Grid-scale forcing and its effect on larger scales in environmental flows

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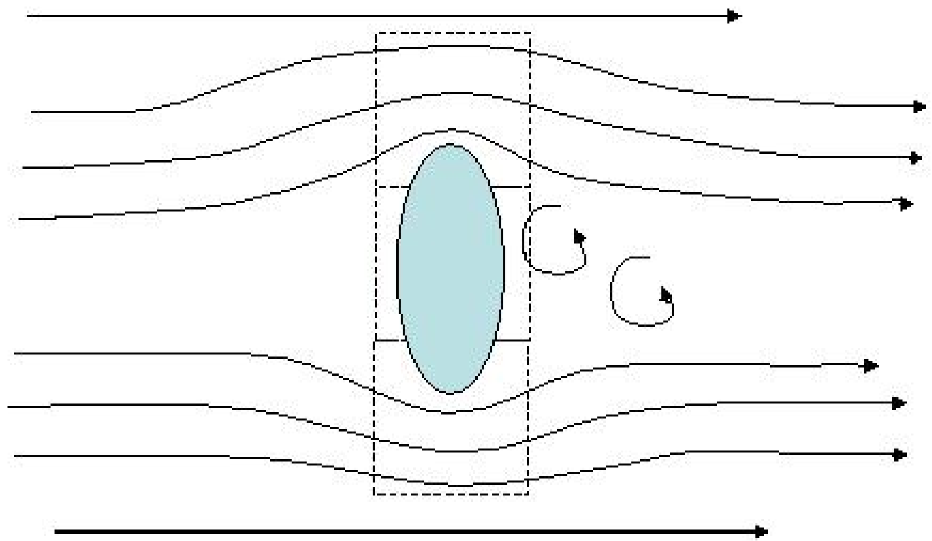
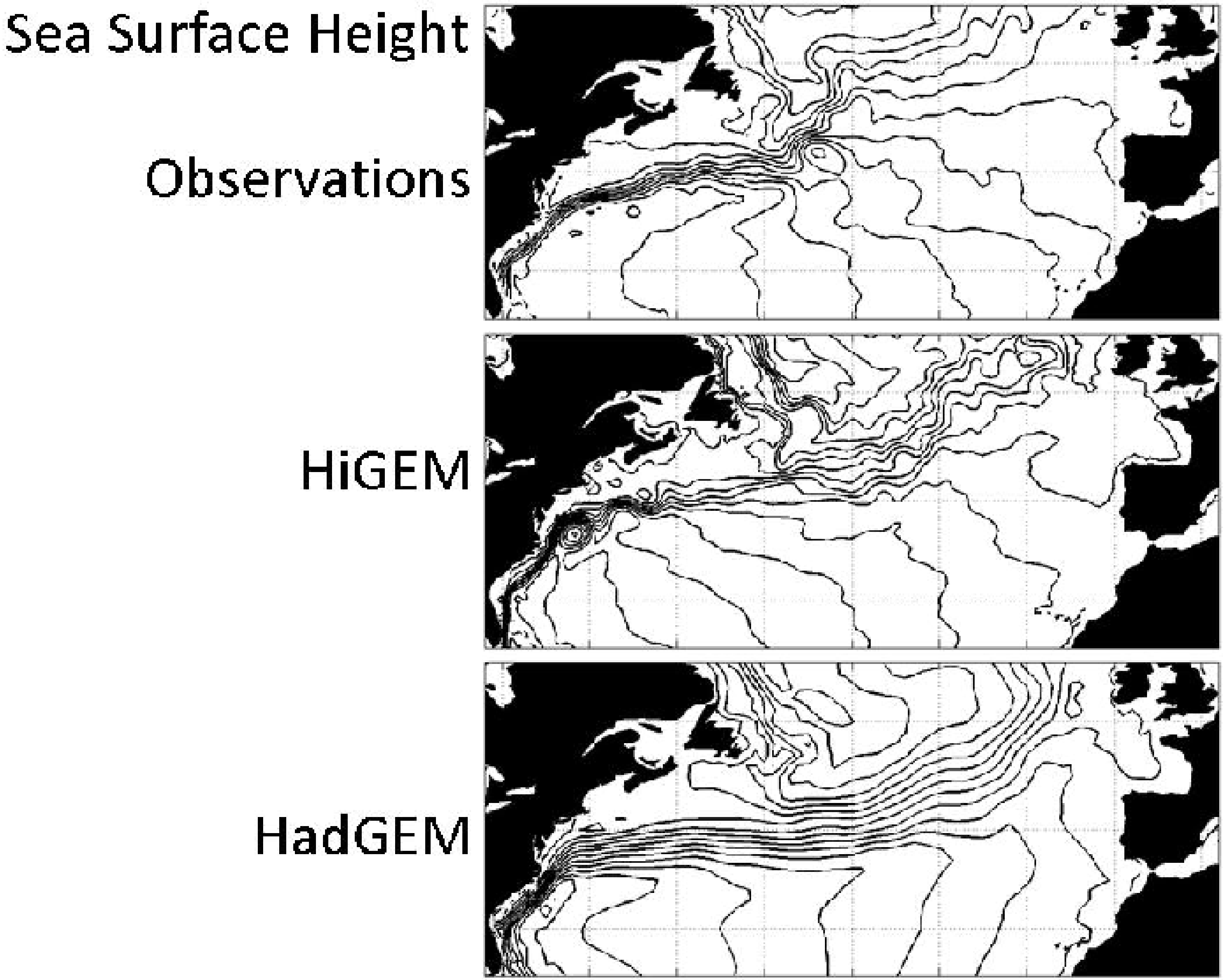
Under-resolved flows

- Relevant processes at and below the grid-scale are typically parametrized. i.e. generally a column calculation using the large scale variables, model parameters and empiricism for a fixed time step.The model dynamics cannot properly represent processes at or below the grid-scale (in fact below scales of several grid lengths) and yet the coupling between dynamics and physics is essentially at the grid-scale.
- Models of real flows over obstacles are always under resolved unless the obstacle has a smooth shape i.e. the minimum variation on the surface of the obstacle is several grid-lengths.
- Terrain and coasts are “fractal-like”. Increasing resolution introduces new features.
- Edges and corners are always under-resolved.

FLOW AROUND SUB-GRID OROGRAPHY

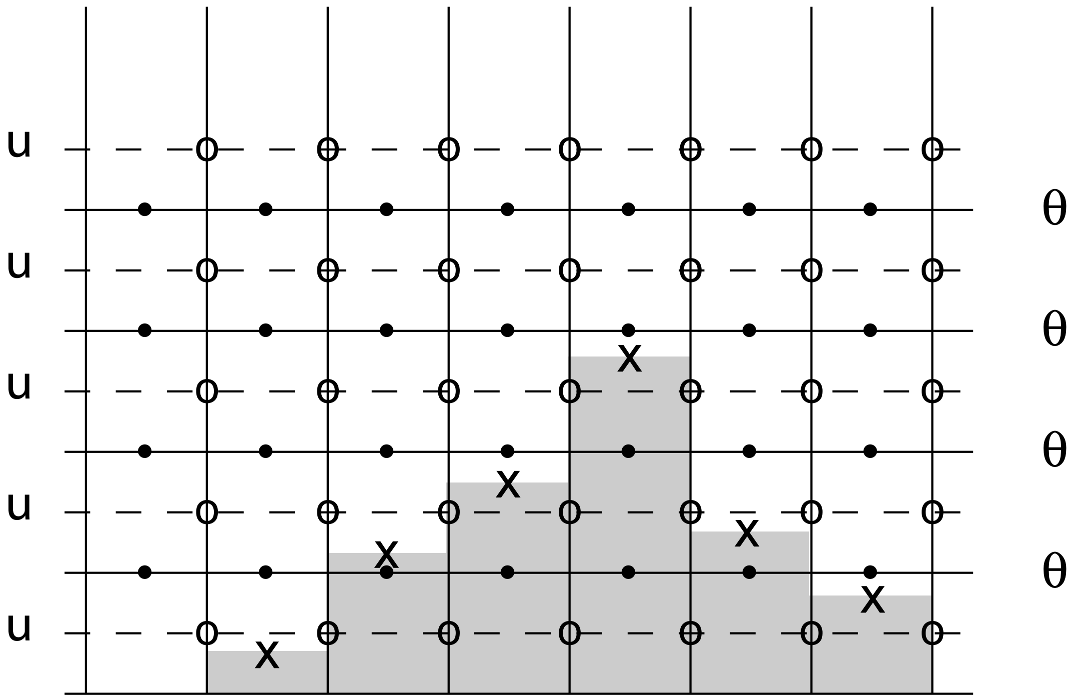



Sea Surface Height
Observations
HiGEM
HadGEM

Terrain-following coordinates

- Most widely used in NWP and climate models.
- Easy to apply lower boundary condition but....
- Horizontal pressure gradient terms consists of 2 parts which for steep slopes are of similar magnitude but opposite signs. i.e. actual horizontal pressure gradient is a small residual between 2 terms. Flattening the coordinates so that they are horizontal above a certain altitude helps (Hybrid coordinates).
- For C-grid staggering, the horizontal winds in the vertical Coriolis terms need to be averaged and near steep slopes the horizontal wind components on the same model (η) levels will be at different heights.
- Terrain-following coordinates not suitable for large aspect ratios hence interest in shaved/cut cells etc. But these have yet to make it into operations.

VERTICAL GRID



How to block flow (consistently)

Consider

$$\frac{\partial u}{\partial t} = -K\left(\mathbf{x}, \mathbf{u}\right) u, \qquad (1)$$

where $K = K\left(\mathbf{x}, \mathbf{u}\right)$, i.e K depends on position and flow (and resolution!).

$K = 0$ represents the undisturbed state.

The solution is $u = u_0 \exp\left(-K\left(\mathbf{x}, \mathbf{u}\right) t\right)$.

For very large K , $u \to 0$ and equation (1) can be recast as

$$u = -\frac{\partial u}{\partial t} / K$$

which has the appropriate asymptotic behaviour.

Thus, adding a term as in equation (1) , provides a mechanism for blocking flow.

Can this be done for the momentum equations and can it be justified?

Time-averaged Navier-Stokes equations

$u = \overline{u} + u'$ where $u' \ll \overline{u}$ and $\overline{u} = \int^T u dt$ and $\int^T u' dt = 0$,

$$\frac{d\overline{u}}{dt} = \frac{1}{\rho}\frac{\partial p}{\partial x} + f_r \overline{v} + \frac{1}{\rho}\frac{\partial}{\partial x}\left(-\rho\overline{u'u'}\right) + \frac{1}{\rho}\frac{\partial}{\partial y}\left(-\rho\overline{v'u'}\right) + \frac{1}{\rho}\frac{\partial}{\partial z}\left(-\rho\overline{w'u'}\right)$$

where, $f_r = 2\Omega sin\phi$ and Ω is the Earth's angular speed of rotation.

Similarly for v, w velocity components.

The over-bar for the time-averaged velocity components will be omitted from subsequent slides.

Additional and/or alternative stress terms

$\frac{1}{\rho}\frac{\partial}{\partial x}\left(-\rho\overline{u'u'}\right) = -K_u\left(\mathbf{x},\mathbf{u},t\right)u$ and $\overline{v'u'} = \overline{w'u'} = 0.$

Similarly for v and w equations.

Then

$$\begin{aligned}
\frac{du}{dt} - f_r v + \frac{1}{\rho}\frac{\partial p}{\partial x} &= -K_u\left(\mathbf{x},\mathbf{u},t\right)u \\
\frac{dv}{dt} + f_r u + \frac{1}{\rho}\frac{\partial p}{\partial y} &= -K_v\left(\mathbf{x},\mathbf{u},t\right)v \\
\frac{dw}{dt} + g + \frac{1}{\rho}\frac{\partial p}{\partial z} &= -K_w\left(\mathbf{x},\mathbf{u},t\right)w
\end{aligned}$$

The interaction coefficients $K = K\left(\mathbf{x},\mathbf{u},t\right)$ depend on time, position and flow so these equations cannot be easily solved in this form.

Met Office

Solving the new equations

Solving over a (short) time interval Δt for which $K = K(\mathbf{x}, \mathbf{u})$, i.e. it depends on position and flow, then

$$\frac{\partial u}{\partial t} = -K(\mathbf{x}, \mathbf{u})\, u$$

has the solution $u = u_0 \exp\left(-K(\mathbf{x}, \mathbf{u})\, \Delta t\right)$.

The time discretized equations can be written as

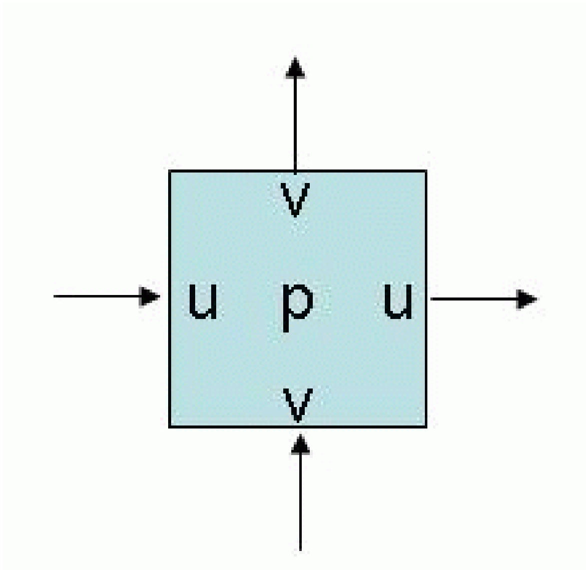
$$(1 + \beta K \Delta t)\ u^{n+1} = (1 - \alpha K \Delta t)\ u^n$$

or

$$u^{n+1} = \frac{(1 - \alpha K \Delta t)}{(1 + \beta K \Delta t)}\, u^n$$

where $\alpha + \beta = 1$. $\alpha = \beta = 1/2$ is $\circ\left(\Delta t^2\right)$, (Crank-Nicholson), whereas $\alpha = 0,\ \beta = 1$ (fully implicit) is $\circ(\Delta t)$. However, for $K \to \infty$, only the implicit discretization behaves as the analytic solution, i.e. $u = 0$.

Blocked flow and staggering



For a blocked flow require that normal velocity at a cell face $= 0$.

C-grid staggering is the natural (only) grid arrangement to do this.

Applying to New dynamics

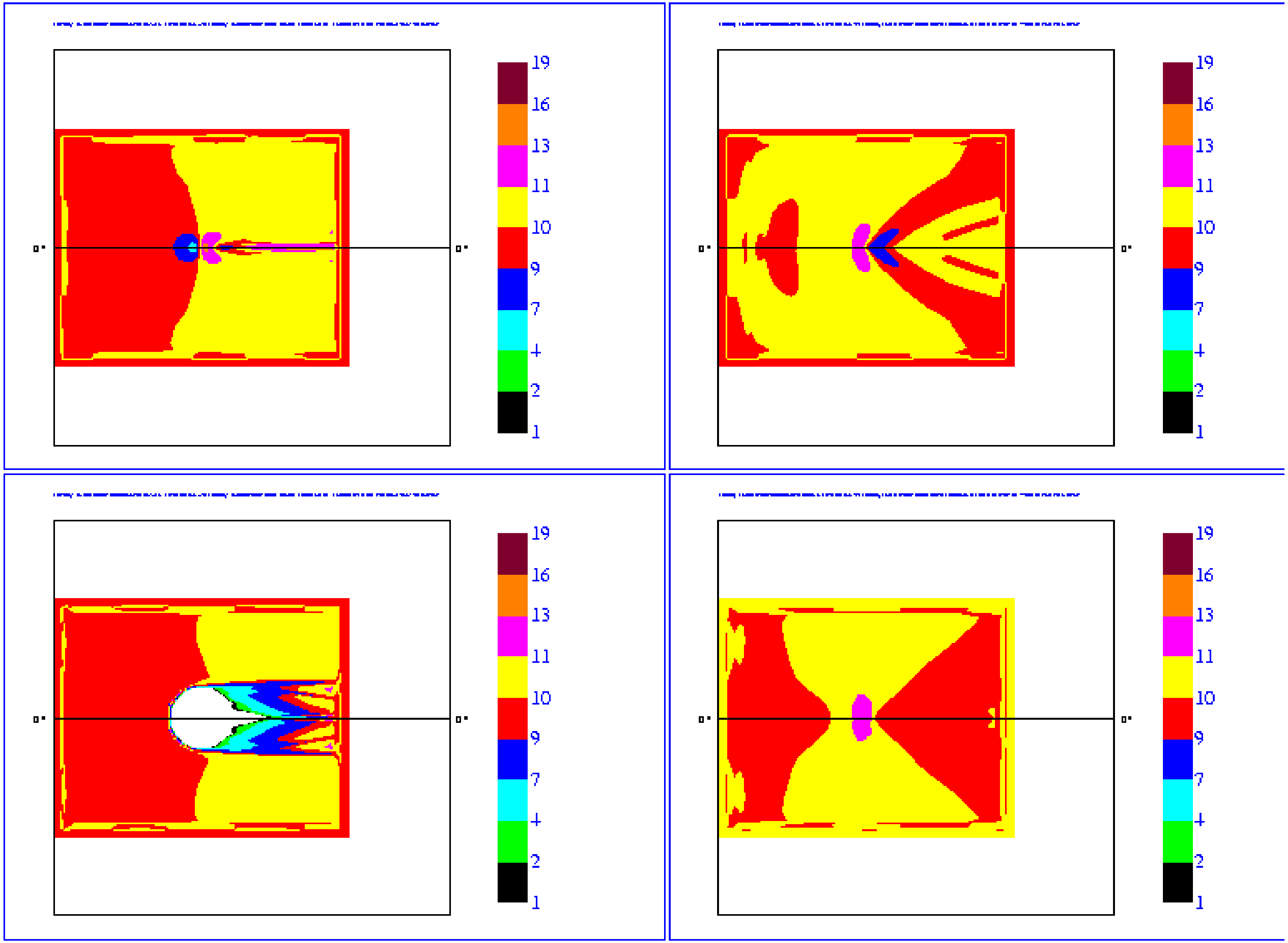
The three stages of the UM predictor-corrector scheme can be written as

$$\begin{aligned}
(1+K\Delta t)\,\mathbf{X}^{(1)} &= \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N})_d^n \\
&\quad +\Delta t\,(\mathbf{S}_1)_d^n + \alpha\Delta t(\mathbf{L}+\mathbf{N})^n \\
\mathbf{X}^{(2)} &= \mathbf{X}^{(1)} + \Delta t\mathbf{S}_2(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}) \\
(1+K\Delta t)\,\mathbf{X}^{(3)} - \alpha\Delta t\mathbf{L}^{(3)} &= \mathbf{X}^{(2)} + \alpha\Delta t(\mathbf{N}^* - \mathbf{N}^n - \mathbf{L}^n) + K\Delta t\mathbf{X}^{(1)}
\end{aligned}$$

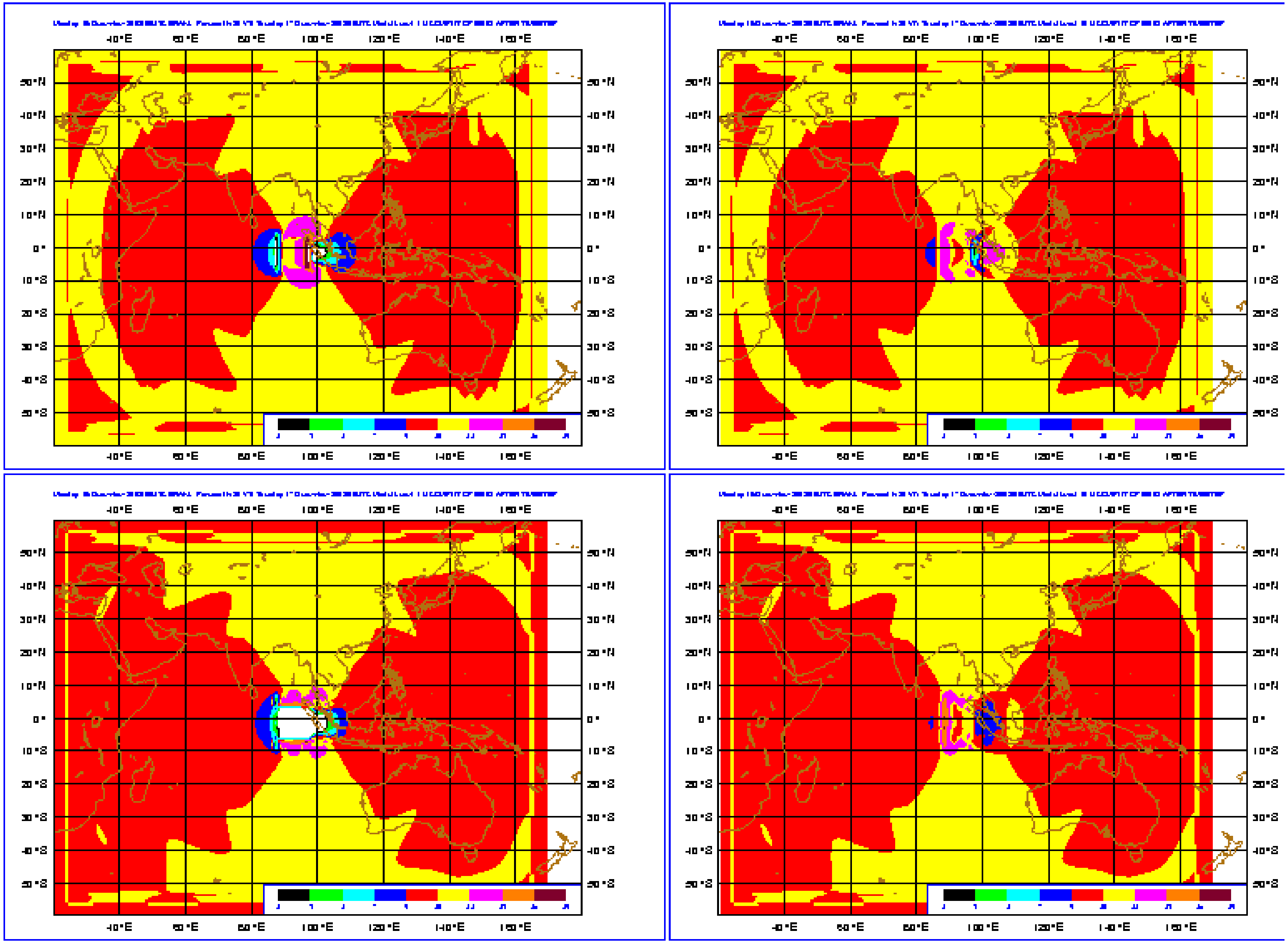
which can be combined to give the target scheme as

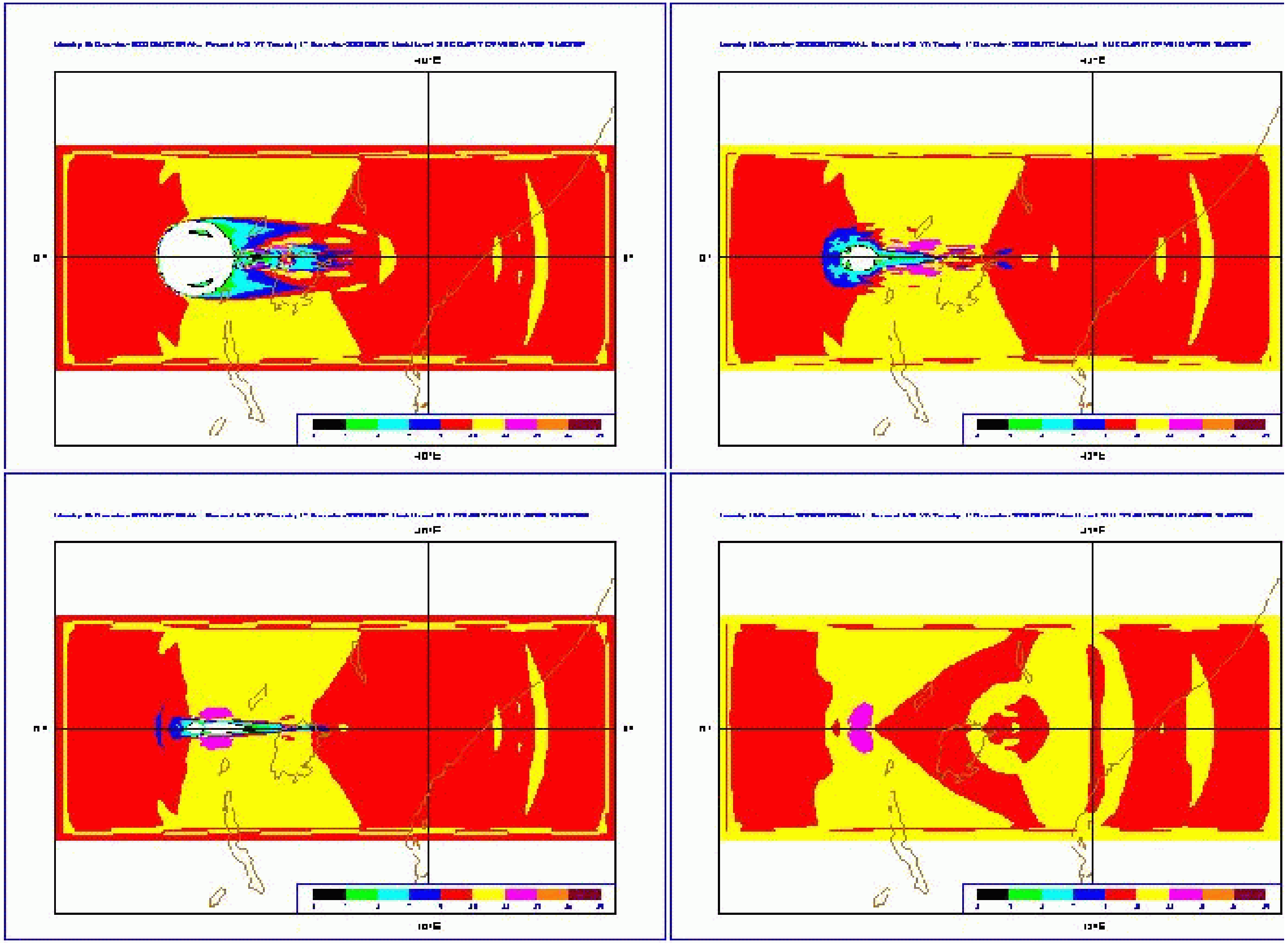
$$\begin{aligned}
\mathbf{X}^{n+1} &= \mathbf{X}_d^n + (1-\alpha)\Delta t(\mathbf{L}+\mathbf{N})_d^n + \Delta t\,(\mathbf{S}_1)_d^n + \alpha\Delta t(\mathbf{L}^{n+1} + \mathbf{N}^*) \\
&\quad +\Delta t\mathbf{S}_2(\mathbf{X}^n, \mathbf{X}^{(1)}, \mathbf{X}^{(2)}) - K\Delta t\mathbf{X}^{n+1}
\end{aligned}$$

500M WITCH OF AGNESI



500M RECTANGULAR CUBE





Closing remarks

1. Couples grid-scale and sub-grid scales to the larger scales by allowing for sub-grid processes without directly modelling them.
2. An alternative treatment of the lower boundary by adding resistance to the flow.
3. Include low-level drag or blocking into the dynamics rather than the parametrizations OR parametrizations can be made 3 dimensional.
4. Handle complex geometries (walls, edges, buildings, trees, etc.) in environmental flows.
5. Easy to build terms into model (UM branch available) and can be used with terrain-following coordinates.

References

- Smolarkiewicz et al (2007) J. Comp. Phys. “Building resolving LES and comparisons with wind tunnel experiments” - the building is the Pentagon!
- Smolarkiewicz and Winter (2010) J. Comp. Phys. “Pores resolving simulation of Darcy flows” - porous media