Deutscher Wetterdienst
Wetter und Klima aus einer Hand

The 'Conservative Dynamical Core' priority project and other current dynamics developments in the COSMO model

33rd EWGLAM and 18th SRNWP meeting
10-13 Oct. 2011, Tallinn, Estonia

M. Baldauf, U. Blahak (DWD, Germany)
B. Rosa, M. Kurowski, M. Ziemianski (IMGW, Poland)

COSMO-Priority Project 'Conservative dynamical core'

Main goals:

- develop a dynamical core with at least conservation of mass, possibly also of energy and momentum
- better performance and stability in steep terrain

2 development branches:

- assess aerodynamical implicit Finite-Volume solvers (*Jameson, 1991*)
  P.L. Vitagliano (CIRA, Italy), L. Torrisi (CNMCA, Italy), M. Baldauf (DWD)
- assess dynamical core of EULAG (e.g. *Grabowski, Smolarkiewicz, 2002*)
  M. Ziemianski, M. Kurowski, B. Rosa, D. Wojcik (IMGW, Poland),
  O. Fuhrer (MeteoCH), M. Baldauf (DWD)

EULAG: anelastic approximated equations (Lipps, Hemler, 1982)
MPDATA for advection, GMRES for elliptic solver
non-oszillatory forward-in-time (NFT) integration scheme

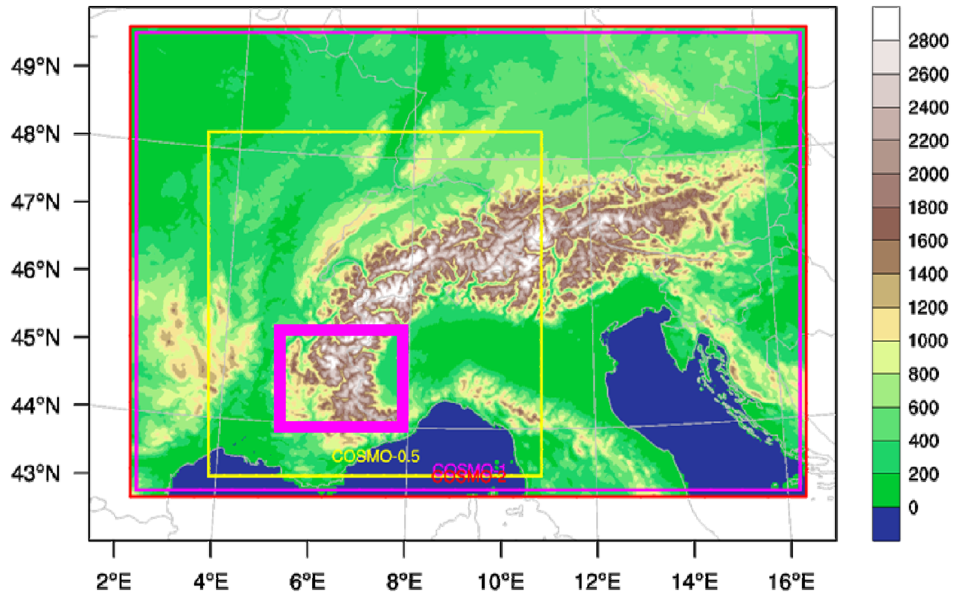
Experiments involve a case study of summer Alpine convection on 12 July 2006.

Simplified parameterizations:

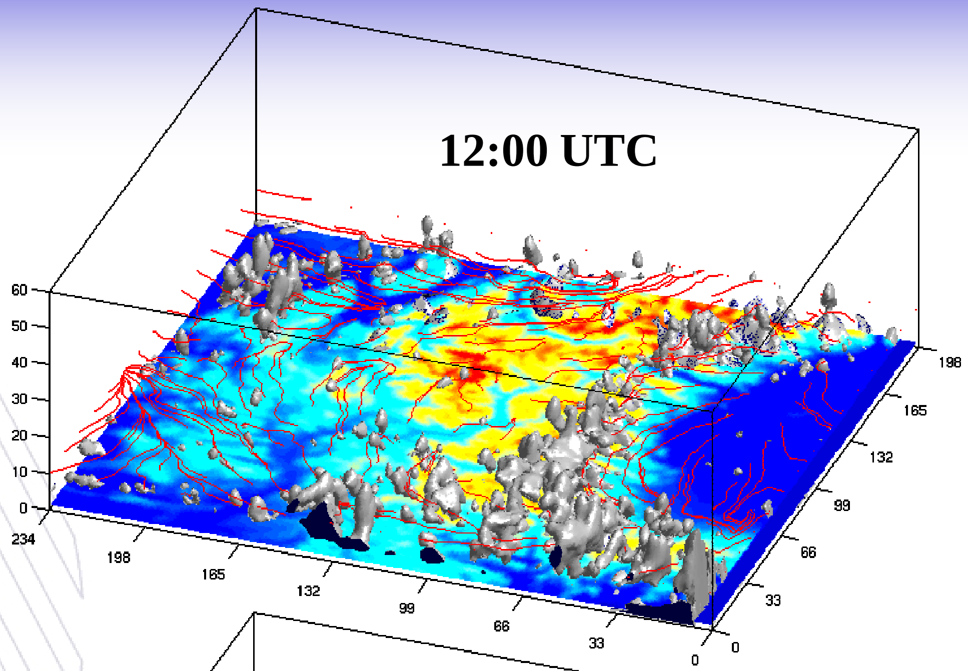
- Boundary layer processes are represented by a 1-eq. TKE (turbulent kinetic energy) model
- Surface fluxes and drag are taken from the operational run of the COSMO2 model for CH
- Simple representation of moist processes (warm rain Kessler-scheme)

Experiment setup:

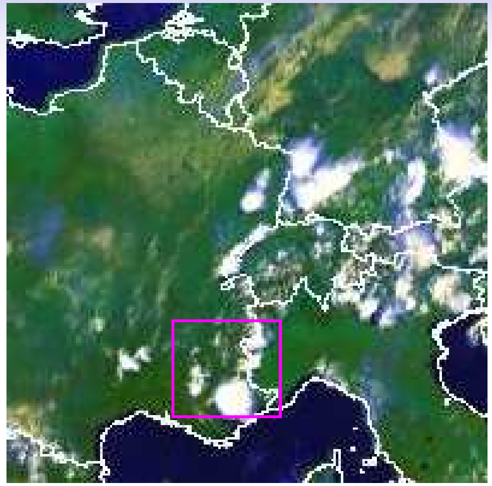
- Horizontal resolution 1.1 km, vertical resolution as in COSMO2
- The computational domain is restricted to 234x198 km and covers the Southern Alps
- Initial, boundary conditions from COSMO2 operational run

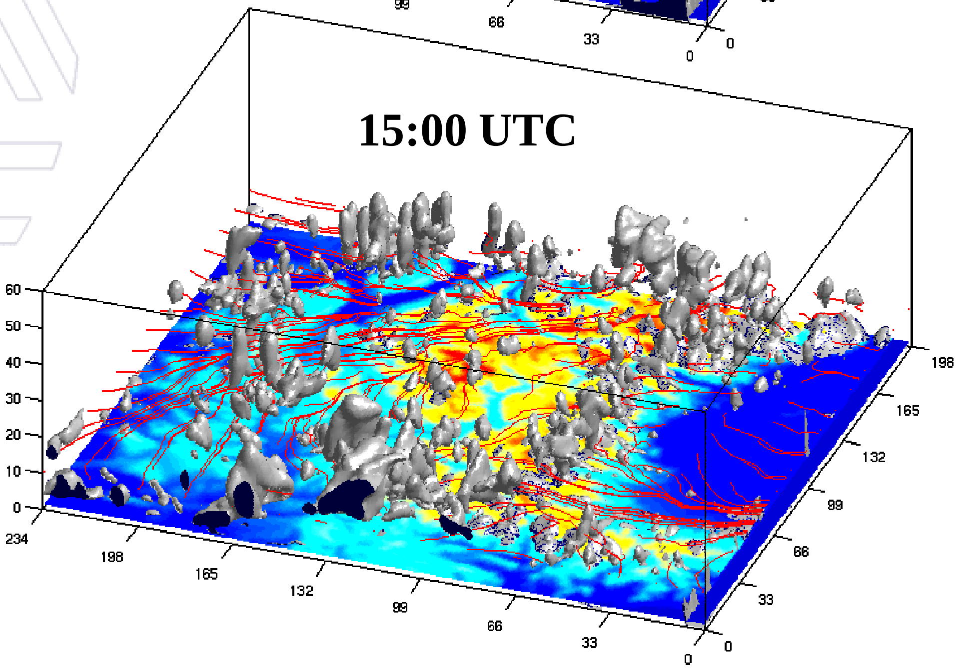


Comparison of the EULAG simulation with satellite images

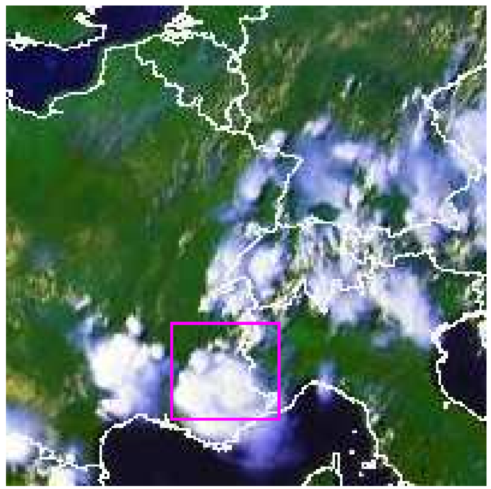


12:00 UTC





15:00 UTC

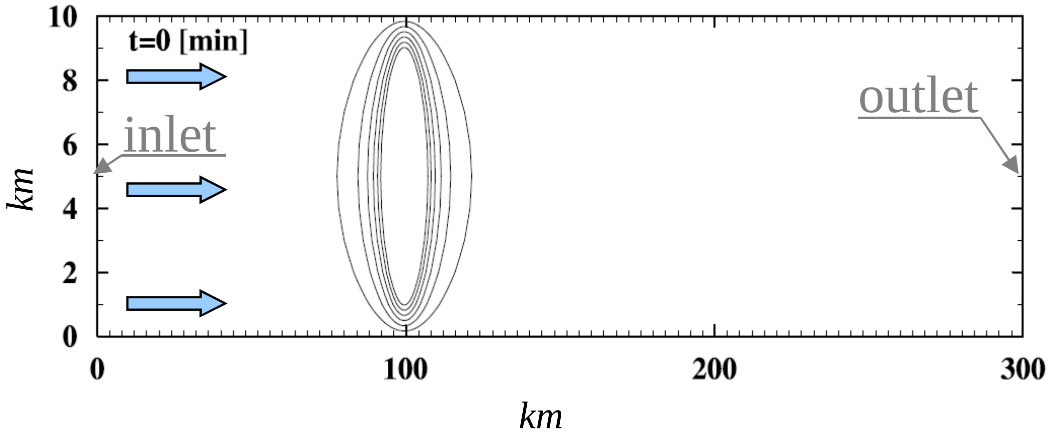


Temporal and spatial structure of the simulated convection in the EULAG experiment closely resembles the actual development

... now test the implementation of the EULAG dynamical core in COSMO

Skamarock W. C. and Klemp J. B. Efficiency and accuracy of Klemp-Wilhelmson time-splitting technique. *Mon. Wea. Rev.* **122**: 2623-2630, **1994**

Initial potential temperature perturbation



Constant ambient flow within channel 300 km and 6000 km long

Initial velocity

$$u(t=0) = 20 m/s$$
$$v(t=0) = 0\ m/s$$
$$w(t=0) = 0\ m/s$$

Initial potential temperature perturbation

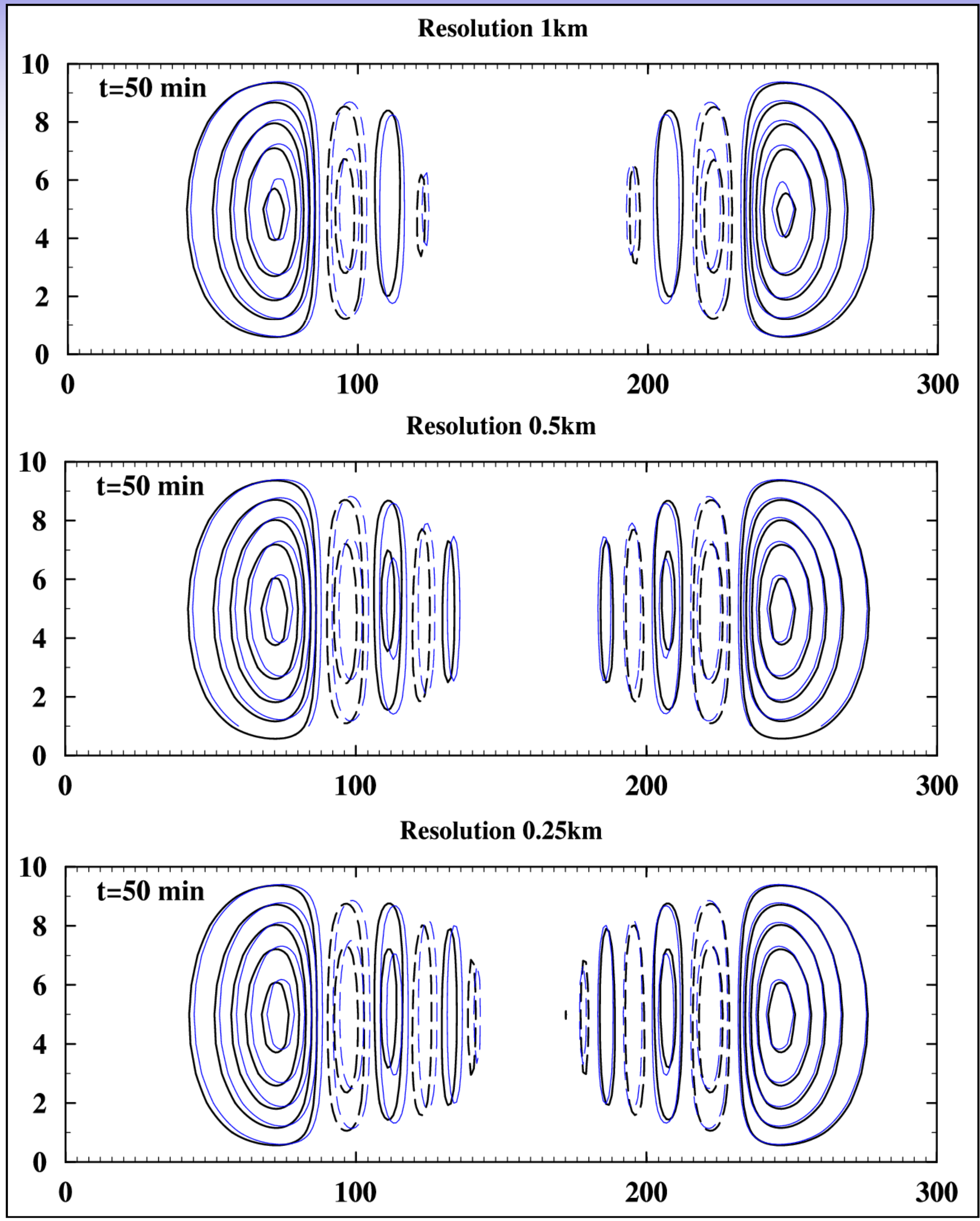
$$\theta(x,z,t=0) = \Delta\theta_0 \frac{\sin(\pi z/H)}{1+(x-x_c)^2/a^2}$$

Setup overview:

- domain size 300x10 km
- resolution 1x1km, 0.5x0.5 km, 0.25x0.25 km
- rigid free-slip b.c.
- periodic lateral boundaries
- constant horizontal flow 20m/s at inlet
- no subgrid mixing
- hydrostatic balance
- stable stratification N=0.01 s^{-1}
- max. temperature perturbation 0.01K
- Coriolis force included

Results - gravity waves in a short channel

$\theta' = \theta - \theta(t=0)$



C&E
Eulag

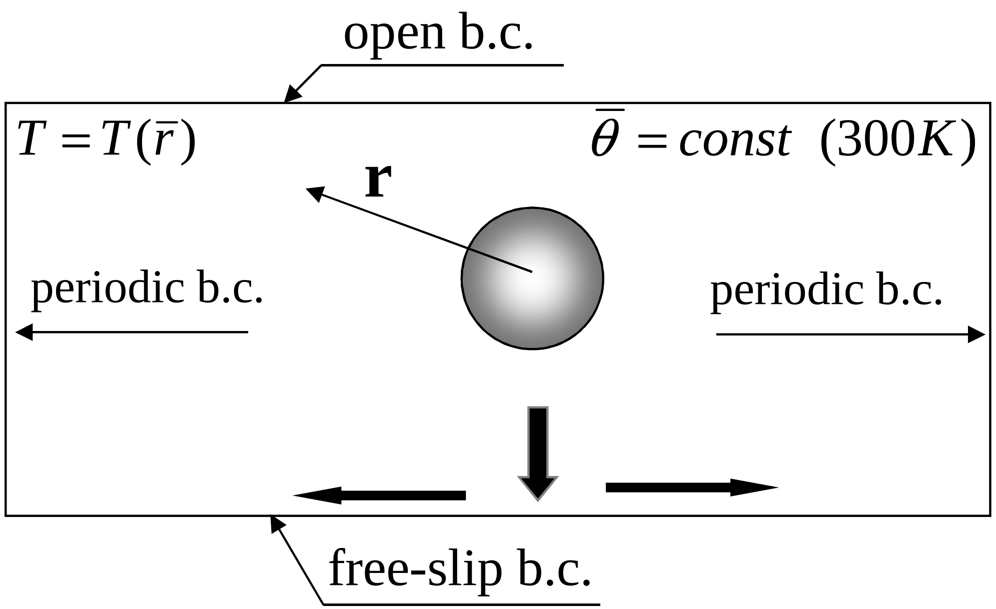
C&E
Eulag

C&E
Eulag

Straka, J. M., Wilhelmson, Robert B., Wicker, Louis J., Anderson, John R., Droegemeier, Kelvin K., Numerical solutions of a non-linear density current: A benchmark solution and comparison *International Journal for Numerical Methods in Fluids*, **(17)**, 1993

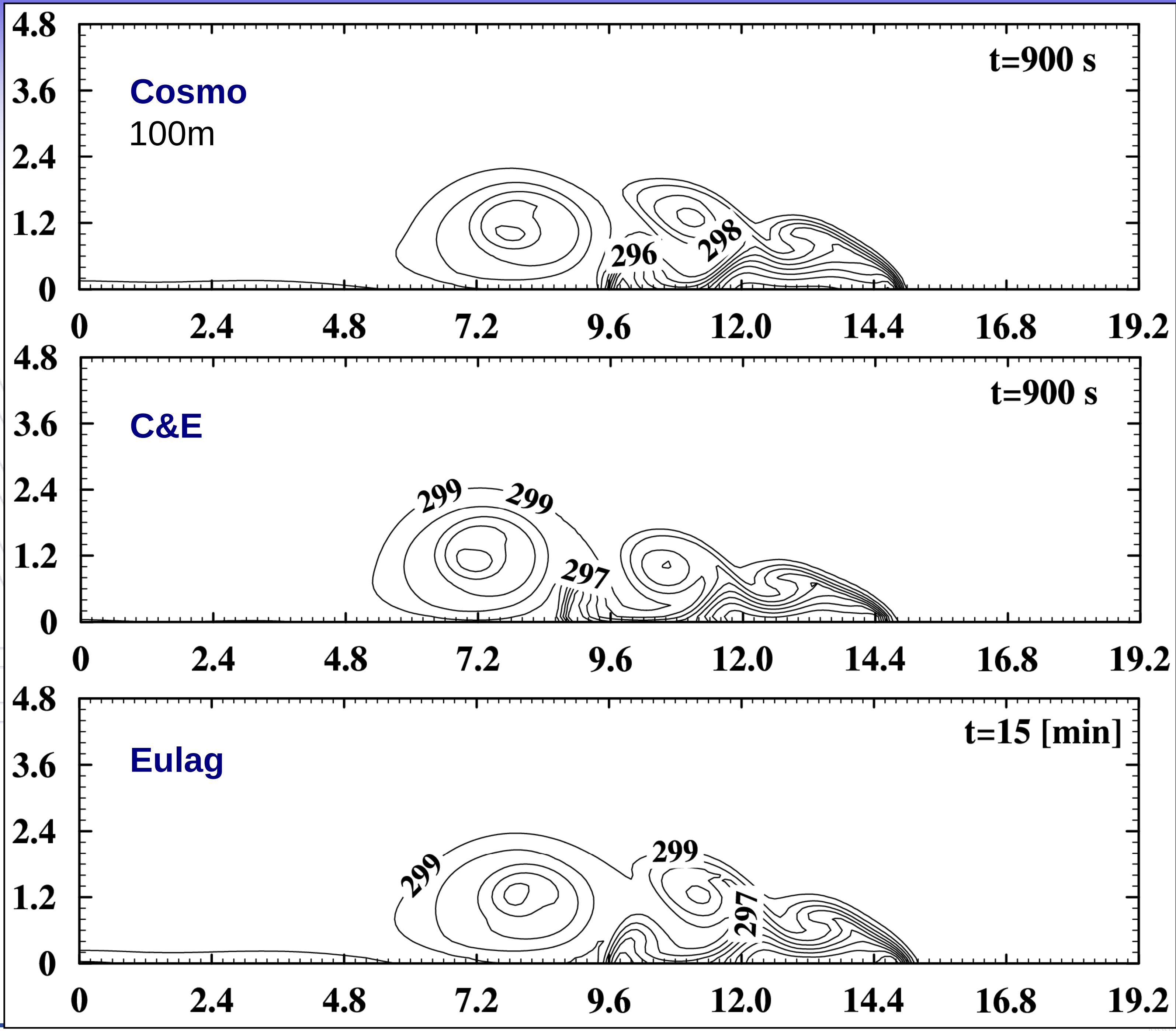
Experiment configuration:

- isentropic atmosphere, $\theta(z)$=const (300K)
- periodic lateral boundaries
- free-slip bottom b.c.
- constant subgrid mixing, $K=75m^2/s$
- domain size 51.2km x 6.4km
- bubble min. temperature -15K
- bubble size 8km x 4km
- no initial flow
- integration time 15min

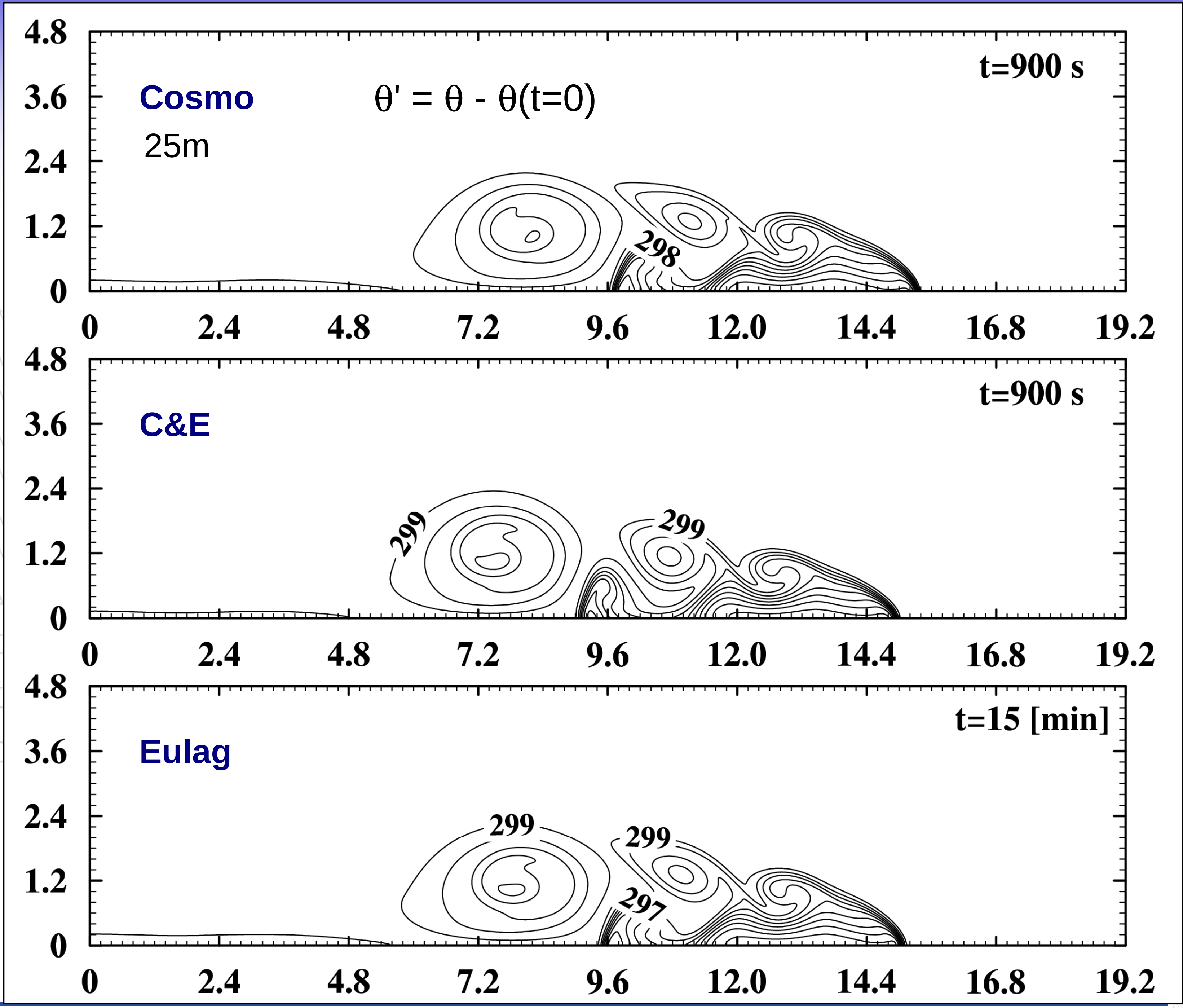


Comparison of the potential temperature distribution

θ' = θ - θ(t=0)



Comparison of potential temperature distribution at resolution 25 m



Conclusions

- for smaller scale models and dry Euler equations the anelastic approximation seems to work quite well
- keep in mind that short sound waves are also strongly damped in our compressible solver (divergence damping)
- all of the idealised tests studied in task 1.1 delivered satisfying results with the anelastic approx.

- what is the meteorological meaning of long sound waves and the Lamb mode?
- are there changes in the assessment when moist processes are studied?

- Temporal and spatial structure of the simulated convection in the EULAG experiment closely resembles the actual development.

- implementation of EULAG code into COSMO currently underway (= 'C&E')
- Results of the idealized tests obtained using the hybrid C&E model are in good qualitative and quantitative agreement both with reference and analytical solutions.
- Small differences indicate the need for further testing and verification of the C&E code.
- Dynamical core of the developed prototype, cooperates correctly with the diffusive forcing from COSMO parameterizations.

- implementation of MPDATA as an alternative tracer advection scheme into COSMO (G. deMorsier, M. Müllner (MeteoCH))

3 publications accepted for **Acta Geophysica 59 (6) , 2011**
(collection of papers for the EULAG workshop, Sopot, Sept. 2010)

B. Rosa, M. J. Kurowski, and M. Z. Ziemiański: Testing the anelastic nonhydrostatic model EULAG as a prospective dynamical core of a numerical weather prediction model. Part I: Dry Benchmarks

M. J. Kurowski, B. Rosa and M. Z. Ziemiański: Testing the anelastic nonhydrostatic model EULAG as a prospective dynamical core of numerical weather prediction model. Part II: Simulations of a supercell

M. Z. Ziemiański, M. J. Kurowski, Z. P. Piotrowski, B. Rosa and O. Fuhrer: Toward very high resolution NWP over Alps: Influence of the increasing model resolution on the flow pattern

M. Baldauf: Non-hydrostatic modelling with the COSMO model,
***proceedings of 'ECMWF workshop on non-hydrostatic modelling'*, 2010**, p. 161-169
← Dispersion relation of sound/gravity waves in filtered equations

goal: prototype of a complete implementation/coupling of EULAG dyn. core into COSMO at end of 2012

Other current numerics developments in COSMO

current dynamical core:

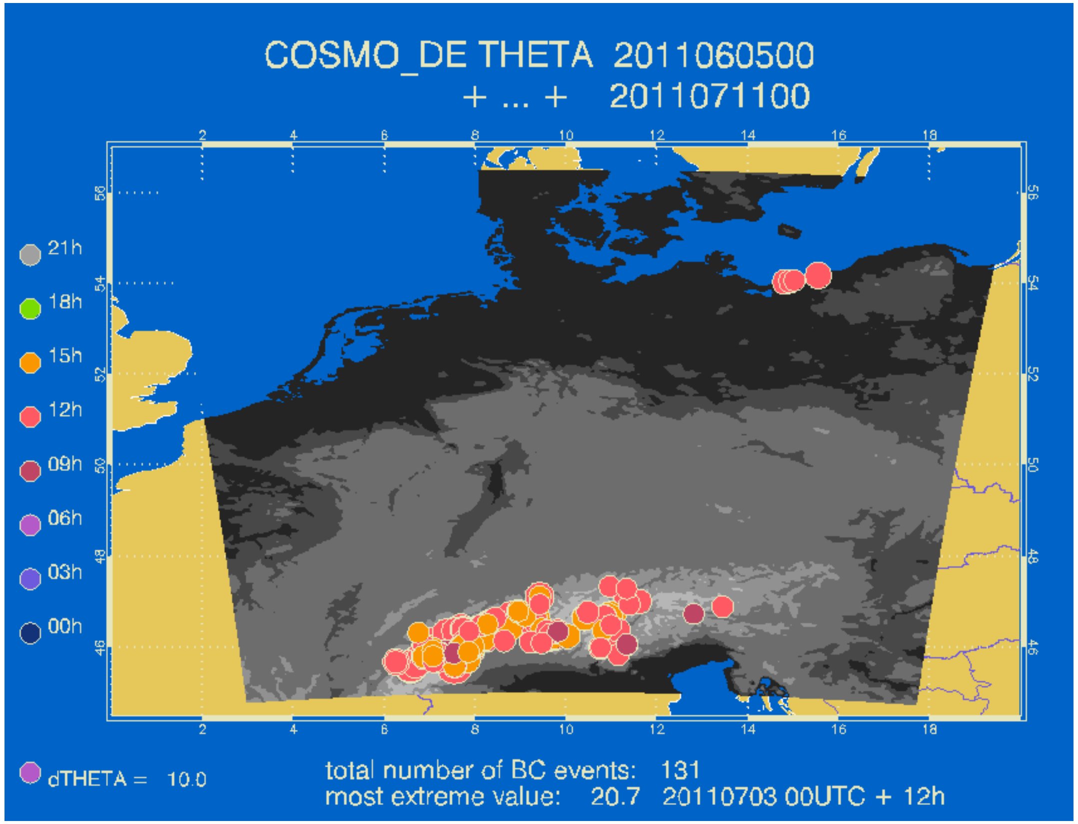
- split-explicit time integration (*Klemp, Wilhelmson, 1978, MWR*) of the fully compressible non-hydrostatic equations
- 5th order horiz. advect. + 3-stage Runge-Kutta (*Wicker, Skamarock, 2002, MWR*), (*Baldauf, 2008, JCP*)
- HE-VI fast waves (sound, gravity waves) (*Baldauf, 2010, MWR*)

why conservation / finite volume discretization is important for a dynamical core … an example

Avoid Θ-Peaks in COSMO

U. Blahak (DWD)

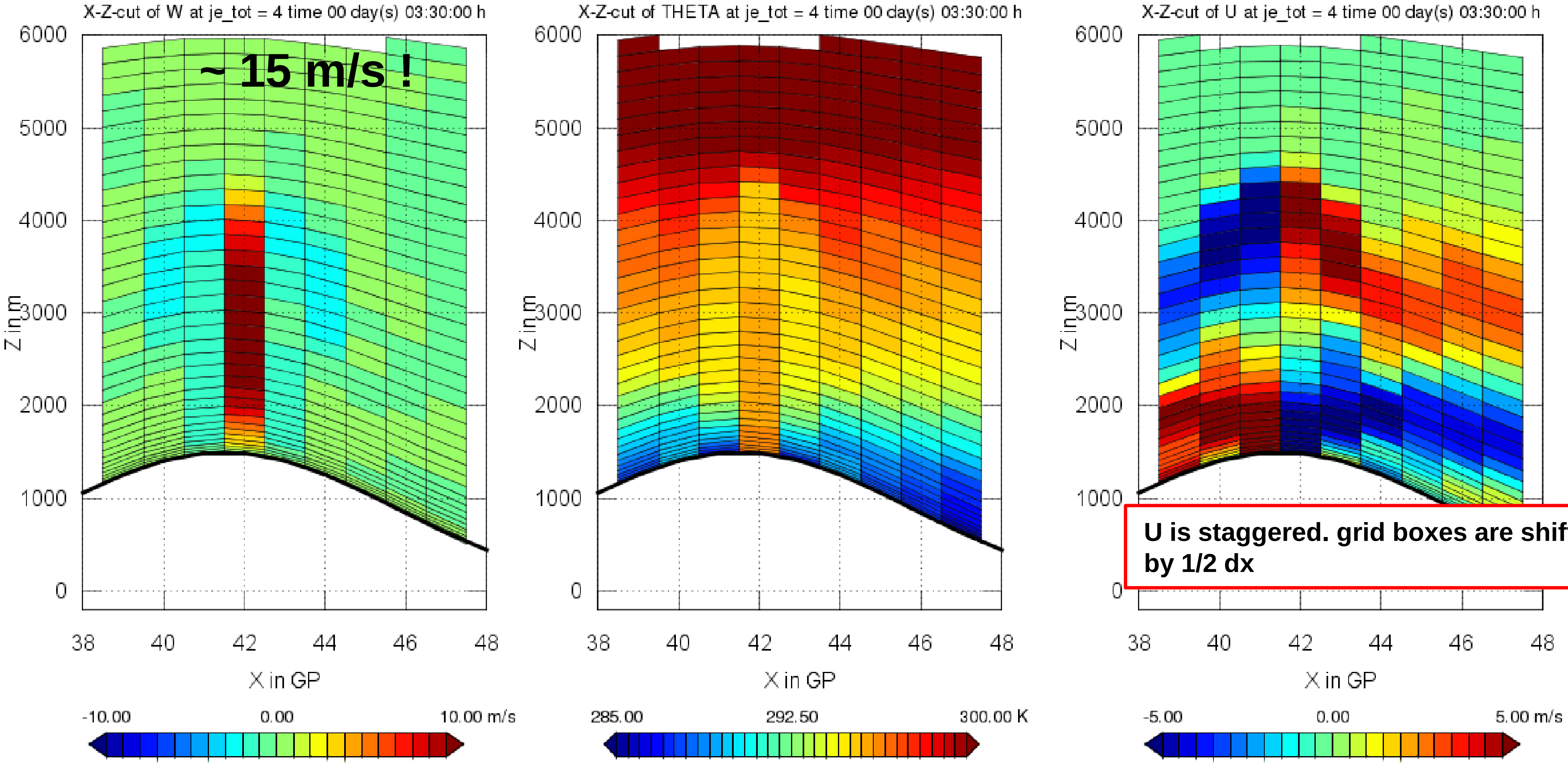
The COSMO-model produces in connection with small-scale, thermically driven circulations strange effects like „Θ-peaks“ in Alpine valleys or grid point storms at mountains or at the coast.



© Routine-Wachhund (B. Ritter)

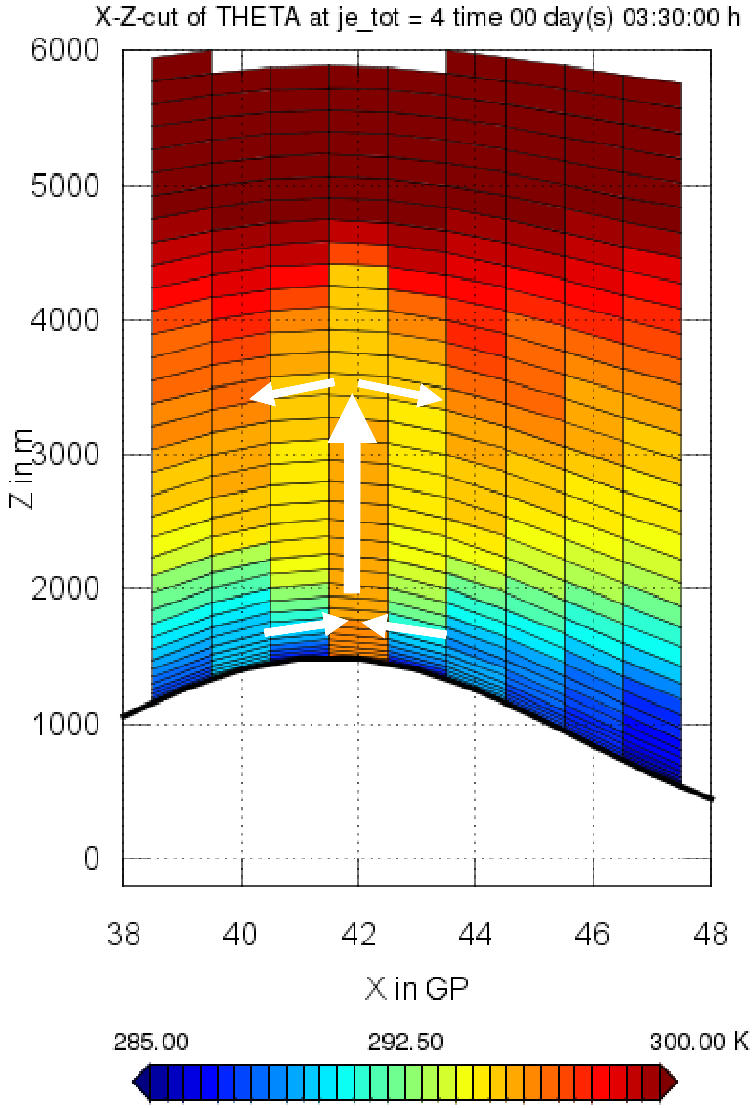
Explanation by an idealised study

2D-simulation (dry), at t=0: V = 0, N = 0.01 s^{-1}, surface is warmer than atmosph



U is staggered. grid boxes are shifted by 1/2 dx

Explanation by an idealised study



$$Adv_{X,T} = -U\frac{dT}{dx} \approx -\left(\frac{U_{i-1/2} + U_{i+1/2}}{2}\right)\Delta_{upwind}(T_i, sign(\overline{U}))$$

$\overline{U}$ **~ 0 !!**

Although correct for the center of the column, it is not representative for the grid box averaged horizontal advection of T (and p') !
... whereas vertical adv. and divergence terms are representatively estimated!

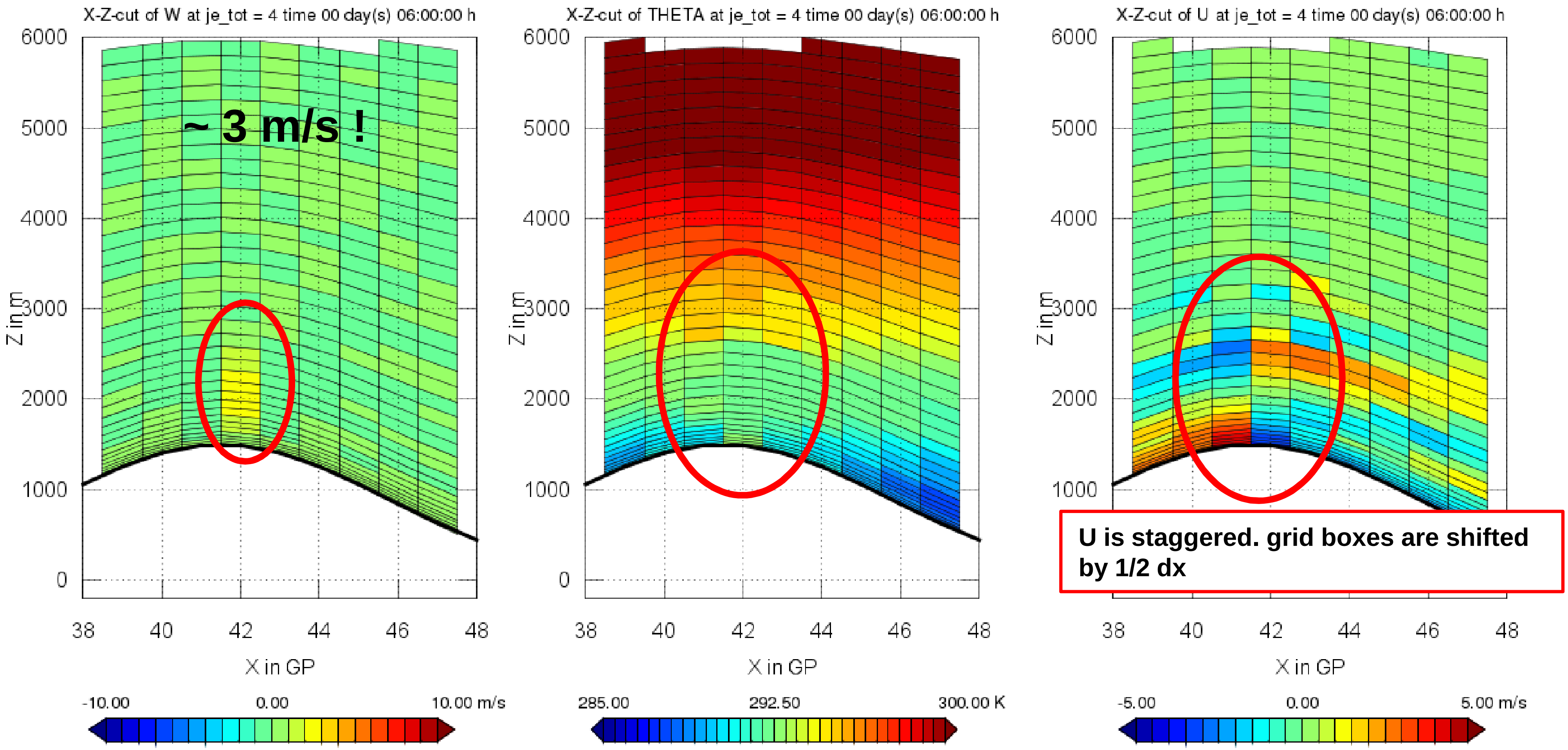
→ too few lateral inflow of cool air into the column
→ **artificial heat source !!!**

Ad-hoc correction:

$$Adv_{X,T} \approx -\frac{1}{2}\left\{ U_{i-1/2}\,\Delta_{upwind}(T_i, sign(U_{i-1/2})) + U_{i+1/2}\,\Delta_{upwind}(T_i, sign(U_{i+1/2})) \right\}$$

Explanation by an idealised study

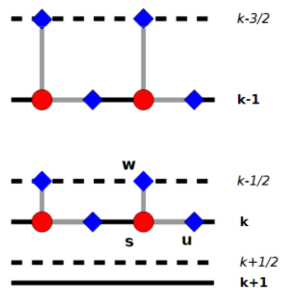
2D-simulation (dry), at t=0: V = 0, N = 0.01 s^{-1}, surface is warmer than atmosph



Development of a new fast waves solver for the Runge-Kutta scheme

M. Baldauf (DWD)

Main changes towards the current solver:



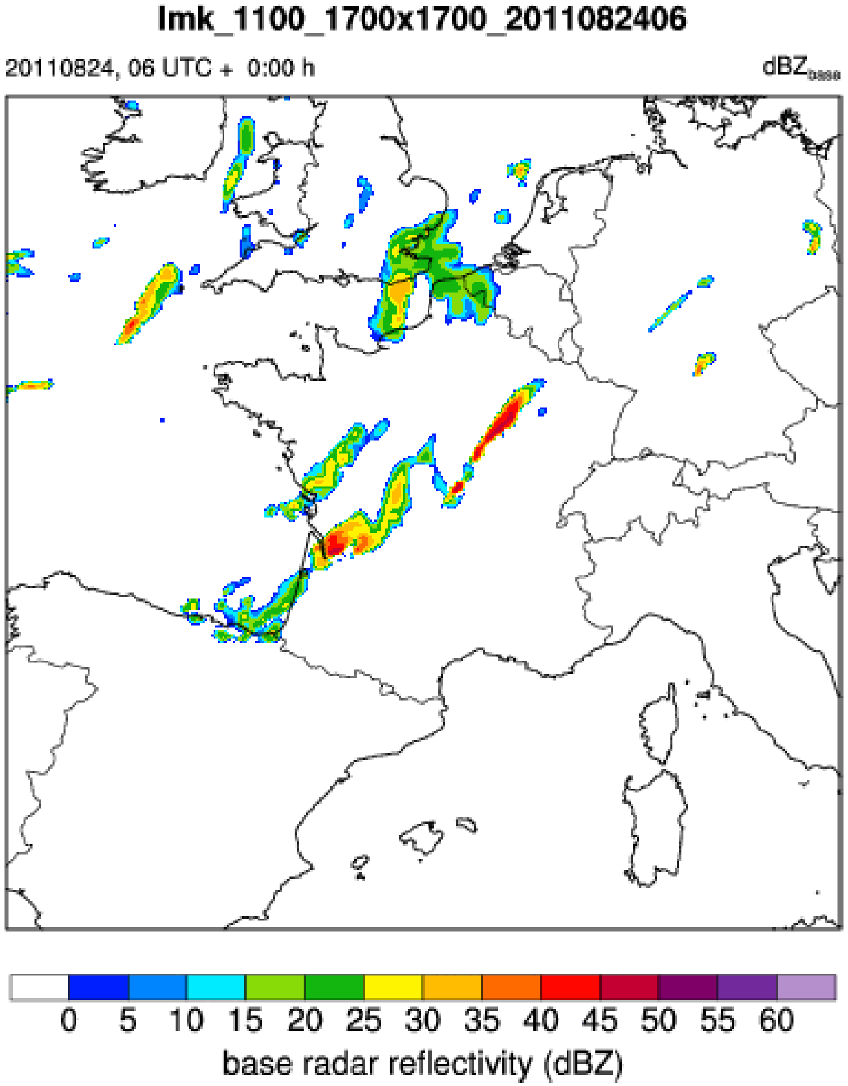
1. improvement of the vertical discretization:
   use of weighted averaging operators for all vertical operations

2. divergence in strong conservation form

$$D = \frac{1}{r\cos\phi}\frac{1}{\sqrt{g}}\left[\frac{\partial}{\partial\lambda}(\sqrt{g}\, u) + \frac{\partial}{\partial\phi}(\cos\phi\sqrt{g}\, v) + \frac{\partial}{\partial\zeta}(J_\lambda u + J_\phi \cos\phi\, v - r\cos\phi\, w)\right]$$

3. optional: complete 3D divergence damping

stability inspected in *Baldauf (2010) MWR* simulated radar reflectivity



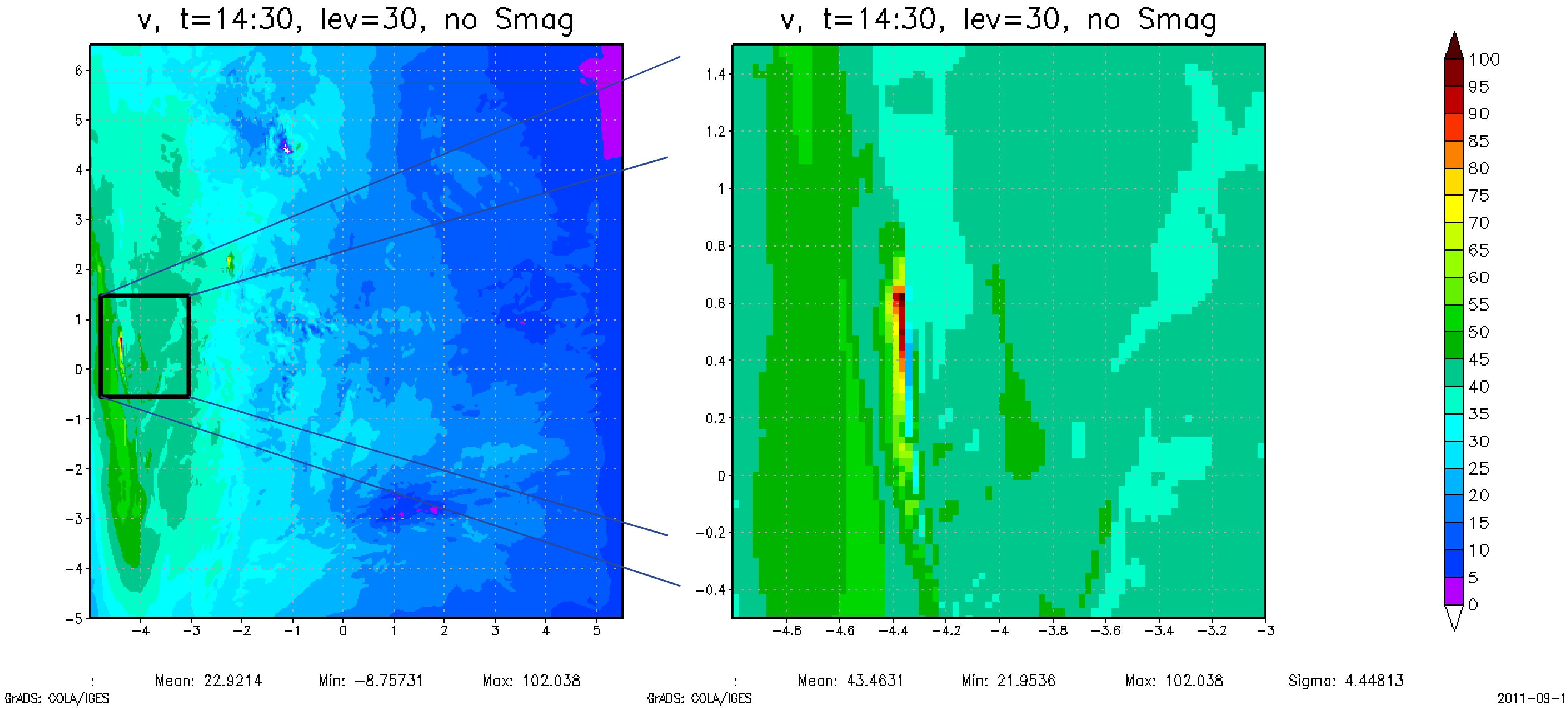
COSMO-run with a resolution of 0.01°(~ 1.1km)
1700 * 1700 grid points

model crash after 10 time steps with the current fast waves solver

stable simulation with the new FW

simulation by Axel Seifert (DWD)

shear instability in COSMO-DE at 26.08.2011, 6 UTC run, after about 14h30 min



Additionally to a 4th order hyperdiffusion ...

$$\frac{\partial \phi}{\partial t} = \ldots - K_4 \Delta\Delta\phi$$

dim.-less diffusion coeff.: $\alpha_4 := K_4 \Delta t / \Delta x^4$
stability for $0 \leq \alpha_4 \leq 1/64 \sim 0.016$
in COSMO-EU: $\alpha_4 = 0.25 / (2\pi^4) \approx 0.0013$ only for v
in COSMO-DE: $\alpha_4 = 0.1 / (2\pi^4) \approx 0.0005$ only for v

... non-linear Smagorinsky-diffusion needed

$$\begin{aligned}
\frac{\partial u}{\partial t} + \mathbf{v} \cdot \nabla u &= \ldots + K_{smag} \Delta u, \\
\frac{\partial v}{\partial t} + \mathbf{v} \cdot \nabla v &= \ldots + K_{smag} \Delta v, \\
\frac{\partial w}{\partial t} + \mathbf{v} \cdot \nabla w &= \ldots + 0,
\end{aligned}$$

Smagorinsky (1963) MWR:

$$\begin{aligned}
K_{smag} &= l_s^2 \cdot \sqrt{T^2 + S^2}, \\
T &= \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \\
S &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.
\end{aligned}$$

$l_s = c\, f(\Delta x, \Delta y)$

dimensionless diffusion coefficient:
$k_{smag} := K_{smag} * \Delta t / l_s^2$
stability → $k_{smag} < ½$

MetStröm

D. Schuster, M. Baldauf (DWD), D. Kröner, S. Brdar, R. Klöfkorn, A. Dedner (Univ. Freiburg)

A new dynamical core based on Discontinuous Galerkin methods
Project 'Adaptive numerics for multi-scale flow', DFG priority program 'Metström'

PhD student (financed by DFG (german research community) for 4 years)

- DG-RK method in a toy model implemented
- currently: implementation of DG solver in the COSMO model (explicit (RK integration), flat terrain)

shallow water equations:

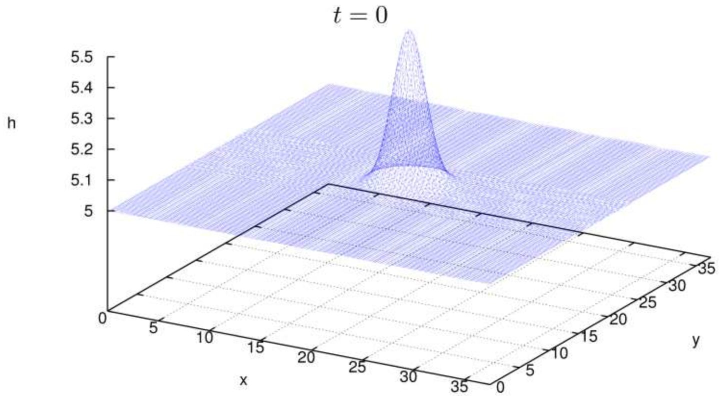
$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = 0$$

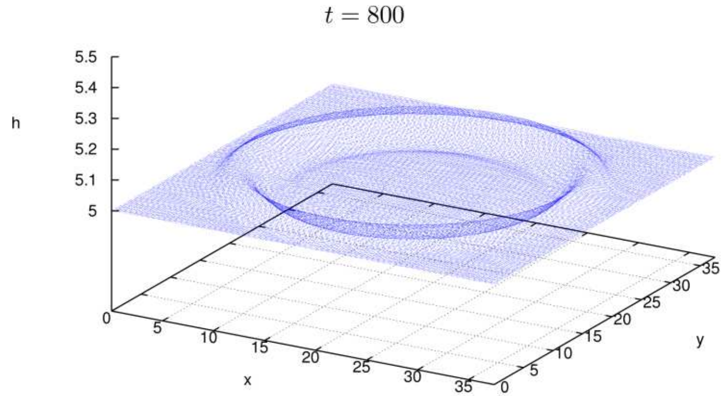
$$\frac{\partial hu}{\partial t} + \frac{\partial (hu^2 + \frac{1}{2}gh^2)}{\partial x} + \frac{\partial (huv)}{\partial y} = 0$$

$$\frac{\partial hv}{\partial t} + \frac{\partial (huv)}{\partial x} + \frac{\partial (hv^2 + \frac{1}{2}gh^2)}{\partial y} = 0$$

2d shallow water equation,
2nd order Runge-Kutta,
2nd order DG, weak formulation

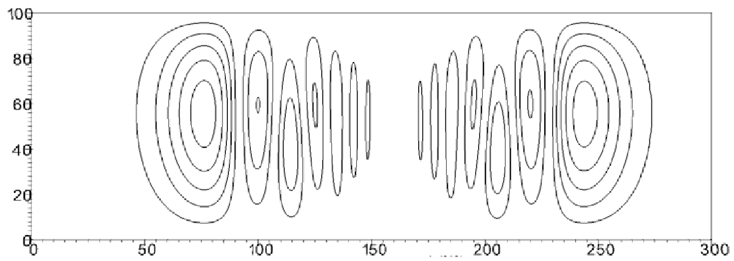
$\Delta x = 0.25$
$\Delta y = 0.25$
$\Delta t = 0.0025$

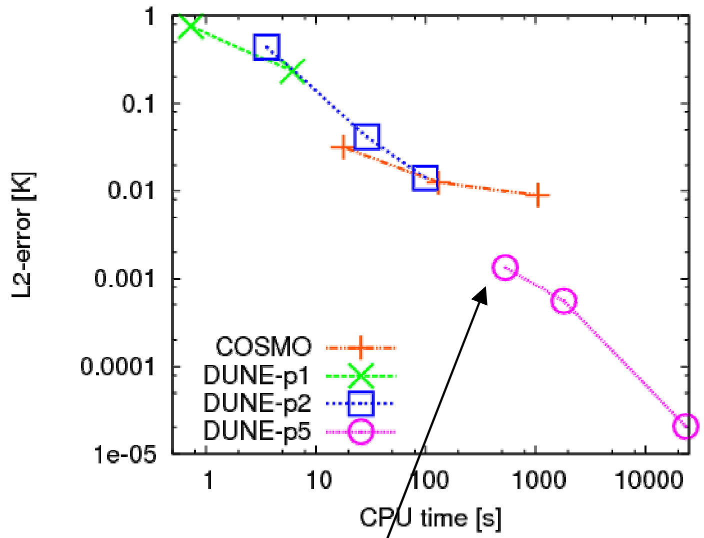




A new dynamical core based on Discontinuous Galerkin methods

Comparison between COSMO and the DUNE library:
test case linear gravity wave
(Skamarock, Klemp (1994) MWR)





higher order DG methods have the potential to be more efficient if the accuracy requirements are high

from: Brdar, Baldauf, Klöfkorn, Dedner:
Comparison of dynamical cores for NWP models,
submitted to *Theor. Comp. Fluid Dynamics*

Wave propagation properties of different equation sets (analytic consideration)

Comparison between the compressible equations and the anelastic approximation; linear analysis (normal modes)

$p=p_0+p'$
$T=T_0+T'$

$$
\begin{aligned}
\frac{\partial u'}{\partial t} + u_0 \frac{\partial u'}{\partial x} &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial x} + \alpha_D \frac{\partial D'}{\partial x} \\
\delta_1 \left(\frac{\partial w'}{\partial t} + u_0 \frac{\partial w'}{\partial x} \right) &= -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \delta_{LH}\frac{N^2}{g}\frac{p'}{\rho_0} - g\frac{\rho'}{\rho_0} + \alpha_{D,v}\frac{\partial D'}{\partial z} \\
\delta_2 \frac{\partial \rho'}{\partial t} + \delta_3 u_0 \frac{\partial \rho'}{\partial x} + \delta_5 w' \frac{\partial \rho_0}{\partial z} &= -\rho_0 D' \\
\frac{\partial p'}{\partial t} + \delta_4 u_0 \frac{\partial p'}{\partial x} + w' \frac{\partial p_0}{\partial z} &= c_0^2 \left(\frac{\partial \rho'}{\partial t} + u_0 \frac{\partial \rho'}{\partial x} + w' \frac{\partial \rho_0}{\partial z} \right) \\
D' &:= \frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z}
\end{aligned}
$$

divergence damping

switches:
- compressible: $\delta_{1..5}=1$
- compr. + div. damp.: $\delta_{1..5}=1$
- anelastic: $\delta_{2,3}=0$, $\delta_{1,4,5}=1$
  OP62, WO72, compr.: $\delta_{LH}=0$
  Lipps, Hemler (1982): $\delta_{LH}=1$

Bretherton-Transformation: $\phi' = \left(\frac{\rho_0}{\rho_s}\right)^{\alpha} \cdot \phi_b$

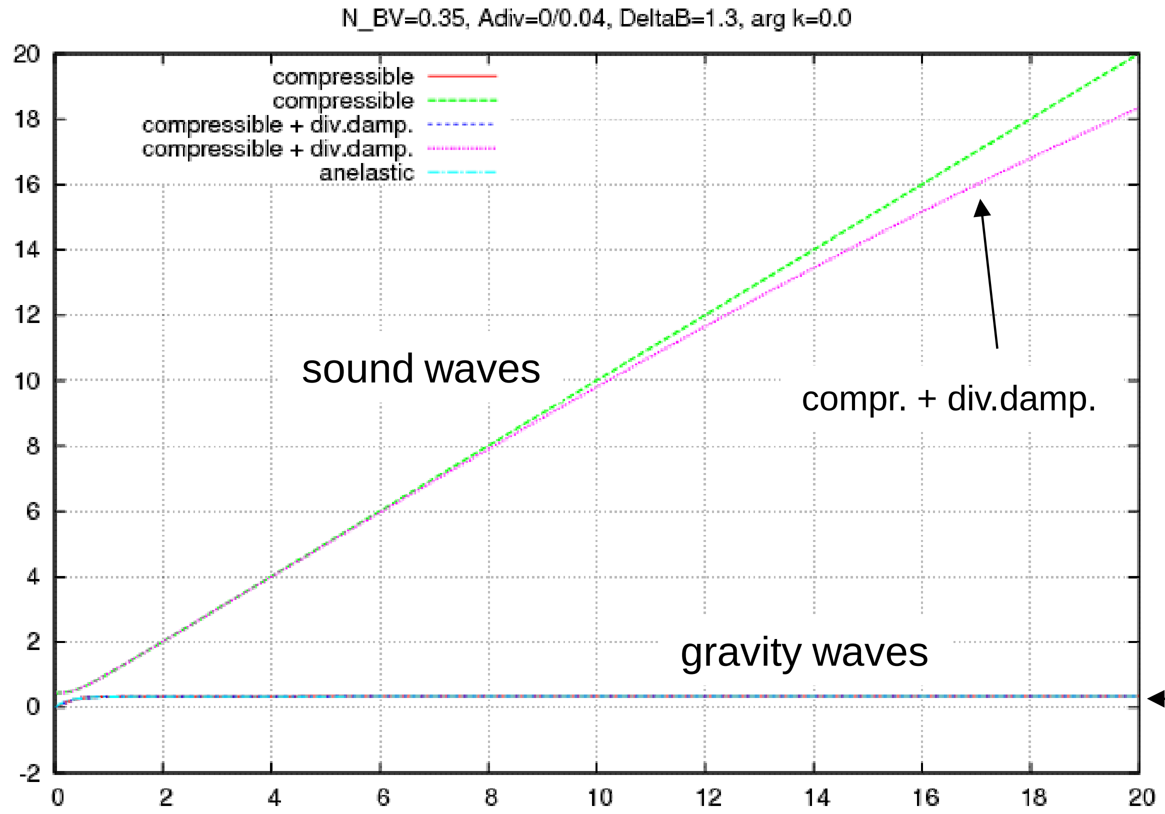
(inverse) scale height: $\frac{1}{H} := -\frac{\partial}{\partial z}\log\frac{\rho_0}{\rho_s}$ ~ (10 km)$^{-1}$

Baldauf (2010) proc. 'ECMWF workshop on non-hydrostatic modelling', ECMWF, p. 161-169
Davies et al. (2003) QJRMS

Dispersion relation $\omega = \omega(k_x, k_z)$ of internal waves

wave ansatz: $u(x,z,t) = u(k_x, k_z, \omega) \; \exp(i\, (k_x x + k_z z - \omega t))$, $w(x,z,t) = \ldots$

Re ω/ω_a



$\omega_a \rightarrow$

$\leftarrow N \cos\beta$

$k * c_s / \omega_a$

λ ~ 7 km

λ ~ 3.5 km

c_s *sound velocity*
(~ 330 m/s)

$$c_s^2 = \frac{c_p}{c_v} RT$$

N Brunt-Vaisala-frequency
(~ 0.01 1/s)

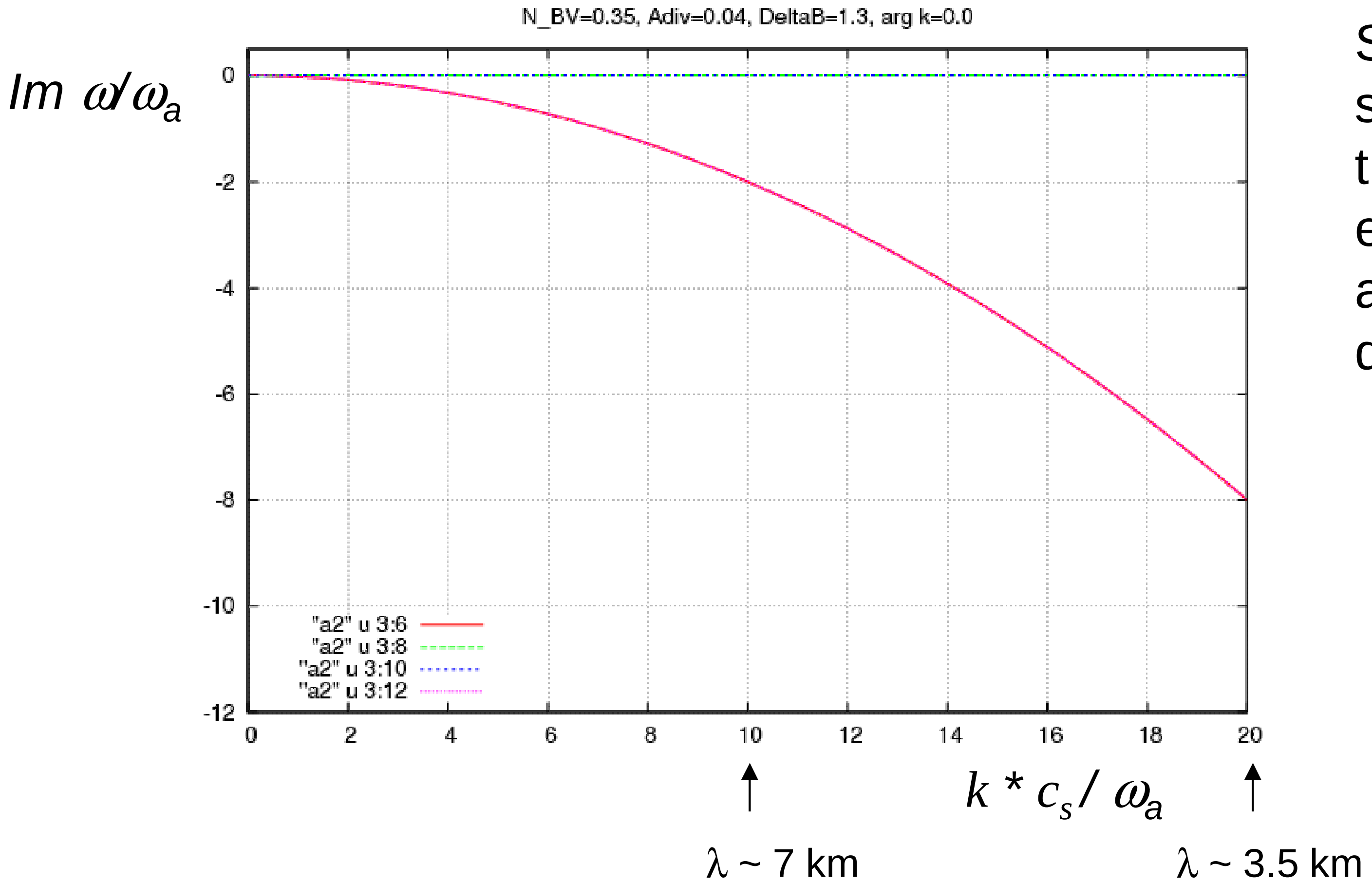
$$N^2 = \frac{g}{T}\left(\frac{\partial T}{\partial z} + \frac{g}{c_p}\right)$$

ω_a acoustic cut off frequency
(~ 0.03 1/s)

$$\omega_a^2 = N^2 + \frac{g^2}{c_s^2}$$

$\beta = \angle(k_z, k_x) = 0°$

Dispersion relation $\omega = \omega(k_x, k_z)$ of internal waves

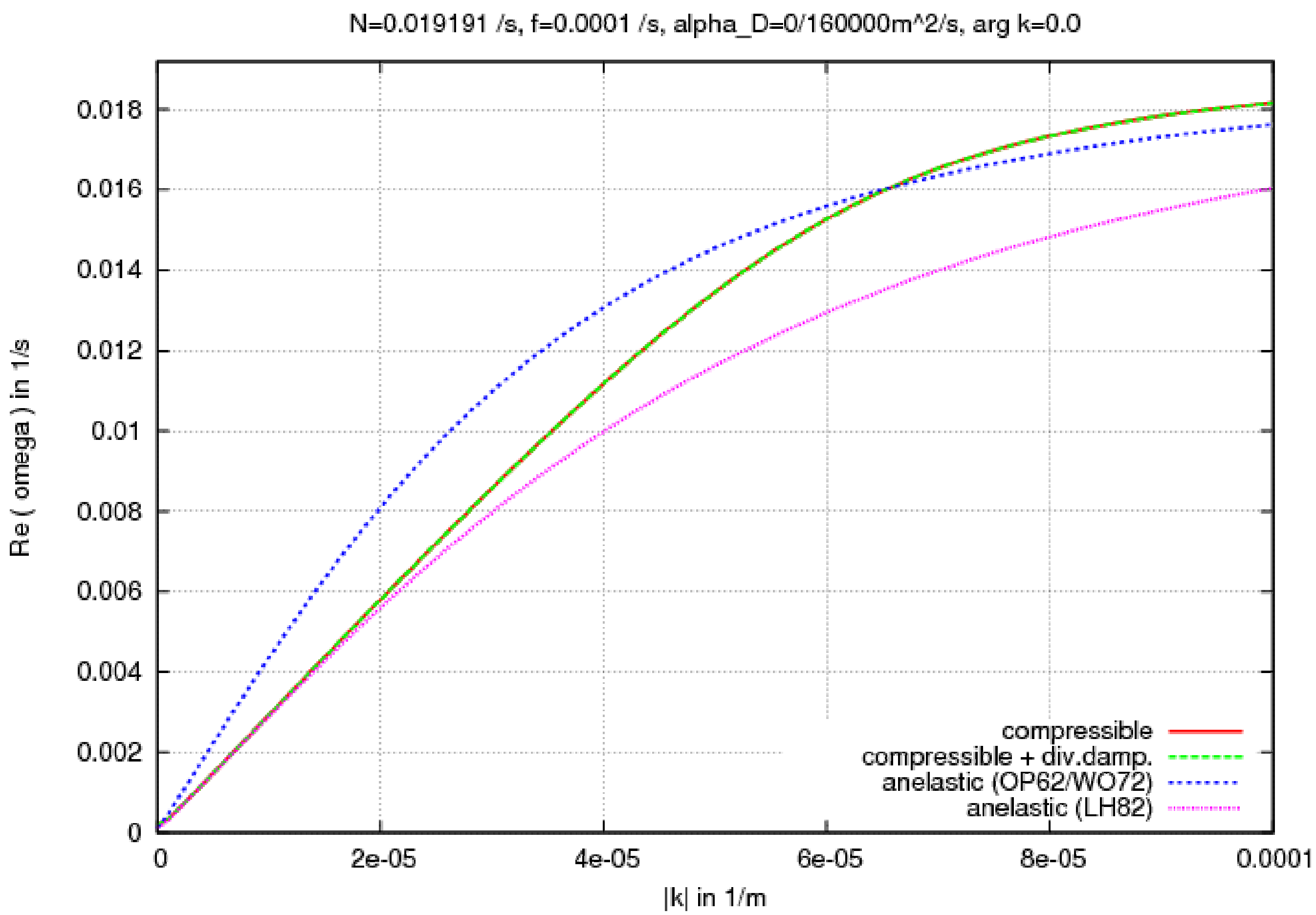


Strong damping of short sound waves in the compressible equations due to artificial divergence damping.

(timescale $1/\omega_a$ ~ 30 sec.)

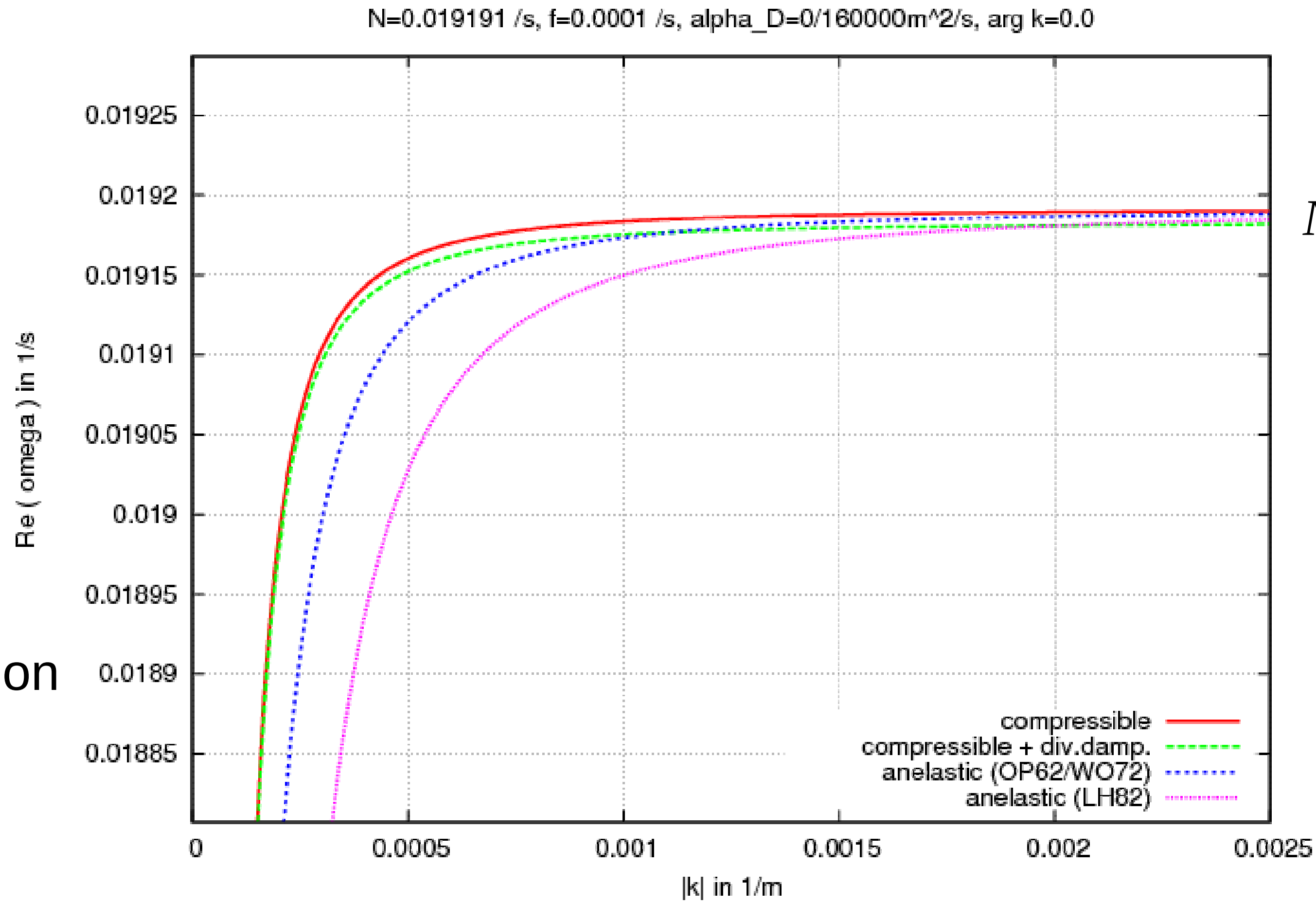
N ~ 0.01 1/s
C_{div} ~0.1

Dispersion relation for horizontally propagating gravity waves
isothermal stratification



Dispersion relation for horizontally propagating gravity waves
isothermal stratification

$\beta = \angle(k_z, k_x) = 0°$

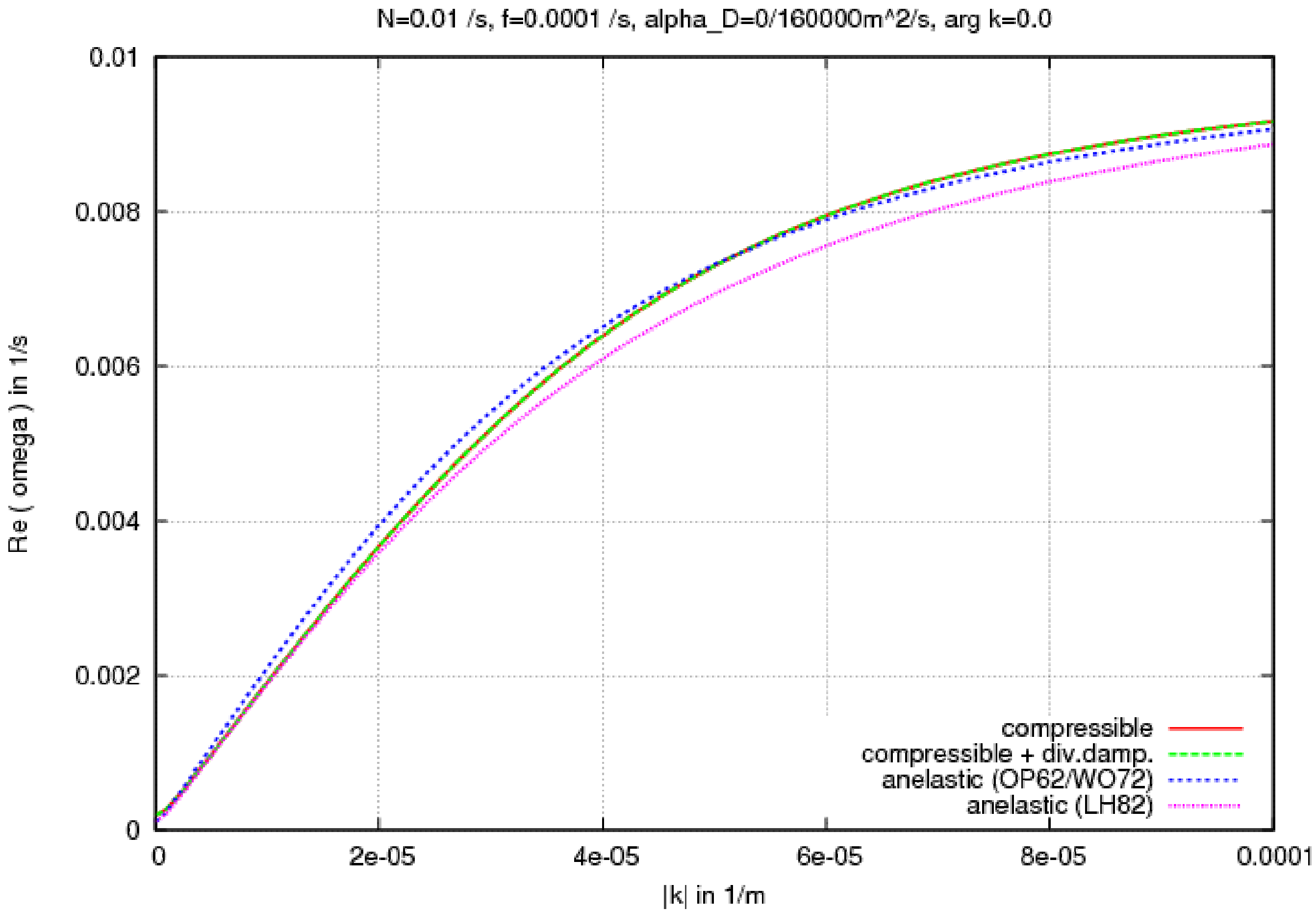


$N \cos \beta$

quite similar dispersion relation for anelastic and compressible eqns.

Dispersion relation for horizontally propagating gravity waves

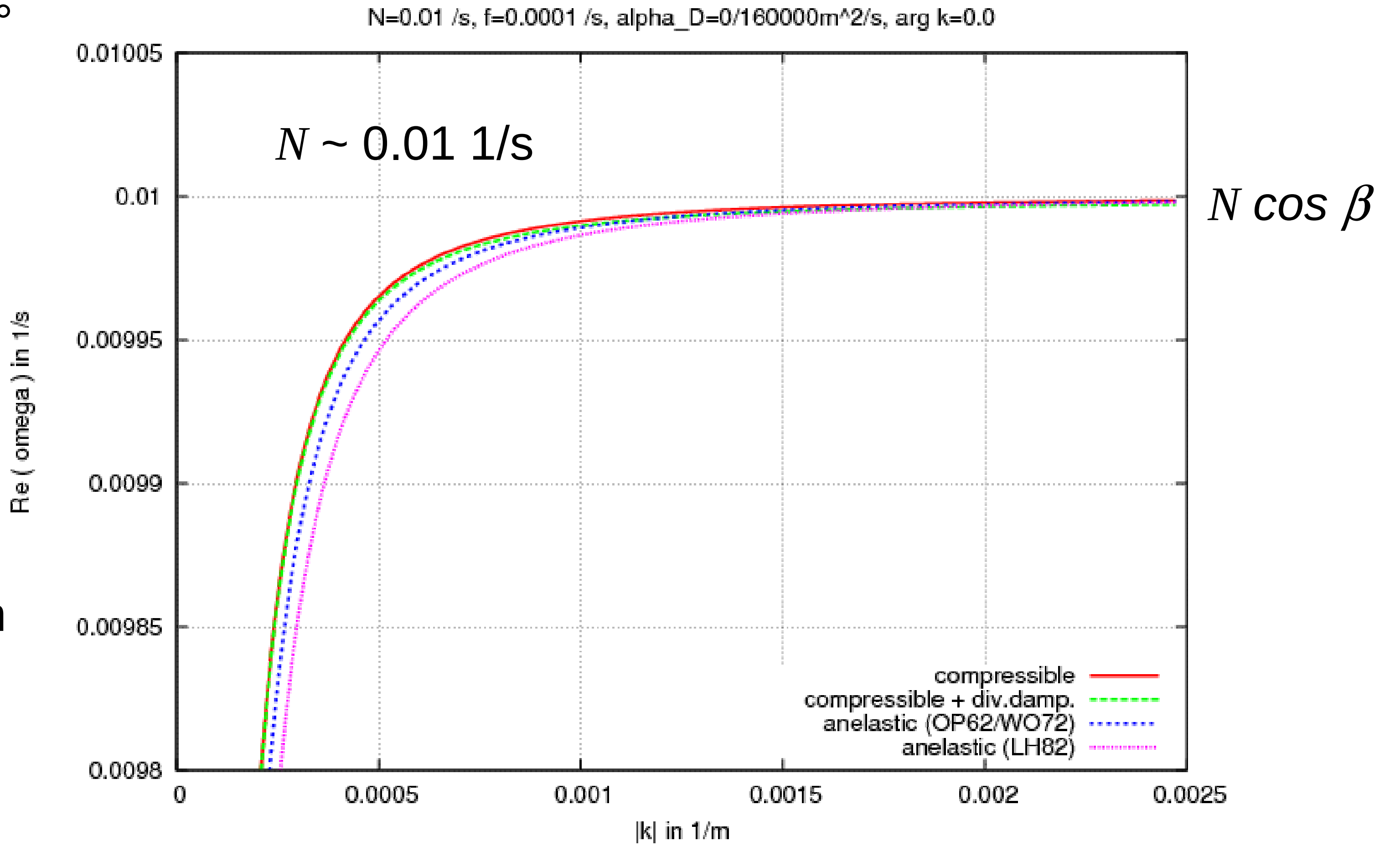
N=0.01 1/s



Dispersion relation for horizontally propagating gravity waves

N=0.01 1/s

$\beta = \angle (k_z, k_x) = 0°$

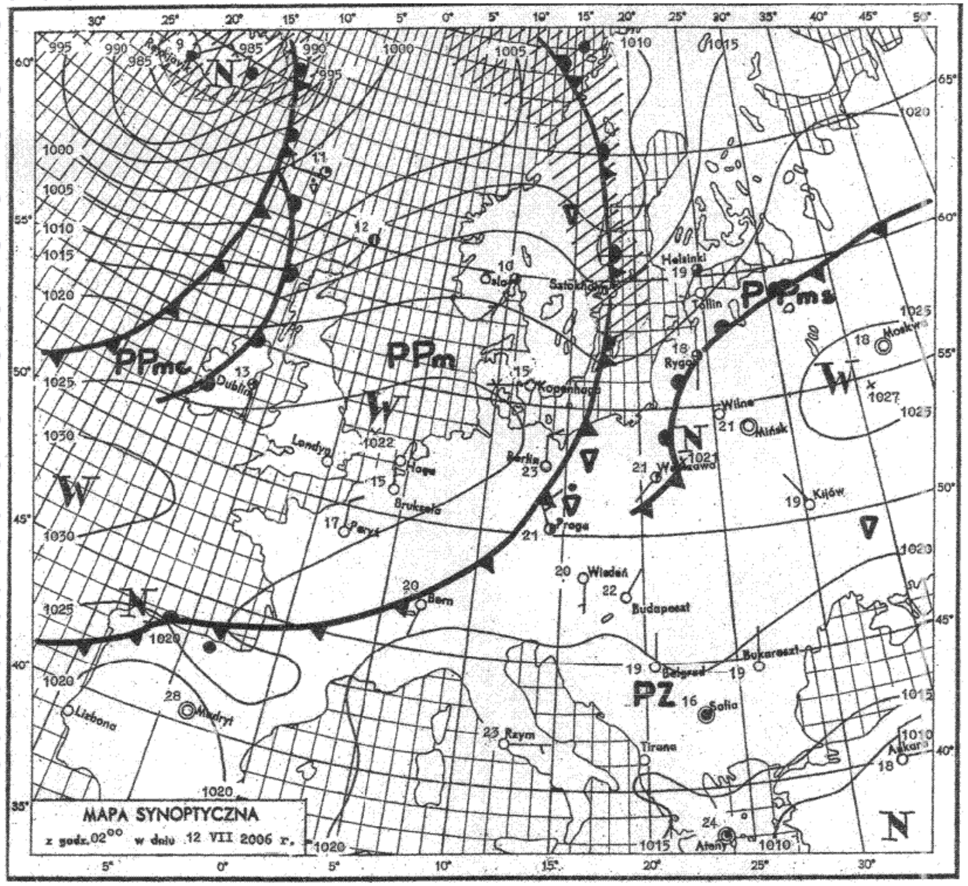


quite similar dispersion relation for anelastic and compressible eqns.

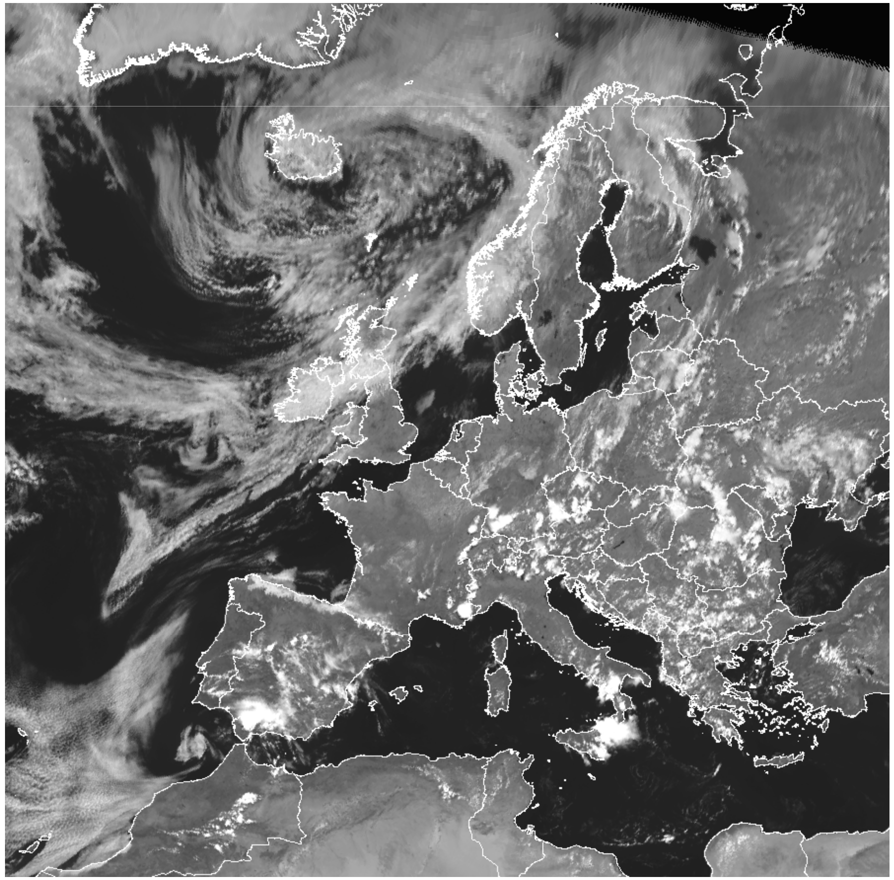
General meteorological situation in the Alpine region - 12 July 2006

Synoptic situation in the area: slow-moving cold front in a shallow surface trough of low pressure

This is representative case study for summer (convective) situations.



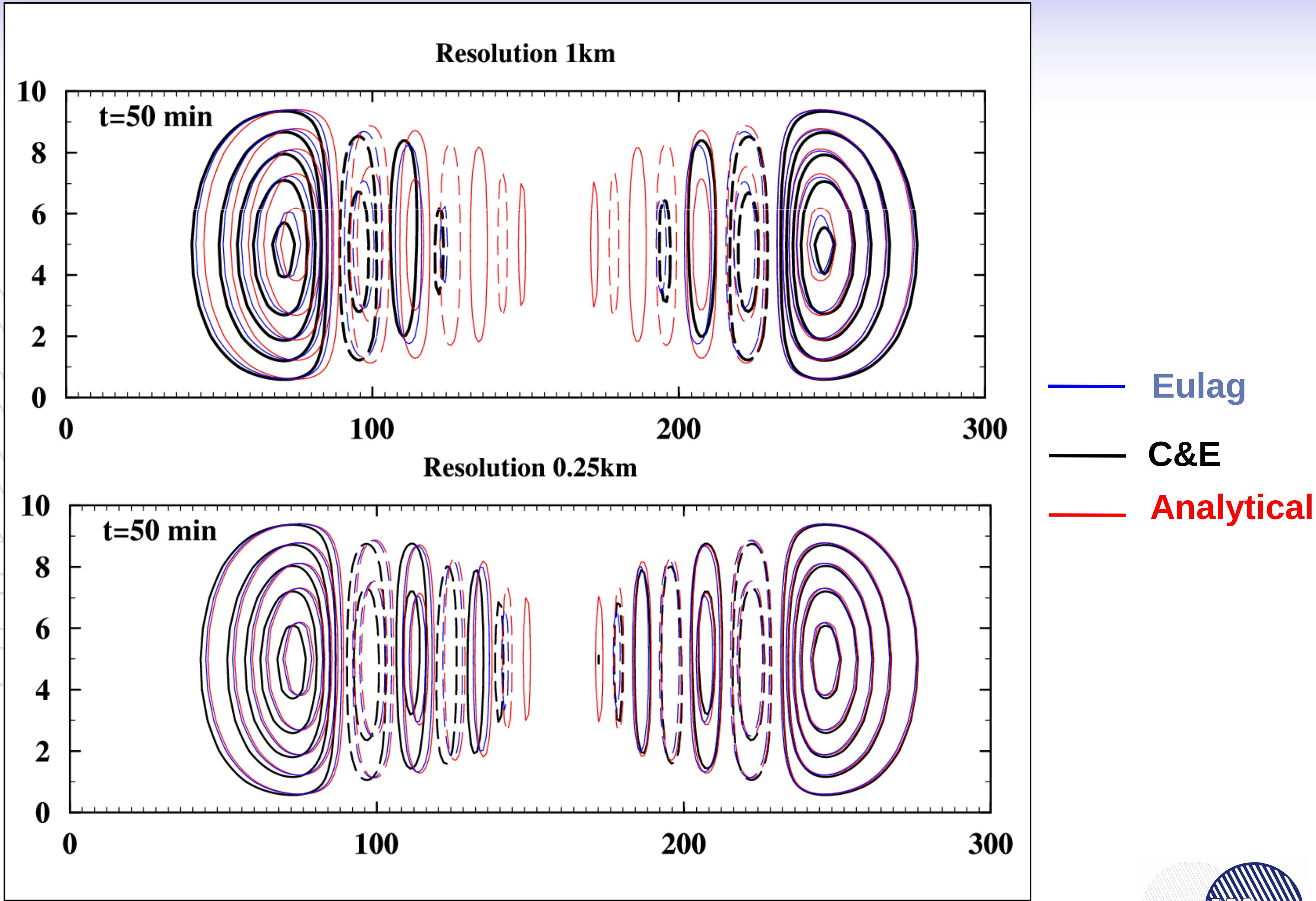
Synoptic map – 2:00 UTC, 12 July 2006



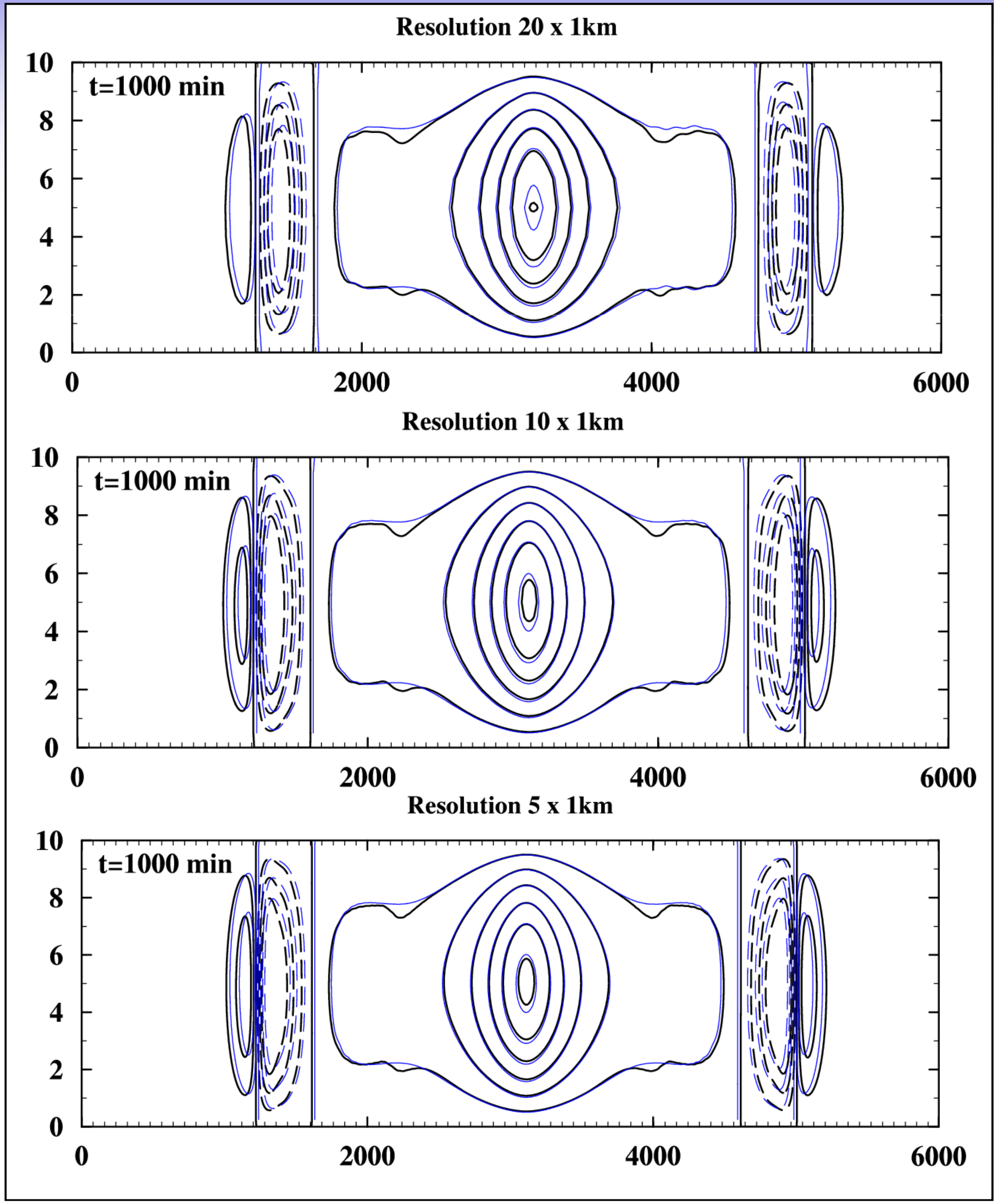
MSG (Meteosat Second Genertion) 12:00 UTC

Comparison with analytical solution

θ' = θ - θ(t=0)



Gravity waves in a long channel



Eulag

C&E

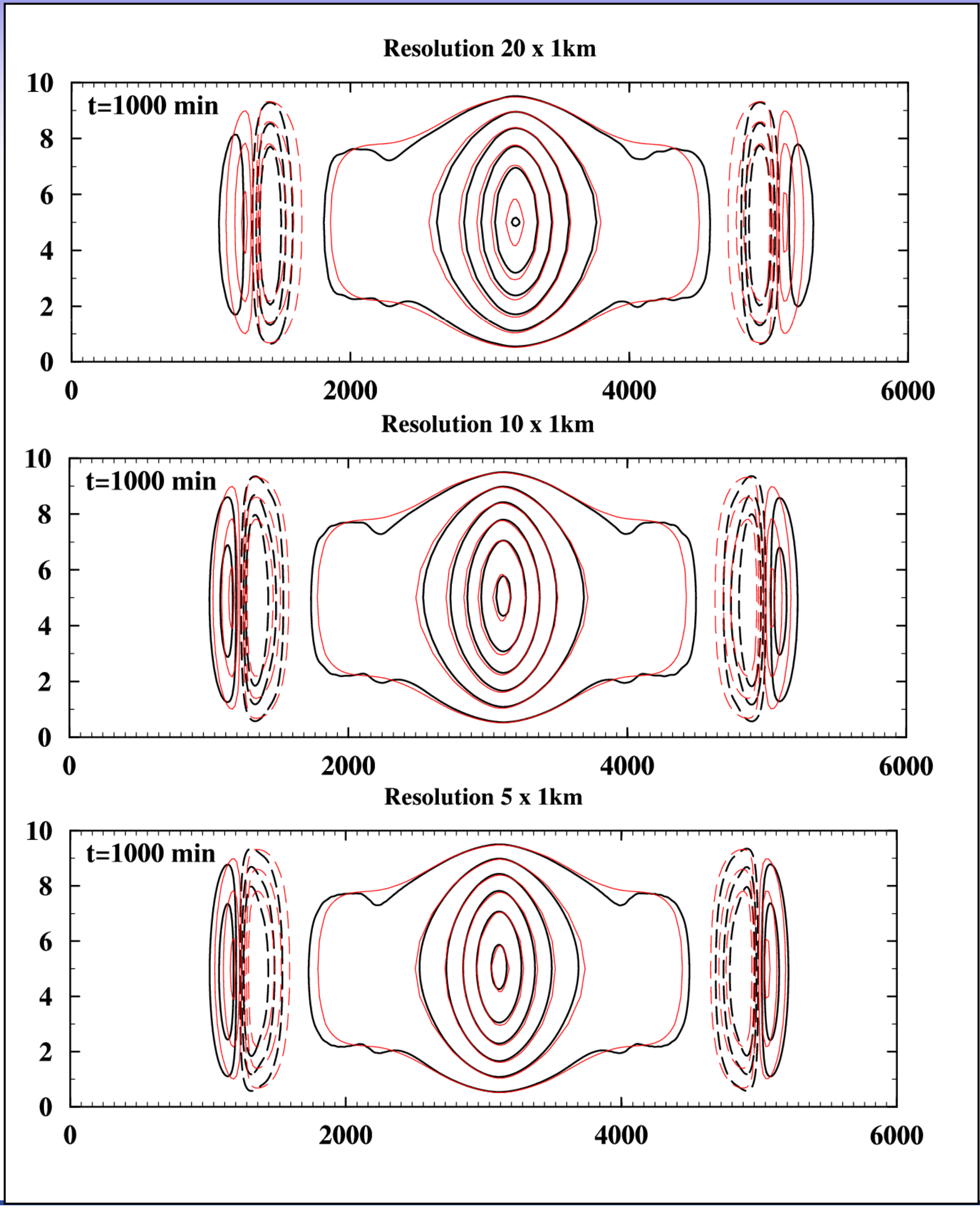
Eulag

C&E

Eulag

C&E

Gravity waves in a long channel



C&E

Analytical

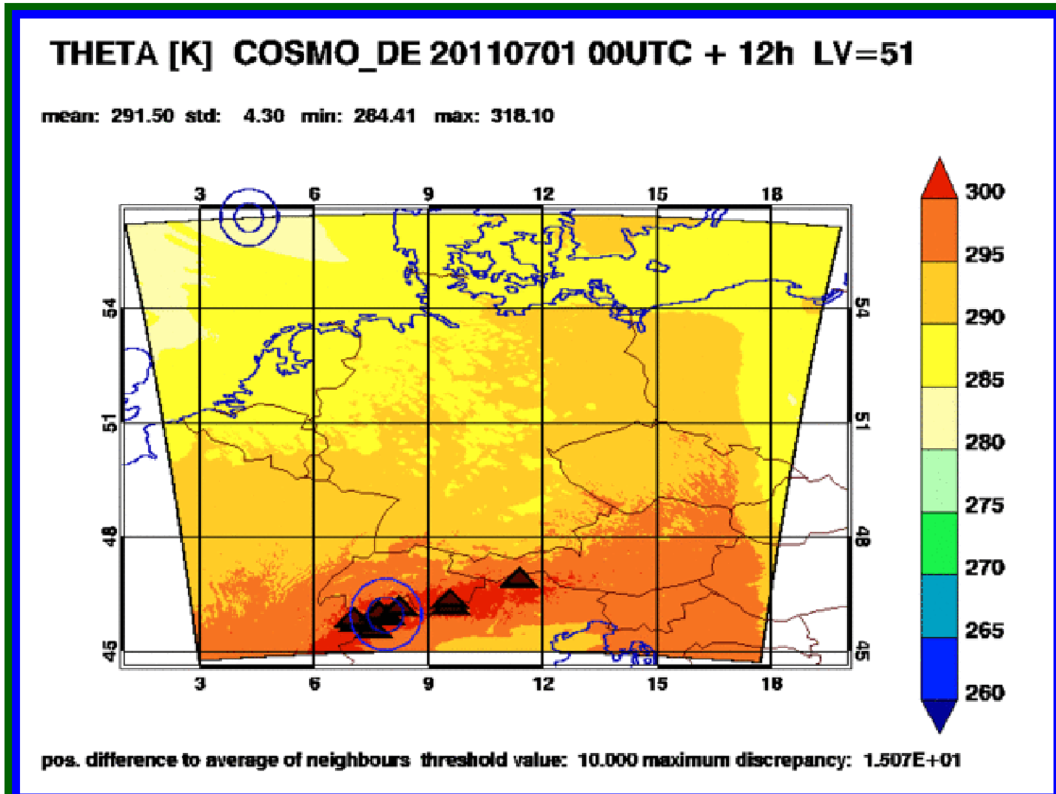
C&E

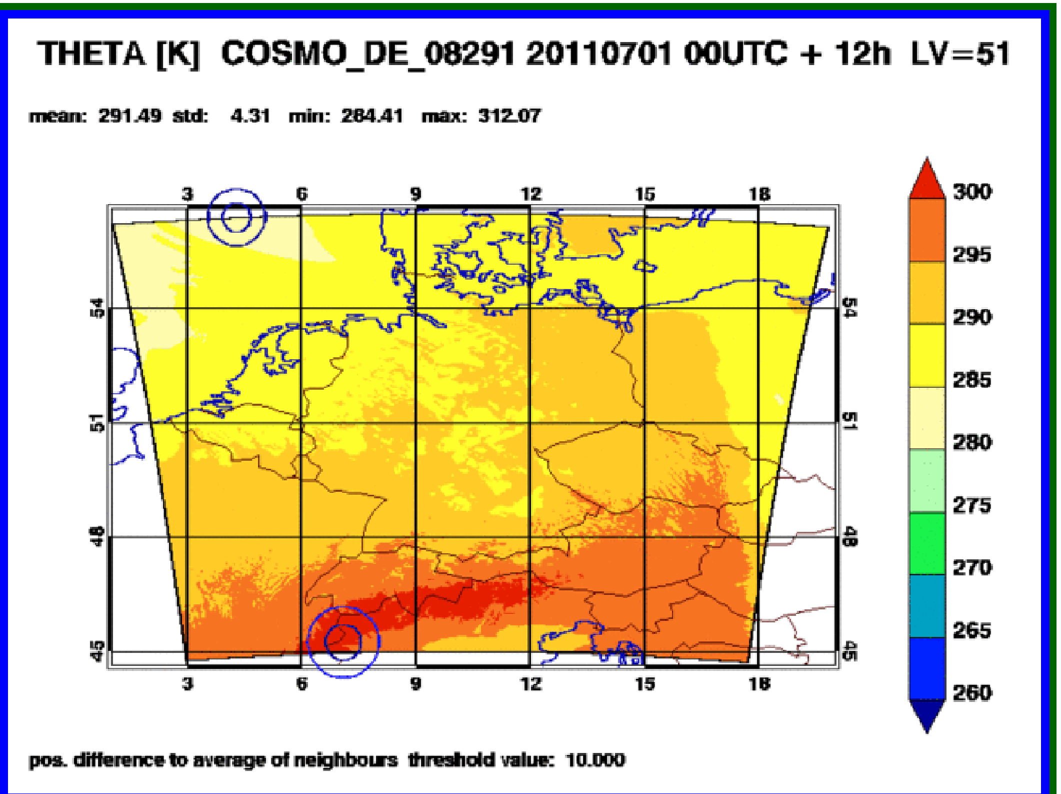
Analytical

C&E

Analytical

Reduction of „Theta-peaks“ in July 2011 (COSMO-DE)





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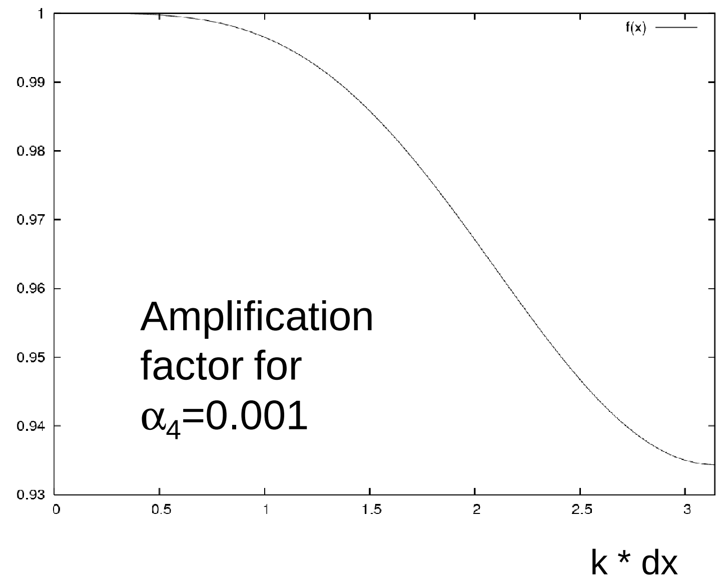
Experiment: 1.6. - ~20.7.2011

Artificial horizontal hyper-diffusion 4th order

$$\frac{\partial \phi}{\partial t} = \ldots - K_4 \Delta\Delta\phi$$

… or dim.-less diffusion coeff.: $\alpha_4 := K_4 \, \Delta t \, / \, \Delta x^4$



stable + 'non-oszillating' sinus-waves:
für $0 \le \alpha_4 \le 1/64 \sim 0.016$
($0 \le \alpha_4 \le 1/128$ for Leapfrog ($2 \Delta t$))

in COSMO-EU: $\alpha_4 = 0.25 / (2\pi^4) \approx 0.0013$ only for v
in COSMO-DE: $\alpha_4 = 0.1 / (2\pi^4) \approx 0.0005$ only for v
(and in a boundary zone for v, p', T', q_v)

- Diffusion 4th order is not monotone! → flux limitation necessary
- additional orography-limitation:
  diffus. flux=0, if slope of a coordinate plane > 250 m / Δx