

A non hydrostatic global spectral model with height based vertical coordinate: formulation and results

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A Hirlam Dynamics Project

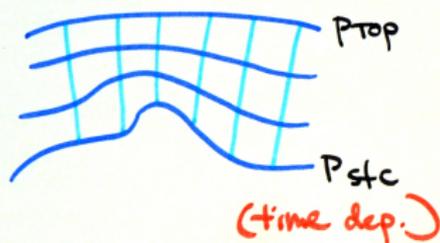
- The objective was to develop a VFE (Vertical Finite Element) scheme for the HARMONIE model
- Due to unsolvable problems in the first proposal, after some time it changed
 - from mass based to height based vertical coordinate
 - from VFE to High Order VFD (Vertical Finite Differences)
- Then we developed and tested a global version, which does not have the problem of the lateral boundary conditions
- Final results are for a global spectral model with High Order VFD using height based vertical coordinate
- The project has finished at this point

Spectral vs. Grid point

- Inherits many of the spectral method pros and cons
- Pros
 - Accurate horizontal spatial discretization
 - Efficient semi-implicit and semi-Lagrangian methods which allow big time steps
 - Exact 3D semi-implicit solver
- Cons
 - Cost of the spectral transformations
 - Low scalability
 - Poor conservative properties
 - Time stepping is at most second order

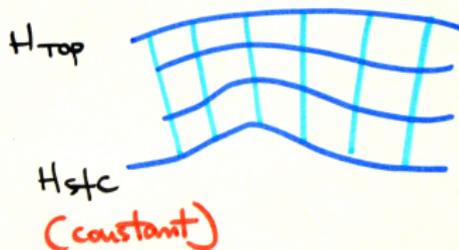
Mass vs. Height based vertical coordinates

Mass based



- Integral and diff. operators
- Constraints
- X term in the 3D divergence
- Lower BC
- SHB stability

Height based



- Only diff. operators
- No constraints
- Conservative 3D divergence
- Simple BC: $w = 0$
- Small Semi-implicit decentering ($\epsilon \sim 0.05$)

Model equations

- The model simulates fully compressible non-hydrostatic air flows on a rotating dry atmosphere
- Prognostic variables are **contravariant velocity components, logarithm of temperature and logarithm of pressure**

Covariant vs. Contravariant basis

given the vector field \vec{a}



$$\int \vec{a} \cdot d\vec{\ell} \approx a_x \delta x$$

Related to circulation, vorticity, momentum eq.

↑ covariant component



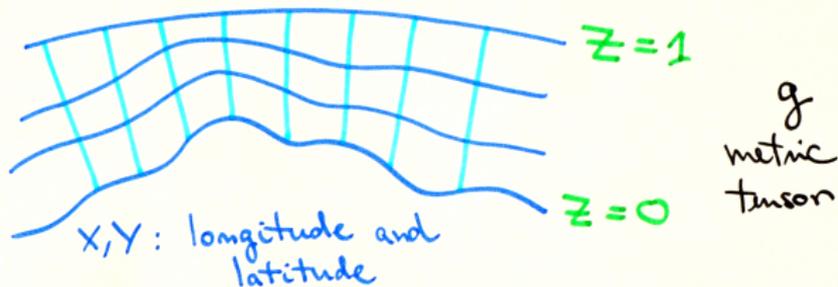
$$\int \vec{a} \cdot d\vec{s} \approx a^x |g|^{1/2} \delta y \delta z$$

Related to flux, conservation, continuity eq.

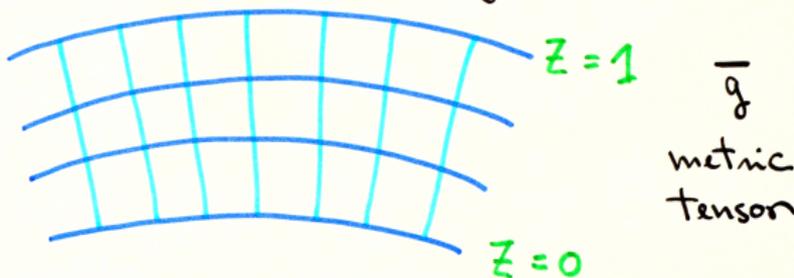
↑ contravariant component

Model coordinates and domain

Nonlinear Model Geometry



Linear Model Geometry

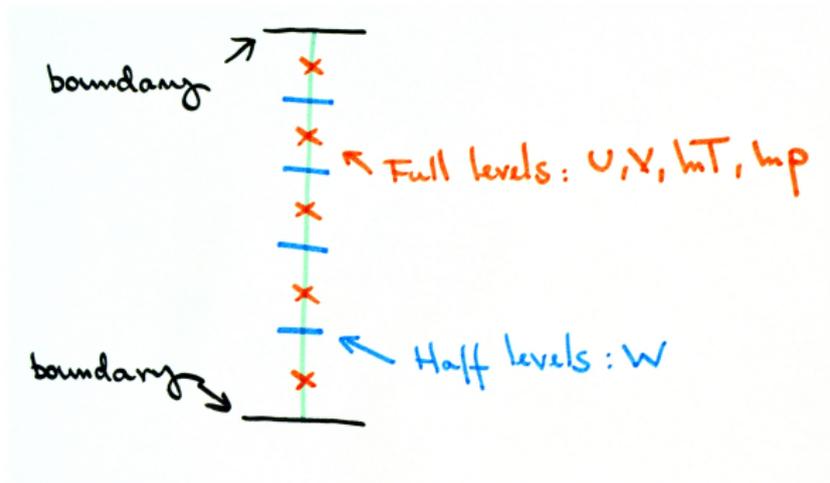


Model coordinates and domain

- Cartesian coordinates fixed to the Earth (x, y, z) are transformed into model coordinates (X, Y, Z) being X and Y the geographical longitude and latitude and Z the terrain-following vertical coordinate
- The relationship between both coordinates is analytical and constant in time
- Surface $Z = 0$ is the bottom boundary or Earth surface whereas surface $Z = 1$ is a rigid spherical surface which represents the top of the atmosphere
- metric tensor is calculated in the model coordinates to find differential operators like divergence, gradient and covariant derivative

Vertical discretization

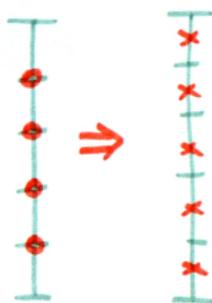
- Here a **High Order VFD** version is presented in 2D planar and 3D spherical geometries
- The prognostic variables are all in **N full levels** except the contravariant vertical velocity which is in **$N - 1$ half levels** plus two boundary levels where it is zero



Vertical discretization

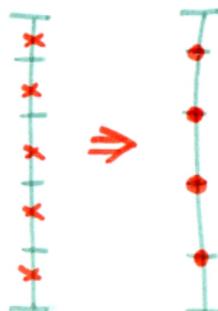
- Vertical **derivative** and vertical **interpolation** operators
- If the input field is given in half levels the result is in full levels and viceversa
- The operators take into account **whether the field is supposed to be zero or not at the boundaries**
- The **stencil** of the operators depends on the order

Half to full



\hat{D}, \hat{I}

Full to half



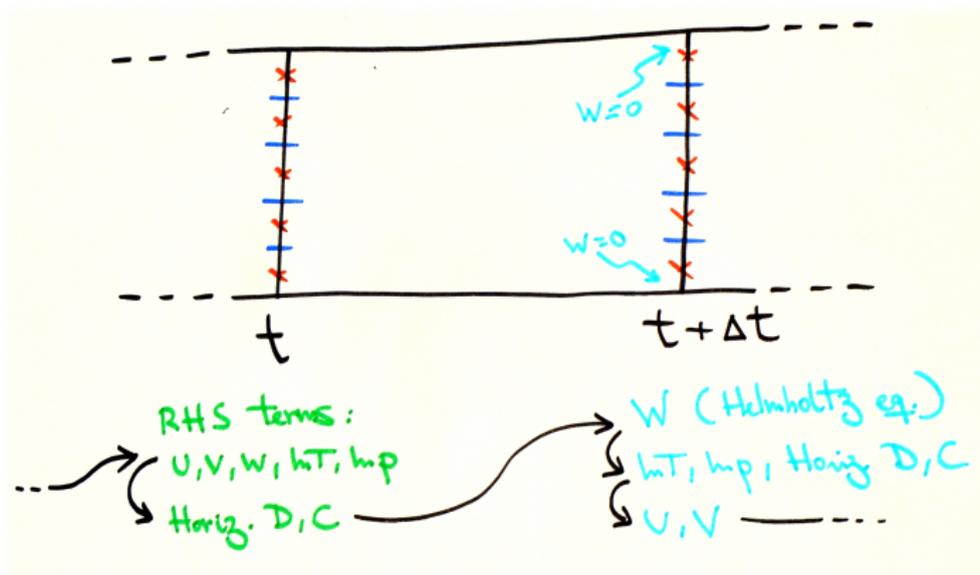
D, I

Semi-implicit time discretization

- The semi-implicit formulation follows closely the formulation used in ALADIN with the **mass-based vertical coordinate**
- The equations are linearized around an **isothermal hydrostatic balanced atmosphere at rest** with **flat orography**
- It is found a **structure equation** for finding the value of the **contravariant vertical velocity** at the next time step

Semi-implicit time discretization

- There is **no boundaries conditions** at next time step between unknown horizontal velocity and vertical velocity at: $W = 0$ at any time
- Horizontal divergence and curl operators **decouple horizontal momentum equations**, similar to Temperton (1991)

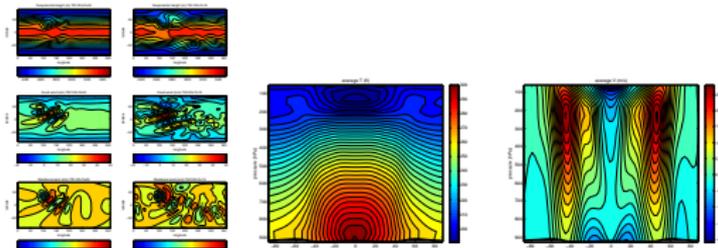
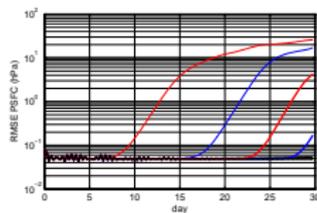
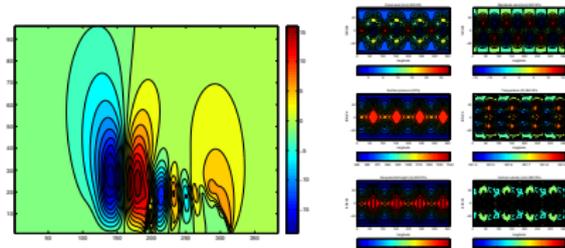
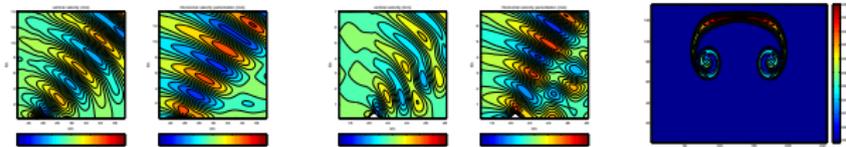


Semi-implicit time discretization

- Divergence in the non linear model is expressed as the sum of the divergence in the linear model plus a geometric constant in time advection term

$$\underbrace{\vec{\nabla} \cdot \vec{V}}_{\substack{\text{nonlinear} \\ \text{model} \\ \text{3D divergence}}} = \underbrace{\vec{\nabla} \cdot \vec{V}}_{\substack{\text{linear} \\ \text{model} \\ \text{3D divergence} \\ \uparrow \\ \text{IMPLICIT}}} + \underbrace{\frac{d}{dt} \left(\ln \frac{|\mathbf{g}|^{\frac{1}{2}}}{|\bar{\mathbf{g}}|^{\frac{1}{2}}} \right)}_{\substack{\text{metric advection} \\ \text{term} \\ \uparrow \\ \text{Eulerian / Semi-Lagr.}}}$$

Next section some 2D and 3D tests



2D Tests

Linear non-hydrostatic flow (Bubnová et al. 1995)

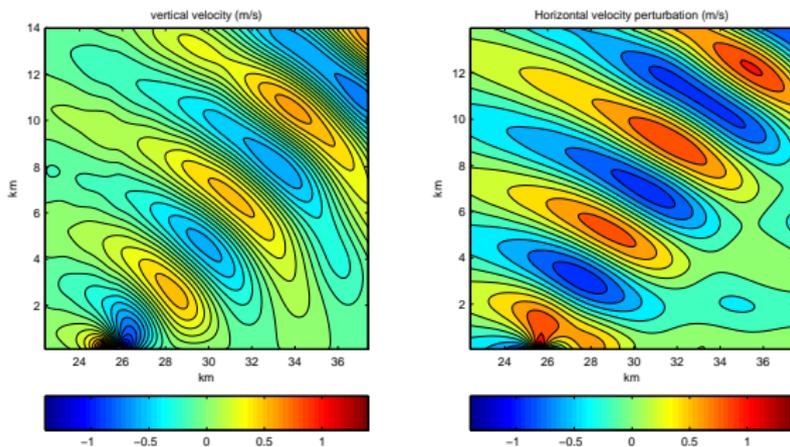


Figure: Vertical and horizontal velocities

2D Tests

Non-linear non-hydrostatic flow (Bubnová et al. 1995)

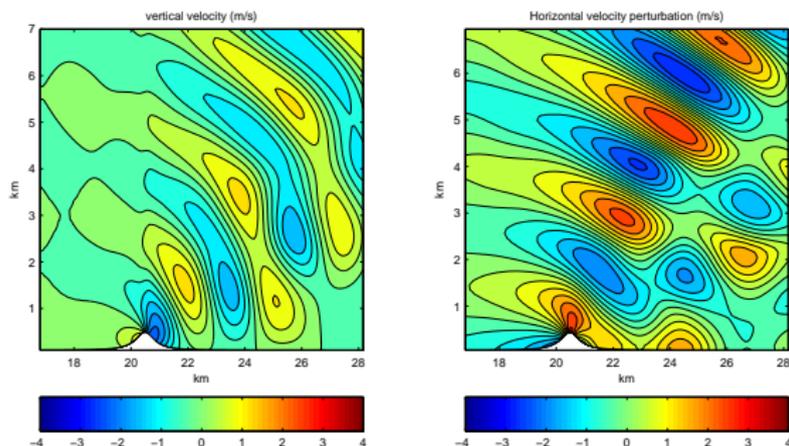


Figure: Vertical and horizontal velocities

2D Tests

Warm bubble (Janjic et al. 2001)

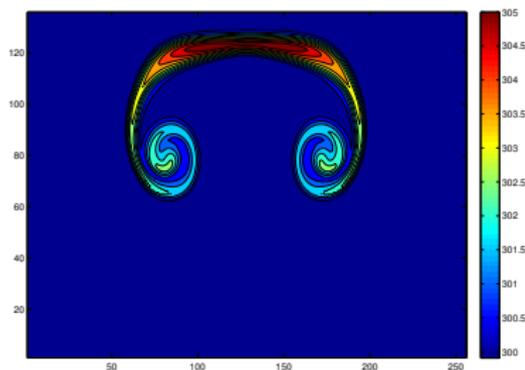


Figure: Potential temperature

Cold bubble (Straka et al. 1993)

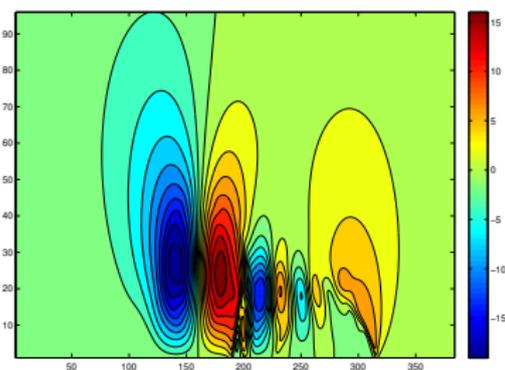


Figure: Vertical velocity

3D Tests

3D Rossby-Haurwitz wave (Jablonowski et al. 2008)

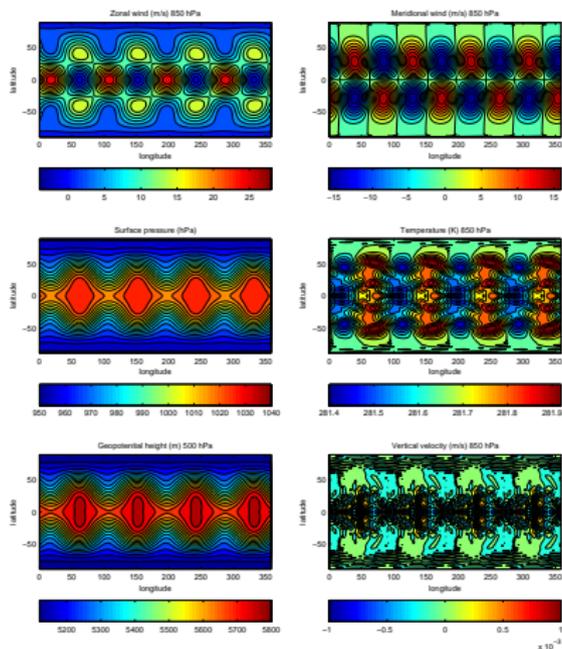


Figure: Integration at day 15.

3D Tests

Steady Jablonowski baroclinic instability rotated test: the flow should stay steady indefinitely (Jablonowski and Williamson 2006)

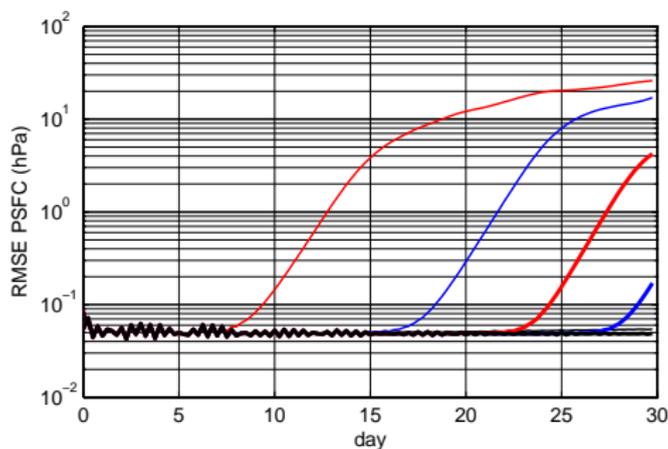


Figure: RMSE of surface pressure for rotated angles of 0° (black), 45° (red) and 90° (blue). Bold lines and thin correspond to T85 and T42 respectively

3D Tests

Mountain induced Rossby wave train: flow across a 2000 *m* height mountain (Jablonowski et al. 2008)

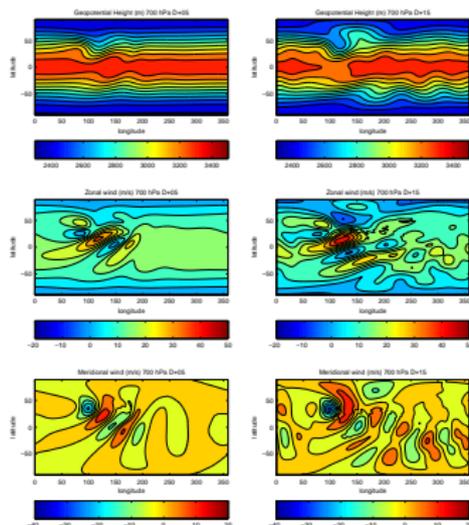


Figure: 2000 *m* mountain induced Rossby wave train at day 5 and 15: geopotential height, zonal and meridional wind at 700 *hPa*

3D Tests

Held-Suarez climate (Held and Suarez 1994)

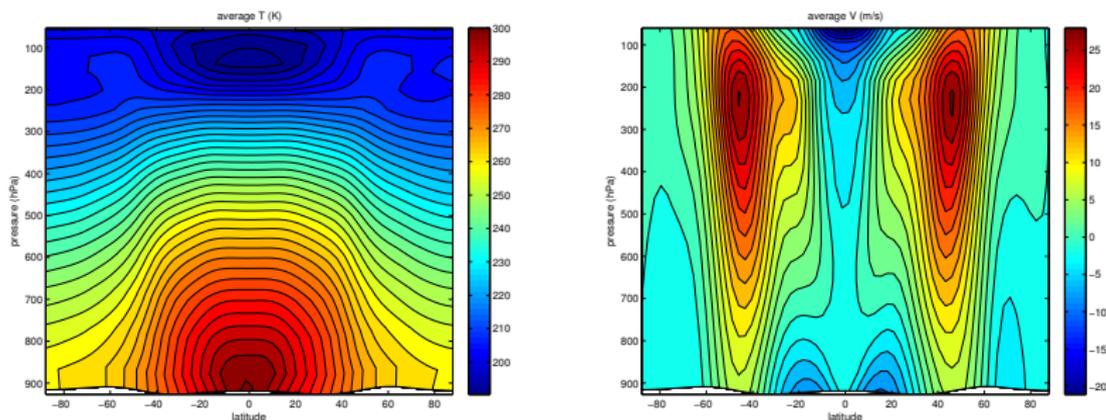
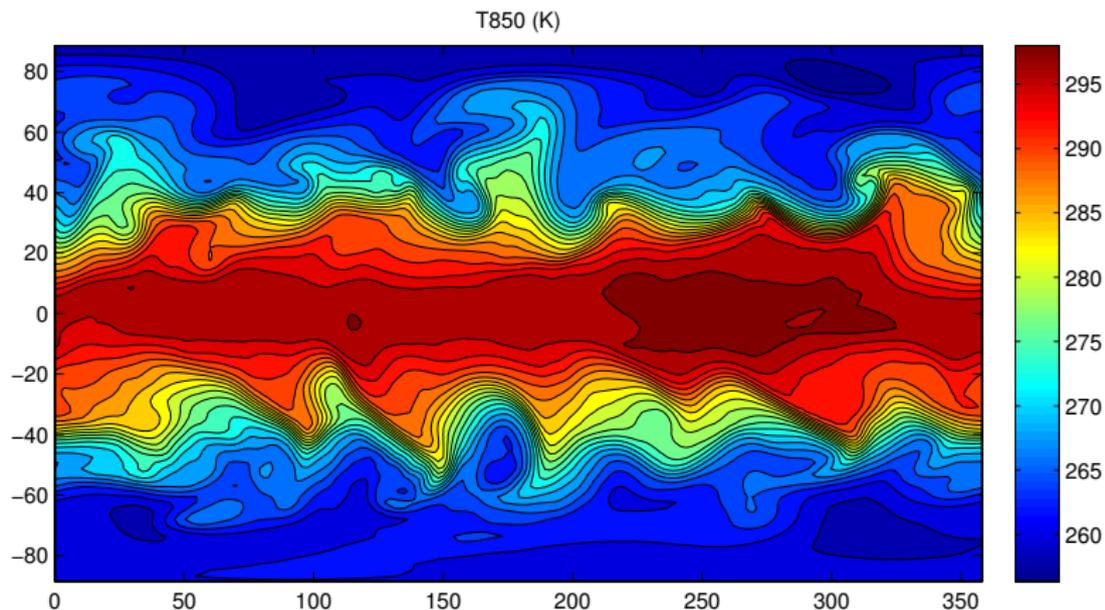


Figure: Held-Suarez test. Mean temperature (K) and mean meridional velocity (m/s) for a 800 days experiment at T42 and 20 vertical levels

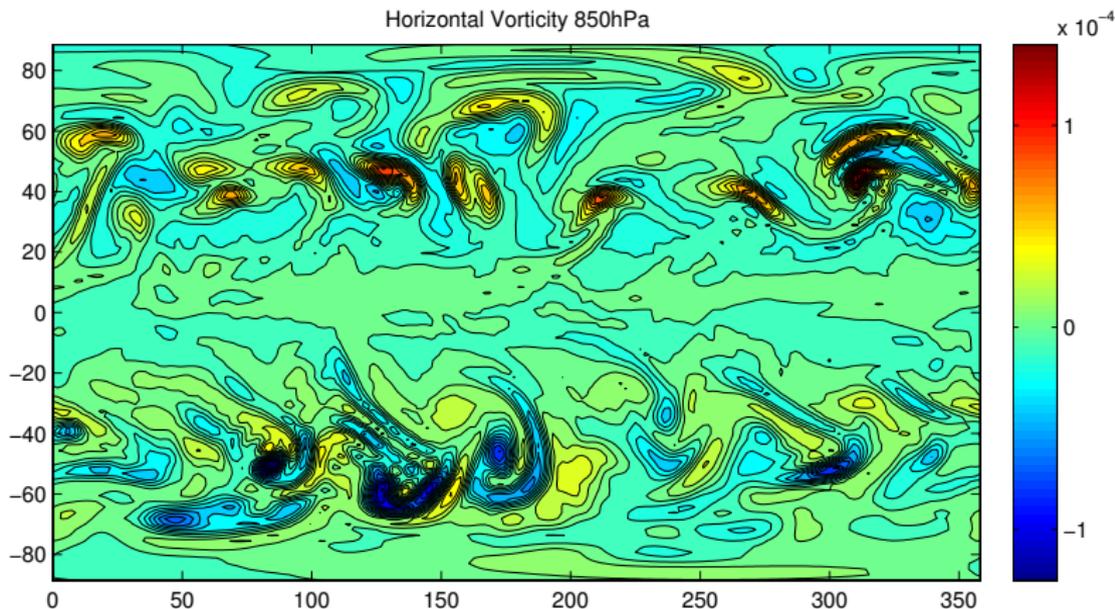
3D Tests

A Held-Suarez planet day (T63)



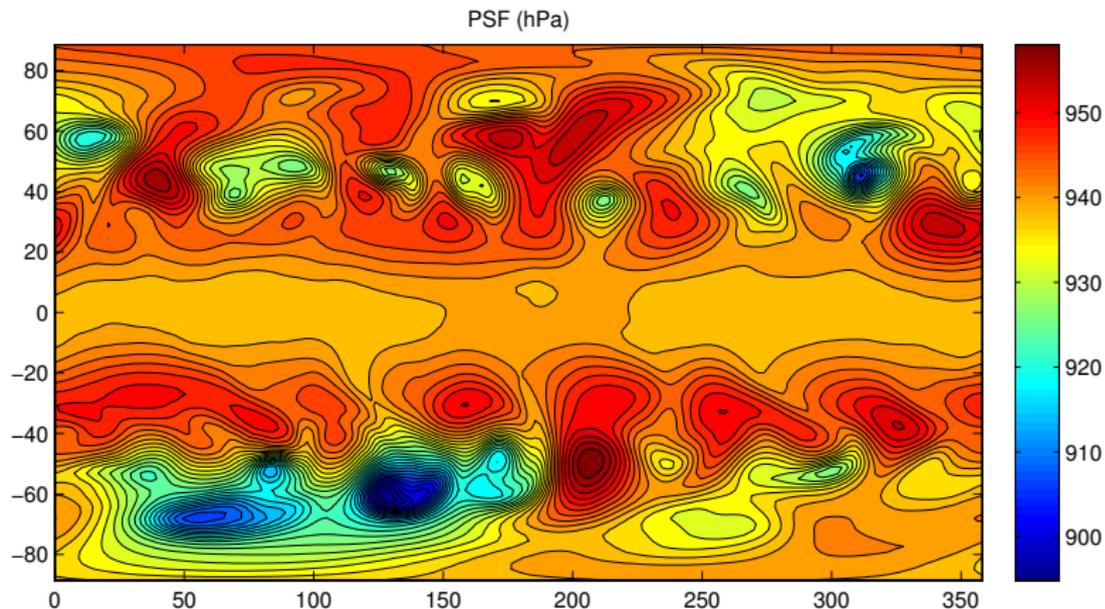
3D Tests

A Held-Suarez planet day (T63)



3D Tests

A Held-Suarez planet day (T63)



Main conclusions of the project

- Implementation of published **2D and 3D tests**: good results in both non-hydrostatic and hydrostatic regimes
- Contravariant velocity: allows **simple boundary conditions** and compact or **conservative expression of the velocity divergence**: this result can be applied to grid point models as well
- **Steep orography and stability**: High Order VFD is competitive with respect other models, it is not the case of the VFE version, which is less stable
- Mentioned **Pros and Cons of the spectral method** in high resolution modelling
- Mentioned **Pros and Cons of mass and height based vertical coordinates**
- Results are good and the scheme **seems to be suitable for another global spectral non-hydrostatic model**

Thank you for your attention



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Mediterranean Sea from Castelldefels (Spain)