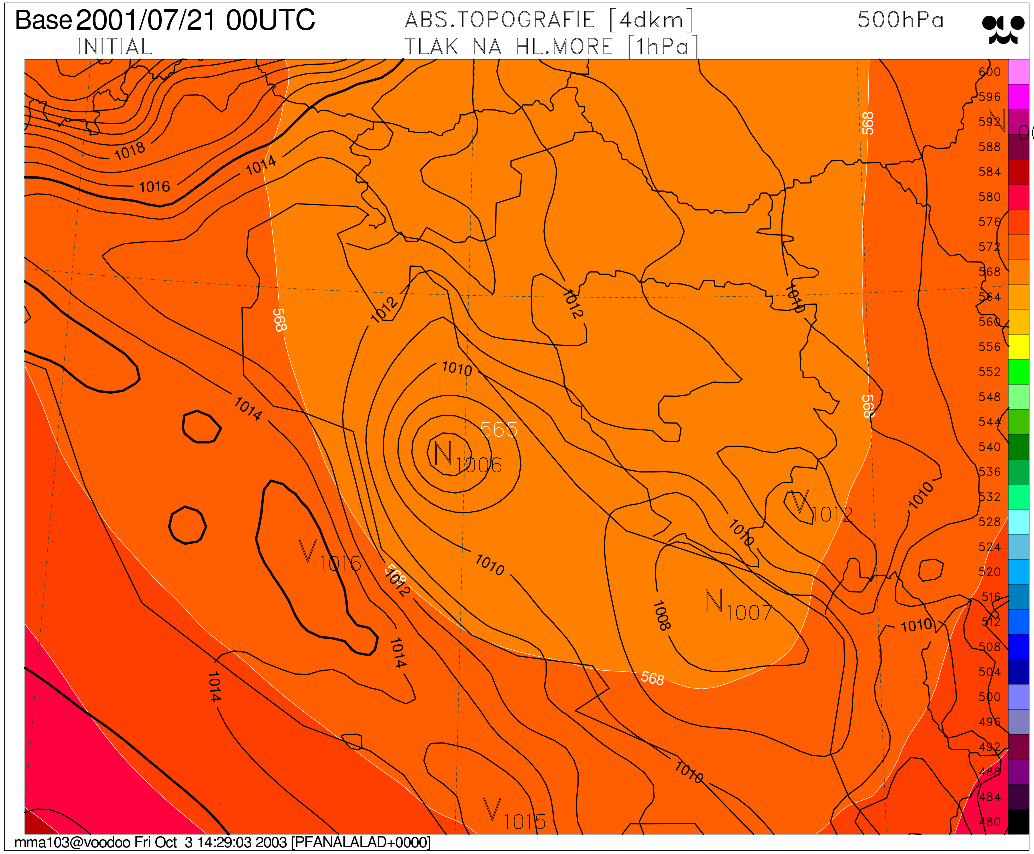


Parallel test, 19 days period

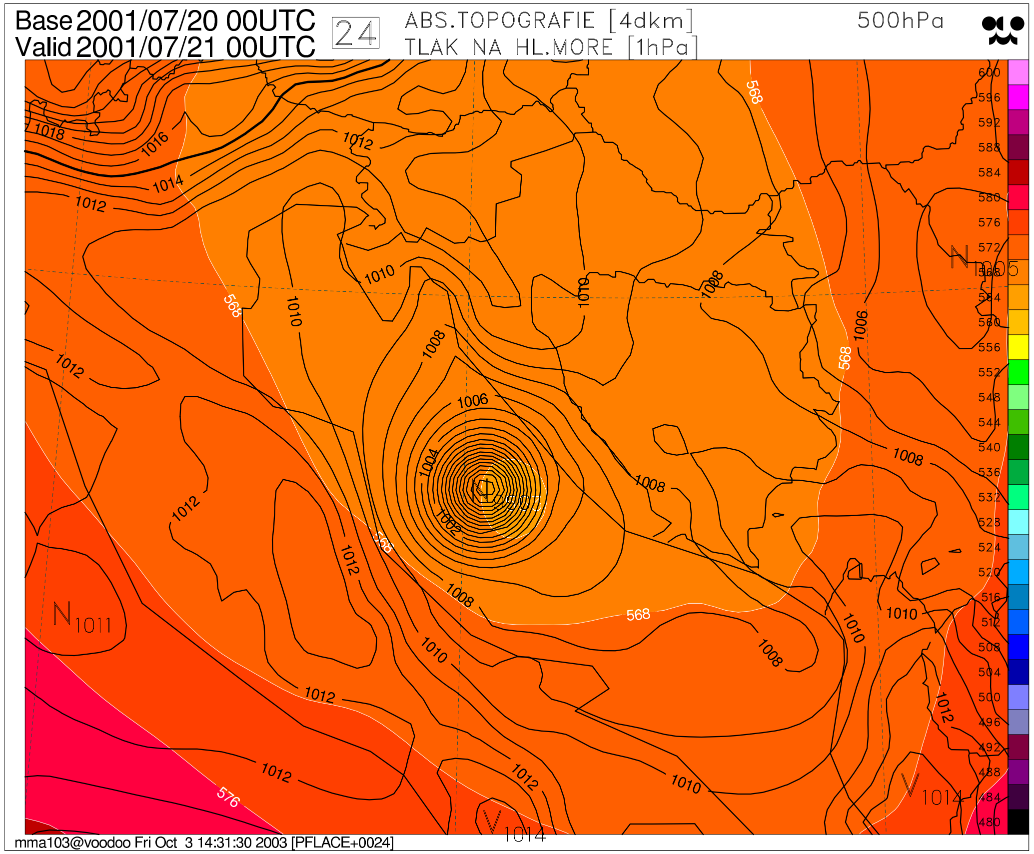
Evolution of geopotential RMSE scores with forecast range



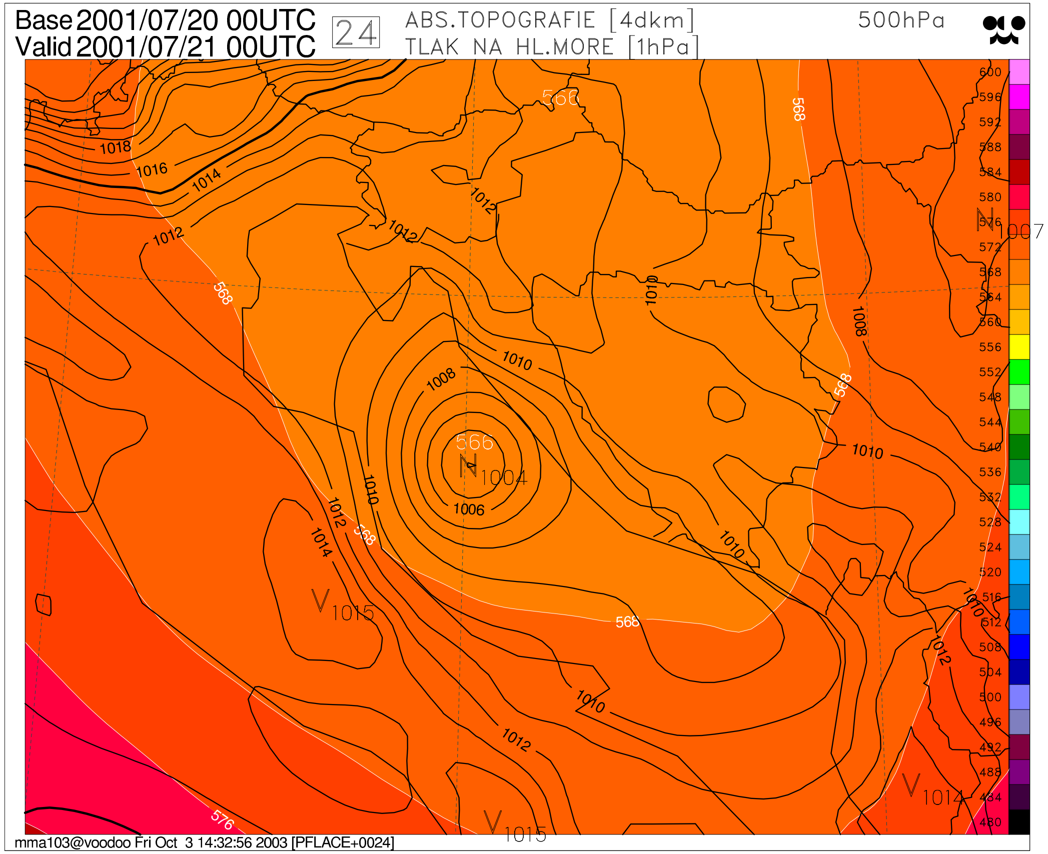
Adriatic storm, ALADIN/LACE analysis



Adriatic storm, operational forecast



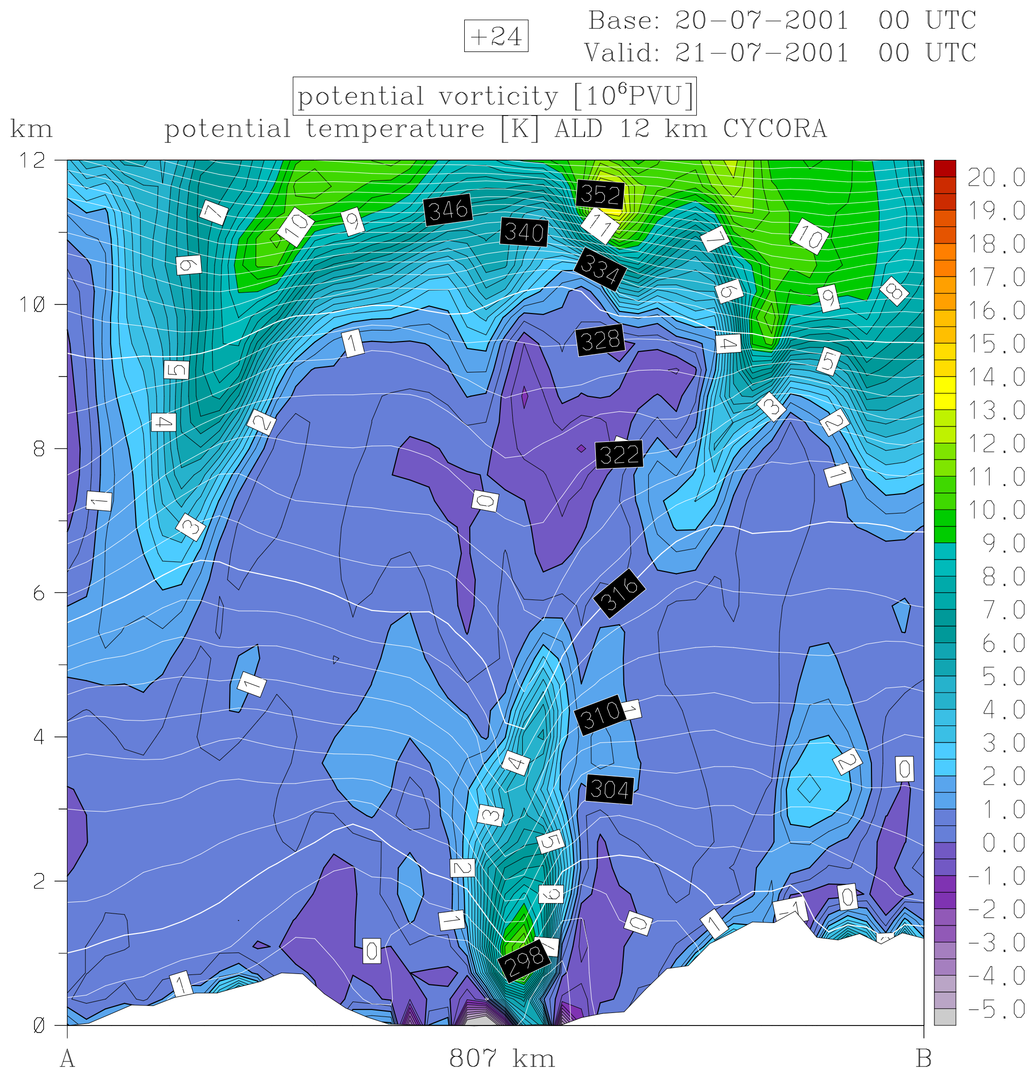
Adriatic storm, SLHD



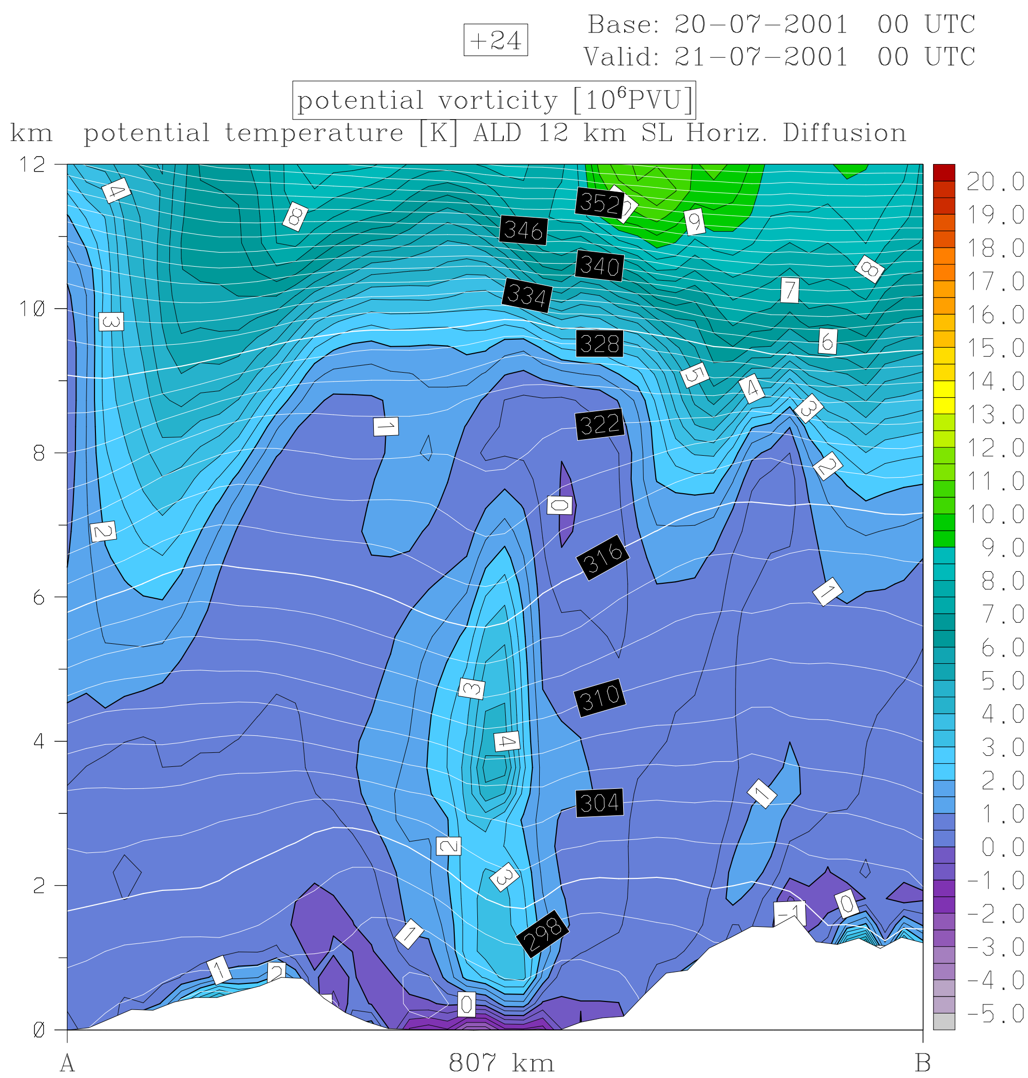
Adriatic storm, vertical cross-section line

ASCS – Aladin Space Cross Section

Adriatic storm, operational forecast



Adriatic storm, SLHD



Conclusion

- General advantages of the SLHD scheme
  - Trivial implementation
  - Cheap non-linear diffusion
  - Stable
- Advantages of the SLHD scheme for a spectral model
  - Physically realistic
  - Real **horizontal** diffusion
  - Scope to affect any prognostic field
  - Simple treatment in a case of non uniform model mesh

- Weak points
  - Unable to control the noise generated by model orography
  - Only one degree of freedom for tuning