Nonhydrostatic modeling on the sphere

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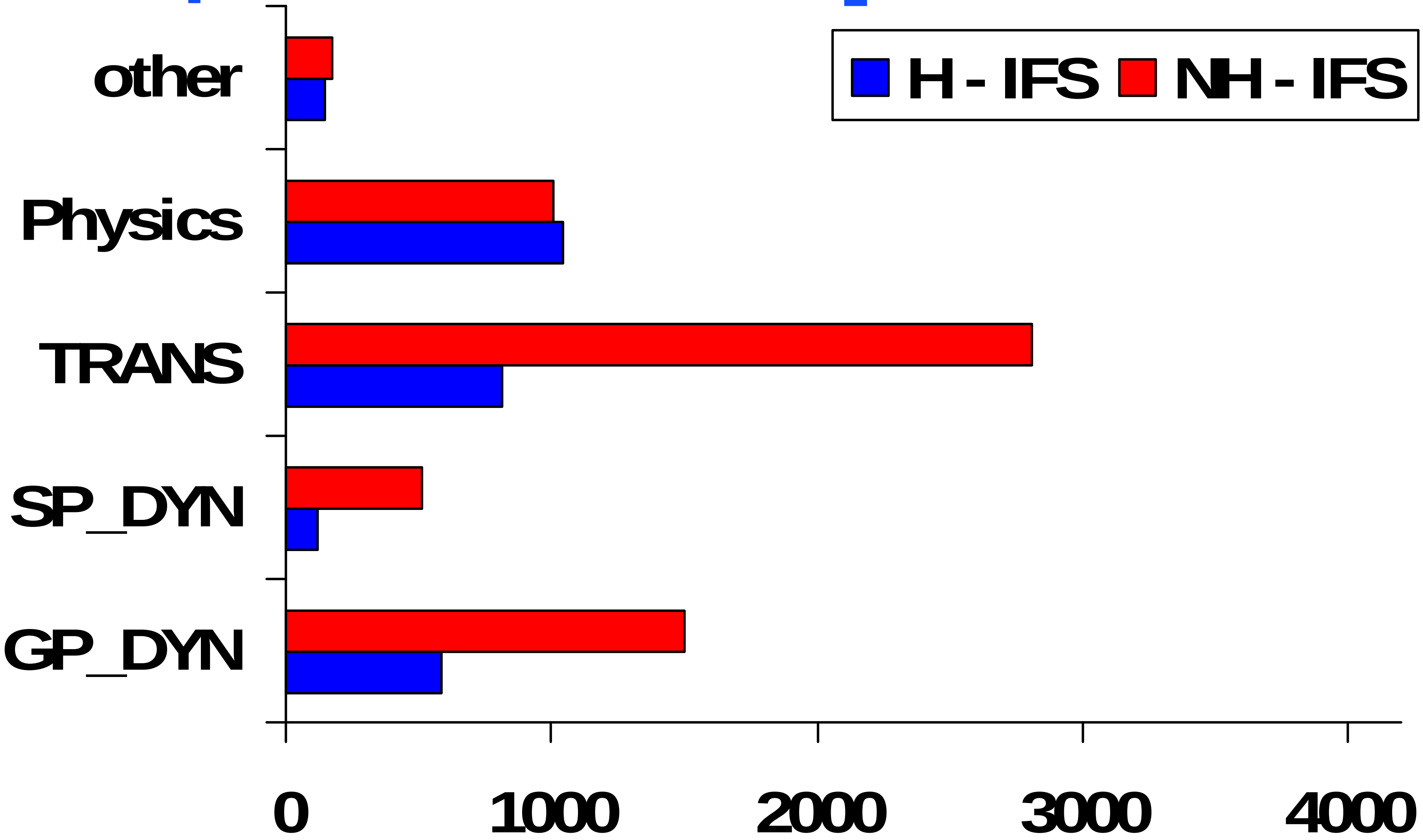
Outline

- (Ultra-)high resolution T3999 simulations highlight some areas of concern:
  - The spectral transform method
  - Cost of the NH dynamics
  - NH dynamics coupling to the physics

Computational Cost at T_L2047



Total cost increase NH – H 106 %

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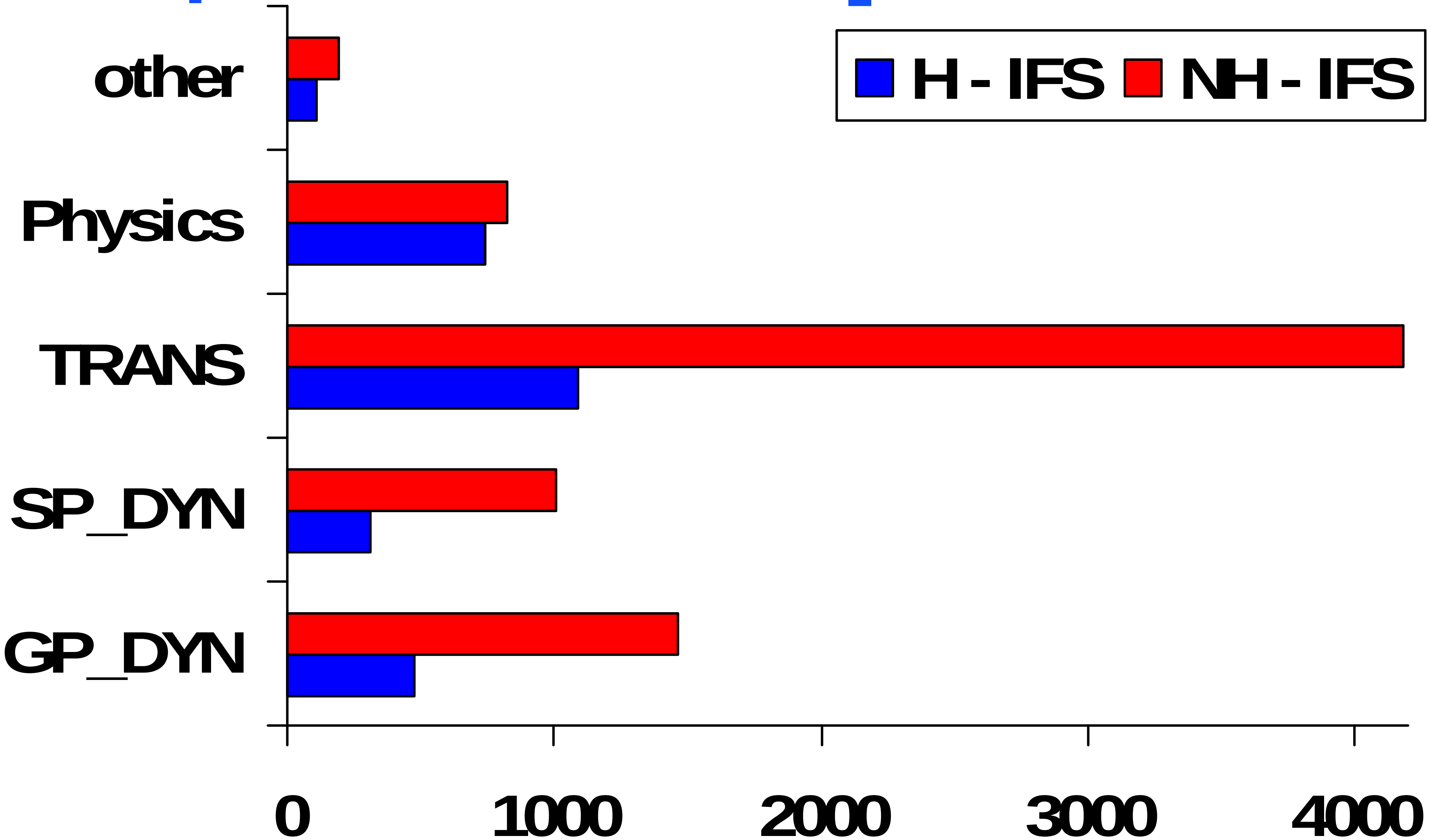
Computational Cost at T_L3999



Total cost increase for 24h forecast: H 50min vs. NH 150min

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Cost of spectral transform method

- The Fourier transform can be computed at a cost of **C*N*log(N)** where C is a small positive number and N is the cut-off wave number in the triangular truncation with the Fast Fourier Transform (FFT).
- Ordinary Legendre transform is **$O(N^2)$** but can be combined with the fields/levels such that the arising matrix-matrix multiplies make use of the highly optimized BLAS routine DGEMM.
- But overall cost of transforms is **$O(N^3)$** for both memory and CPU time requirements.
- On top of the computational cost there is also the cost of message passing associated with the "transpositions" but likely **$O(N^2)$**

⇨ **Desire** for **a fast Legendre transform where the cost is proportional to C*N*log(N)**
and thus overall cost proportional to N^2*log(N)

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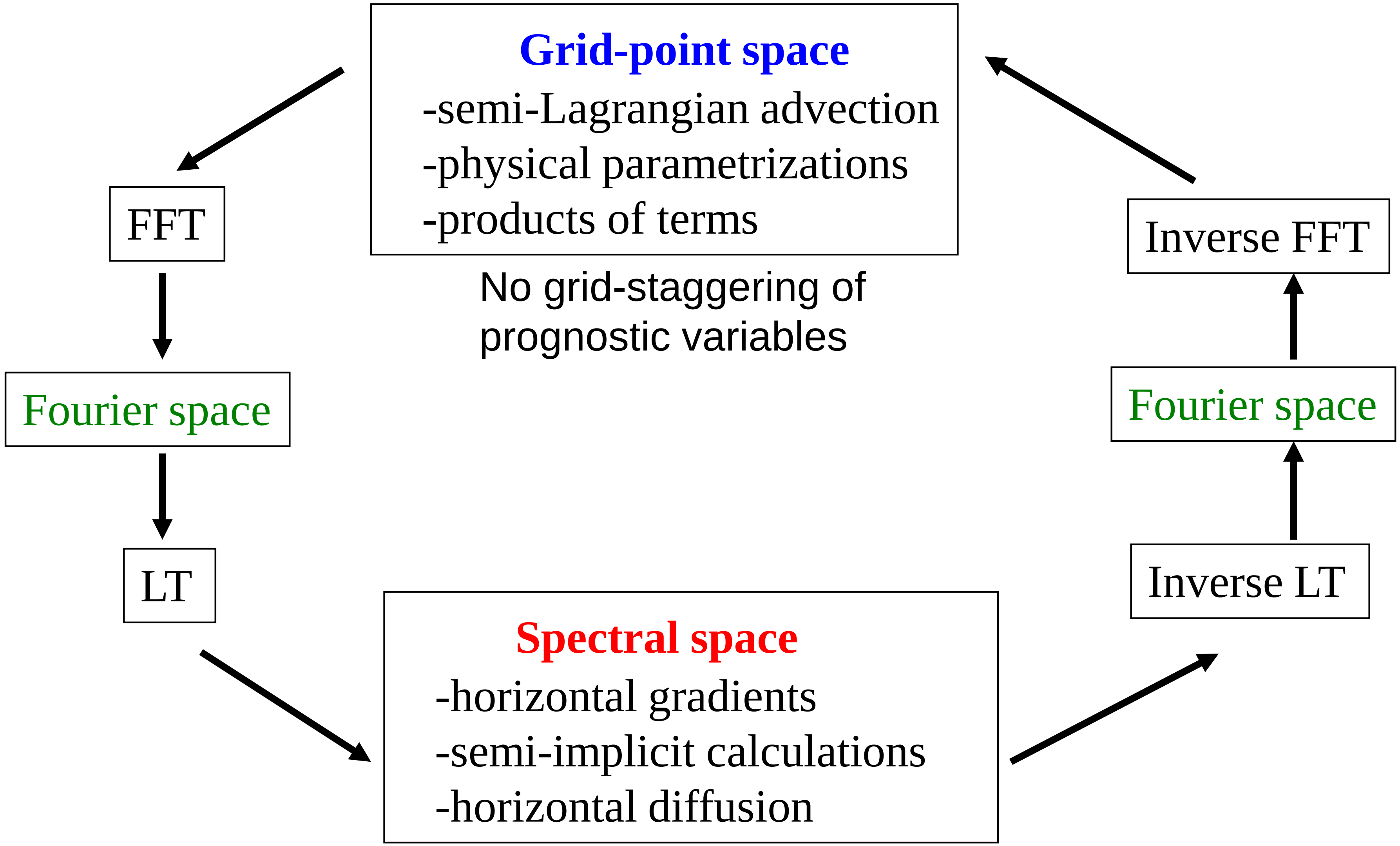
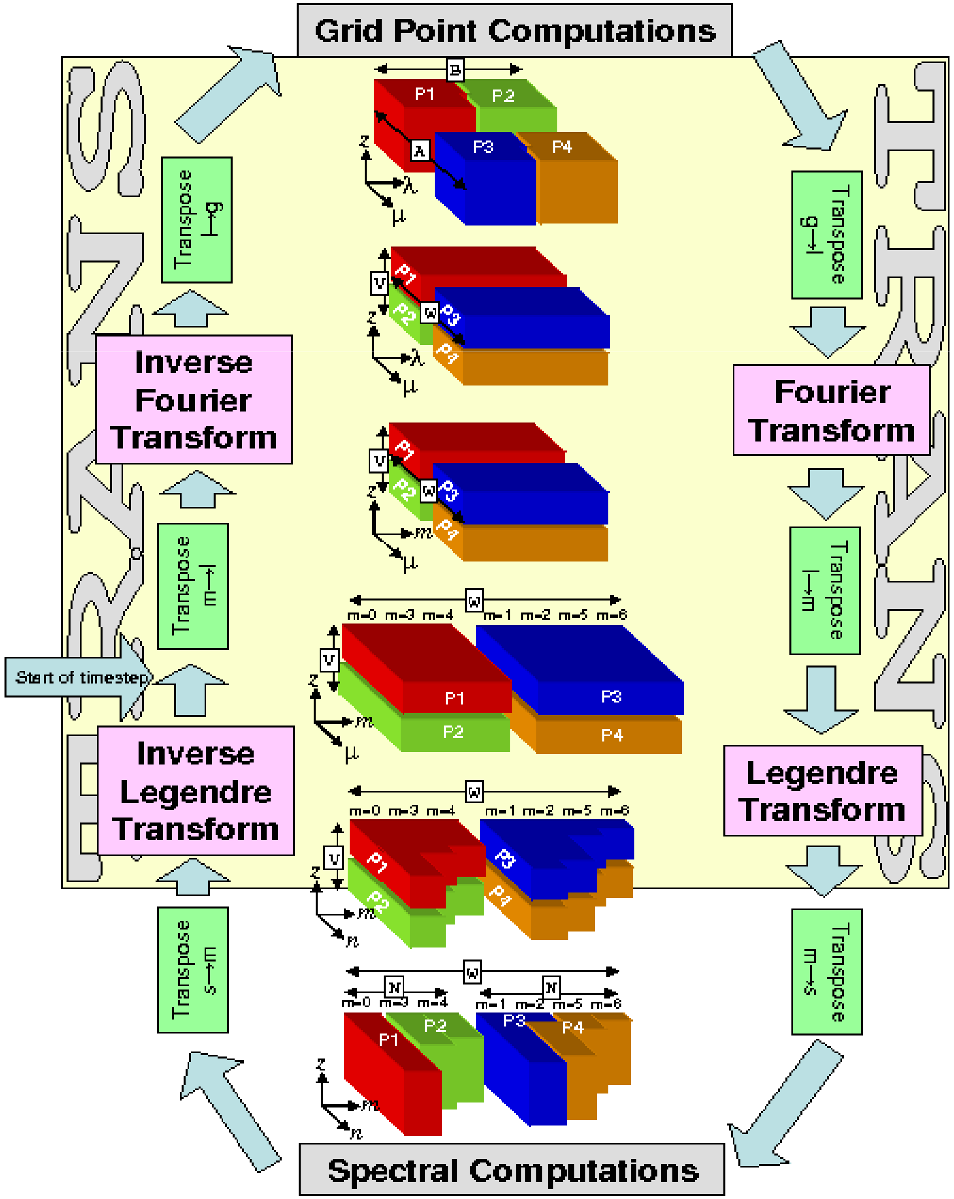
Schematic description of the spectral transform method in the ECMWF IFS model



FFT: Fast Fourier Transform, LT: Legendre Transform

Transpositions within the spectral transforms



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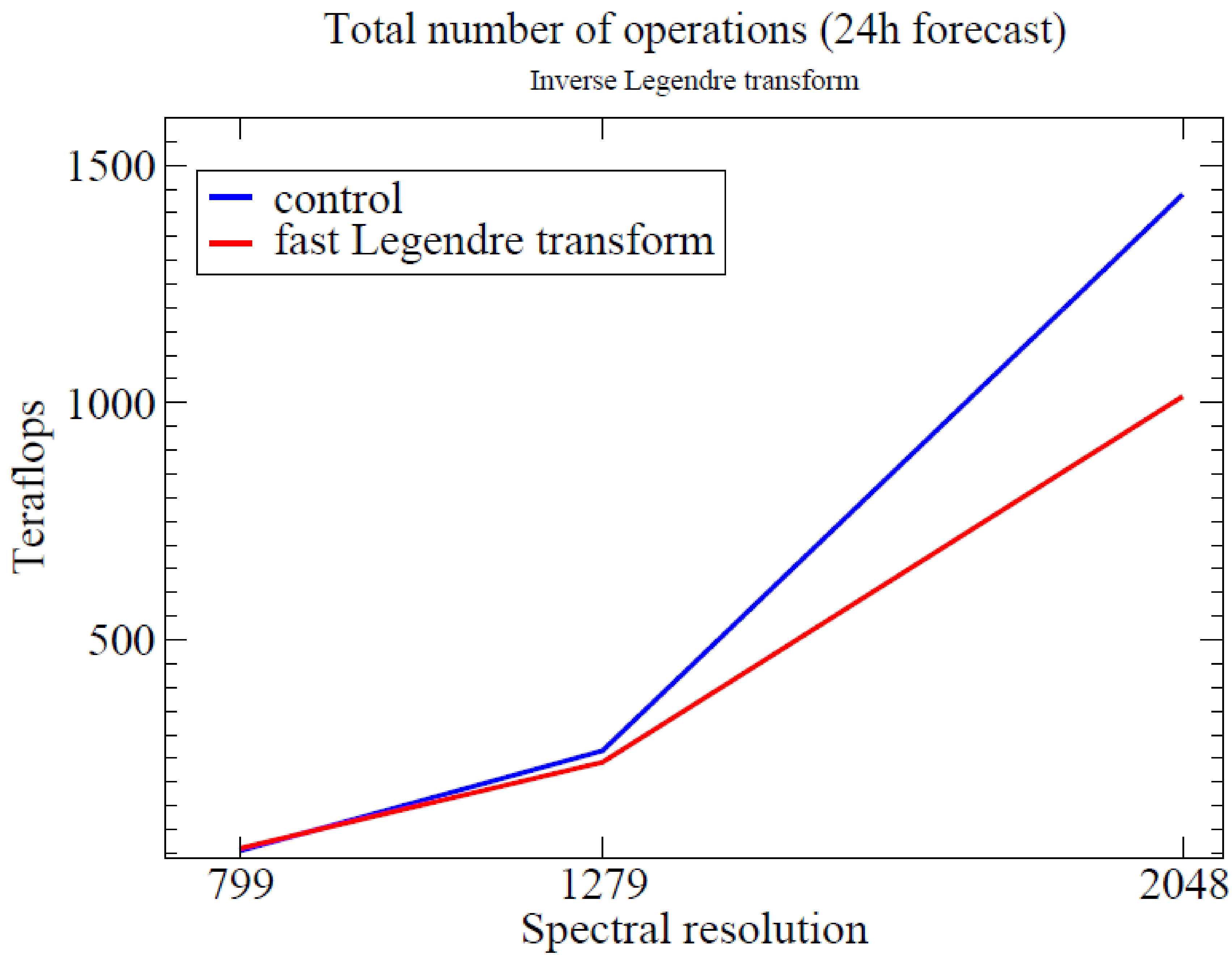
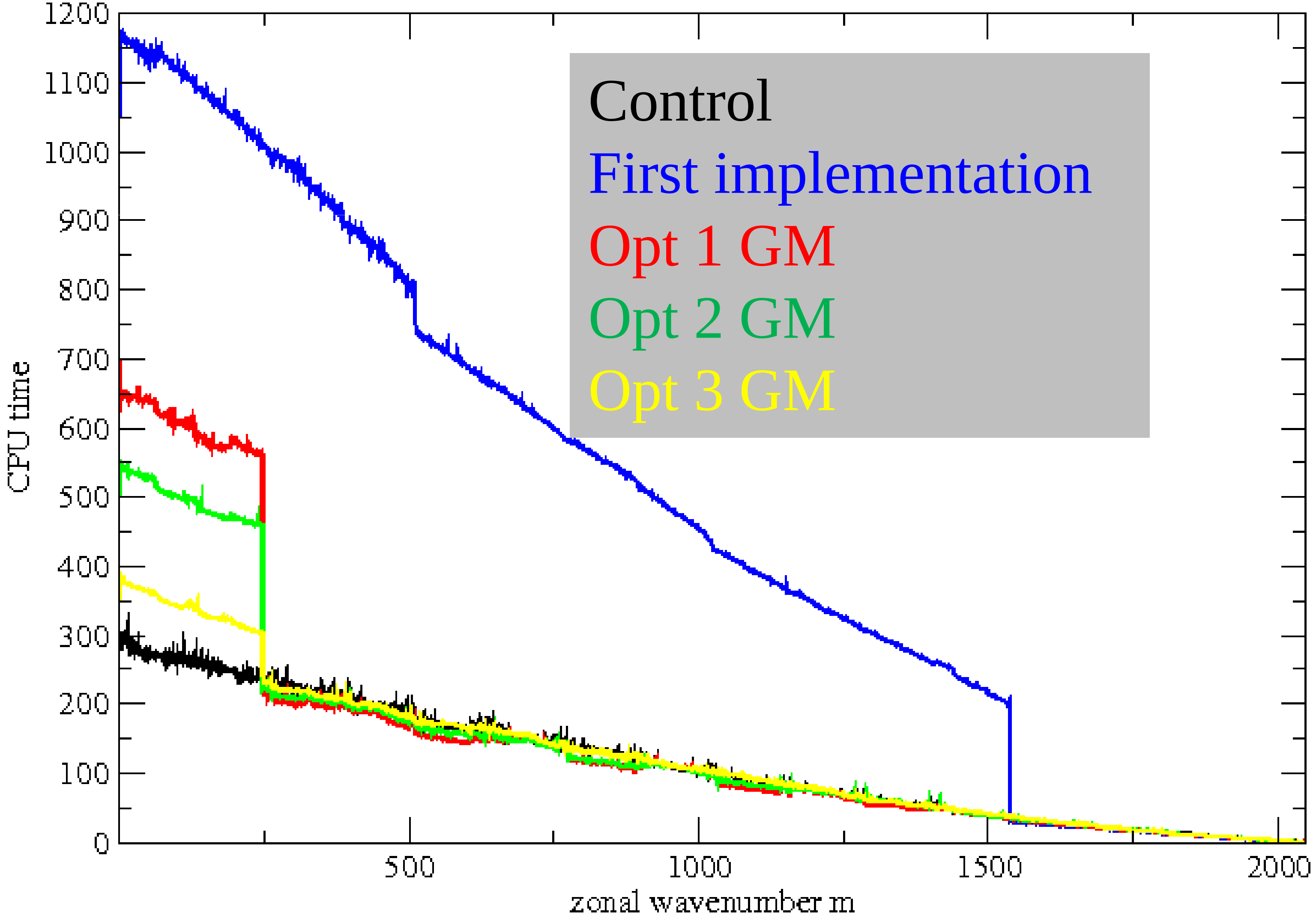

Total number of operations (24h forecast)
Inverse Legendre transform
control
fast Legendre transform
Teraflops
1500
1000
500
799
1279
2048
Spectral resolution

Timings for new Legendre transform



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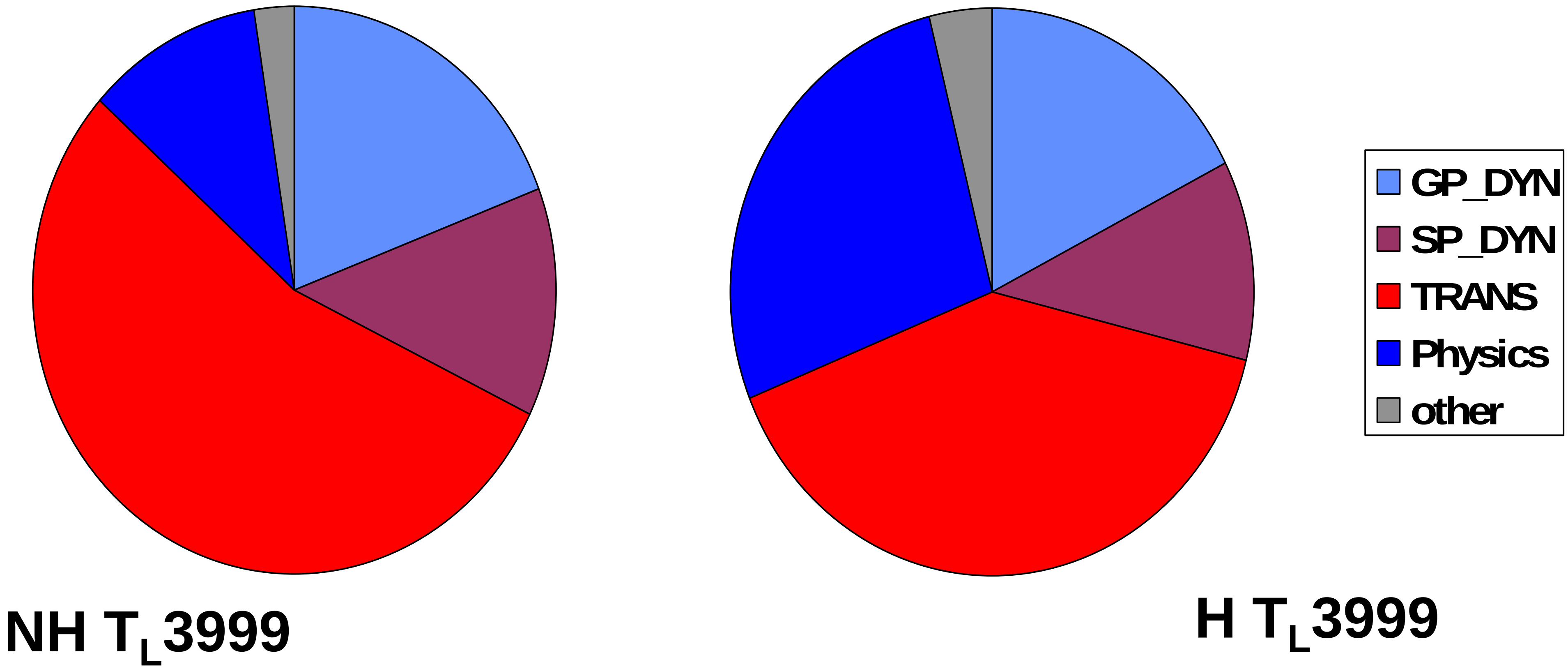
Accuracy of computation of the associated Legendre polynomials

- **Increase of error due to recurrence formulae (Belousov, 1962)**
- **Recent changes to transform package went into cycle 35r3 that allow the computation of Legendre functions and Gaussian latitudes in double precision following *(Schwarztrauber, 2002)* and increased accuracy 10^{-13} instead of 10^{-12}.**
- **Note: the increased accuracy leads in the "*Courtier and Naughton* (1994) procedure for the reduced Gaussian grid" to slightly more points near the poles for all resolutions.**
- **Note: At resolutions > T3999 above procedure needs review!**

Computational Cost at T_L3999 hydrostatic vs. non-hydrostatic IFS



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The IFS NH equations

Two new prognostic variables in the nonhydrostatic formulation

$$\mathcal{Q} \equiv \log(p/\pi)$$

'Nonhydrostatic pressure departure'

$$d \equiv -g(p/mRT)\partial w/\partial \eta$$

'vertical divergence'

Define also: $\mathcal{D} \equiv d + \mathcal{X}$

With residual residual

$$\mathcal{X} \equiv (p/RTm)\nabla_\eta \Phi \cdot \partial \mathbf{v}_h/\partial \eta$$

Three-dimensional divergence writes

$$D_3 = \nabla_\eta \cdot \mathbf{v}_h + \mathcal{X} + d.$$

NH-IFS prognostic equations

$$\frac{d\mathbf{v}_h}{dt} = -\frac{RT}{p}\nabla_\eta p - \frac{1}{m}\frac{\partial p}{\partial \eta}\nabla_\eta \Phi - 2\Omega \times \mathbf{v}_h + P_{\mathbf{v}},$$

$$\frac{d\mathcal{D}}{dt} = \frac{dd}{dt} + \frac{d\mathcal{X}}{dt} = -\frac{gp}{mRT}\frac{\partial P_w}{\partial \eta},$$

$$\frac{dT}{dt} = -\frac{RT}{c_v}D_3 + \cancel{\frac{c_p}{c_v}}P_T,$$

$$\frac{d\mathcal{Q}}{dt} = -\frac{c_p}{c_v}D_3 - \frac{1}{\pi}\frac{d\pi}{dt} + \cancel{\frac{c_p}{c_v T}P_T}.$$

$$\frac{\partial \pi_s}{\partial t} = -\int_0^1 \nabla_\eta \cdot (m\mathbf{v}_h)\,d\eta,$$

'Physics'

"anelastic physics coupling"

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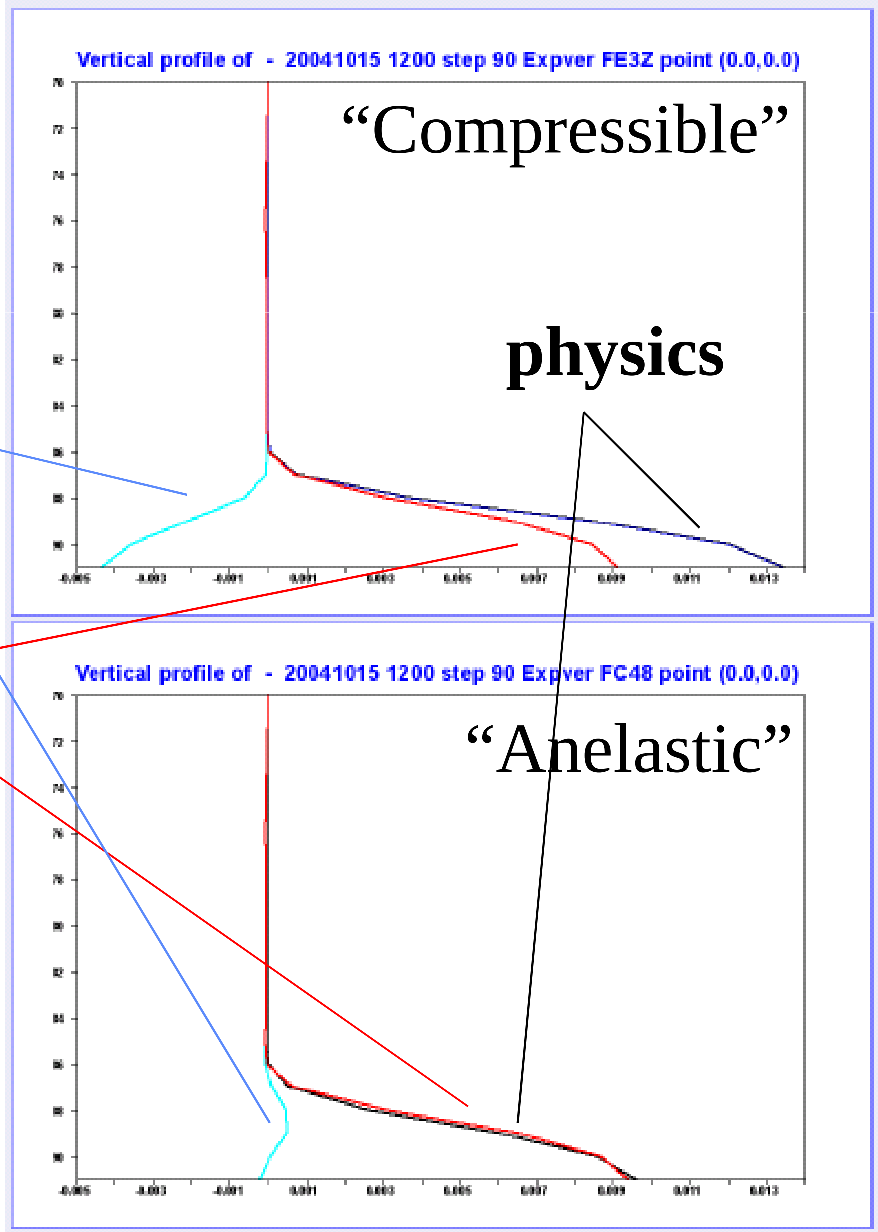
Dynamics – Physics coupling

Sylvie Malardel

The case of constant heating near the surface with dt=10s



Total temperature tendency after 15min is nearly identical !

Dynamics – Physics coupling

- (A) In the “anelastic coupling case” the nonhydrostatic pressure departure is instantaneously converted into volume change (D3) without the ("resolved") computations of the dynamics.
- (B) In the “compressible coupling case” the physics is allowed to change the non-hydrostatic pressure departure, but cannot change the mass distribution. The mass rearrangement can be done only in the dynamics (via advection).

→ Preliminary results suggest that (A) is more stable with large time-steps and the need to retain fully compressible dynamics for cloud-resolving simulations is questionable !

Towards a unified hydrostatic-anelastic system

- Scientifically, the benefit of having a prognostic equation for non-hydrostatic pressure departure is unclear.
- The existence of two reference states with different requirements undesirable.
- The coupling to the physics is ambiguous.
- For stability reasons, the NH system requires at least one iteration, which essentially doubles the number of spectral transforms.
- Given the cost of the spectral transforms, any reduction in the number of prognostic variables will save costs.
- Split-explicit (vertically implicit) schemes essentially damp the pressure perturbation towards the anelastic solution.

Towards a unified hydrostatic-anelastic system

- **There is a recent enhancement of the validity of anelastic models (based on scale analysis) to temperature perturbations of 30-50K which substantially extends the original work by Ogura and Philipps *(Klein et al, 2010)***

Unified system

(Arakawa and Konor, 2009)

here in the context of IFS

$$\rho_{qs} \equiv \frac{\pi}{R\tilde{T}}$$

$$\frac{\partial \pi}{\partial z} \equiv -\rho_{qs} g$$

$$\frac{1}{\rho_{qs}} \frac{d\rho_{qs}}{dt} = -\nabla \cdot \mathbf{v} + \epsilon,$$

$$\epsilon = \frac{(\kappa - 1)}{1 + q^x} \frac{dq^x}{dt},$$

$$\tilde{T} \equiv T(1 + q^x)^{-R/c_p},$$

$$q^x = (p-\pi)/\pi$$

The linear system

$$\frac{\partial D'}{\partial t} = -\Delta\left[\gamma\tilde{T}' + RT^* q' + \frac{RT^*}{\pi_S^*}\pi'_S\right],$$

$$\frac{\partial w'}{\partial t} = g(\kappa + \partial^*)q',$$

$$\frac{\partial \tilde{T}'}{\partial t} = -\tau D',$$

$$\frac{\partial \phi'}{\partial t} = gw' - c_p\tau D' + RT^* B(\eta)\left(\frac{\pi_S^*}{\pi^*}\right)\nu D',$$

$$\frac{\partial \pi'_S}{\partial t} = -\pi_S^*\nu D',$$

Describes the small deviation from a hydrostatically balanced, isothermal, and resting reference state.

Unified system – the linear system

$$\frac{1}{H_*}\partial^* \left\{ \left[\Delta + \frac{1}{H_*^2}\partial^*(\partial^* + 1) \right] \frac{\partial^2}{\partial t^2} + N_*^2 \Delta \right\} w' = 0,$$

The structure equation is identical to the Lipps and Hemler system, or in other words, small perturbations from a hydrostatically balanced reference state show the *same behaviour in EULAG* and *the unified system* ! (and with Coriolis only small difference for baroclinic modes.)

At large scales the unified system collapses to the existing hydrostatic system.

There may be a remaining concern for high altitude wave-breaking in the stratosphere ***(U. Achatz + Klein et al, 2010)***

Executive Summary

- **For the hydrostatic model not too many worries until 2015 !**
- **Nonhydrostatic IFS: Computational cost (almost 3 x at T_L3999) is a serious issue !**
- **Progress in the application of Fast Legendre Transforms.**
- **“anelastic” Dynamics-Physics coupling more stable with large time-steps and leading to the same result.**
- **Exploring possibilities towards a unified IFS hydrostatic-anelastic system (*Arakawa and Konor, 2009*).**

Additional slides

Horizontal discretisation of variable X (e.g. temperature)

$$X(\lambda,\mu,\eta,t)=\sum_{m=-N}^{N}\sum_{n=|m|}^{N}X_n^m(\eta,t)Y_n^m(\lambda,\mu)$$

Triangular truncation (isotropic)

Spherical harmonics

$$X_n^m(\eta,t)=\frac{1}{4\pi}\int_{-1}^{1}\int_{0}^{2\pi}X(\lambda,\mu,\eta,t)P_n^m(\mu)e^{-im\lambda}\,d\lambda\, d\mu$$

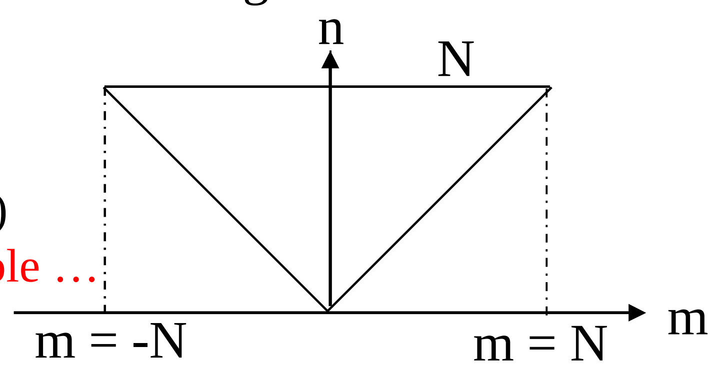
Fourier functions

associated Legendre polynomials

Legendre transform
by Gaussian quadrature
using $N_L \geq (2N+1)/2$
"Gaussian" latitudes (linear grid)
((3N+1)/2 if quadratic grid)
"fast" algorithm desirable …

FFT (fast Fourier transform)
using
$N_F \geq 2N+1$
points (linear grid)
(3N+1 if quadratic grid)
"fast" algorithm available …

Triangular truncation:



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