A Limited Area Stochastic Pattern Generator (with an application in COSMO)

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Model (tendency) error simulation

- Goal: Develop a universal tool for model error simulation in LAM EPS and LAM EDAs.
- Status: Now, we have a stochastic pattern generator (SPG) of univariate Gaussian spatio-temporal fields on 2-D and 3-D limited area spatial domains with **meaningful and tunable structure**.
- Current research: Extension of the SPG to the non-Gaussian humidity and cloud fields *(using the univariate Gaussian SPG as a building block)*.

Motivation

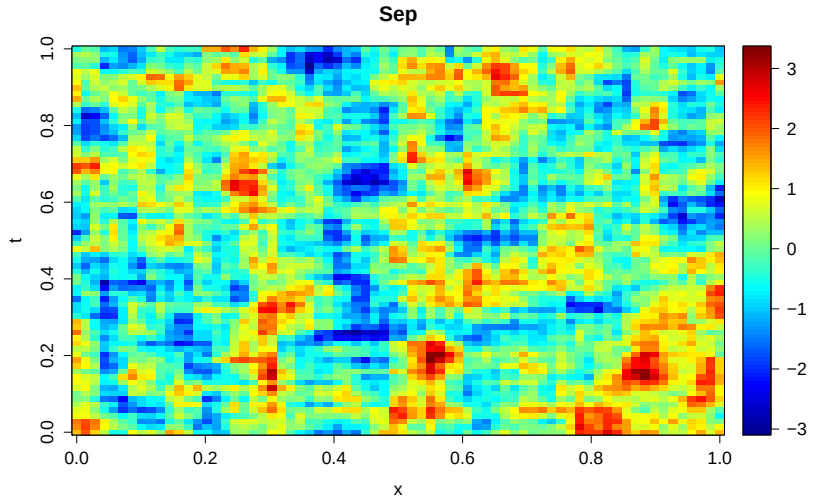
The existing pattern generators for ensemble applications produce *separable* space-time correlations:

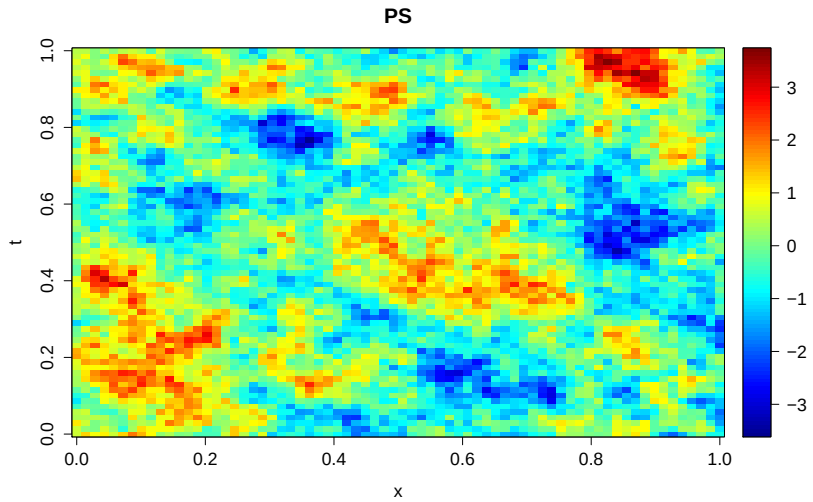
$$C(t, \mathbf{s}) = C_t(t) \cdot C_{\mathbf{s}}(\mathbf{s})$$

But: no space-time interactions.

In reality, longer spatial scales 'live longer' than shorter spatial scales, which 'die out' quicker. This 'proportionality of scales' is widespread in geophysical fields (Tsyroulnikov QJRMS 2001) and other media.

Separable *vs.* non-separable correlation models





Our approach: Linear evolutionary stochastic partial differential equations

1. Flexibility (local inhomogeneity, non-stationarity, non-Gaussianity).
2. Sparse matrices $\Rightarrow$ fast computations.

Formulation

Domain: the cube with cyclic boundary conditions.

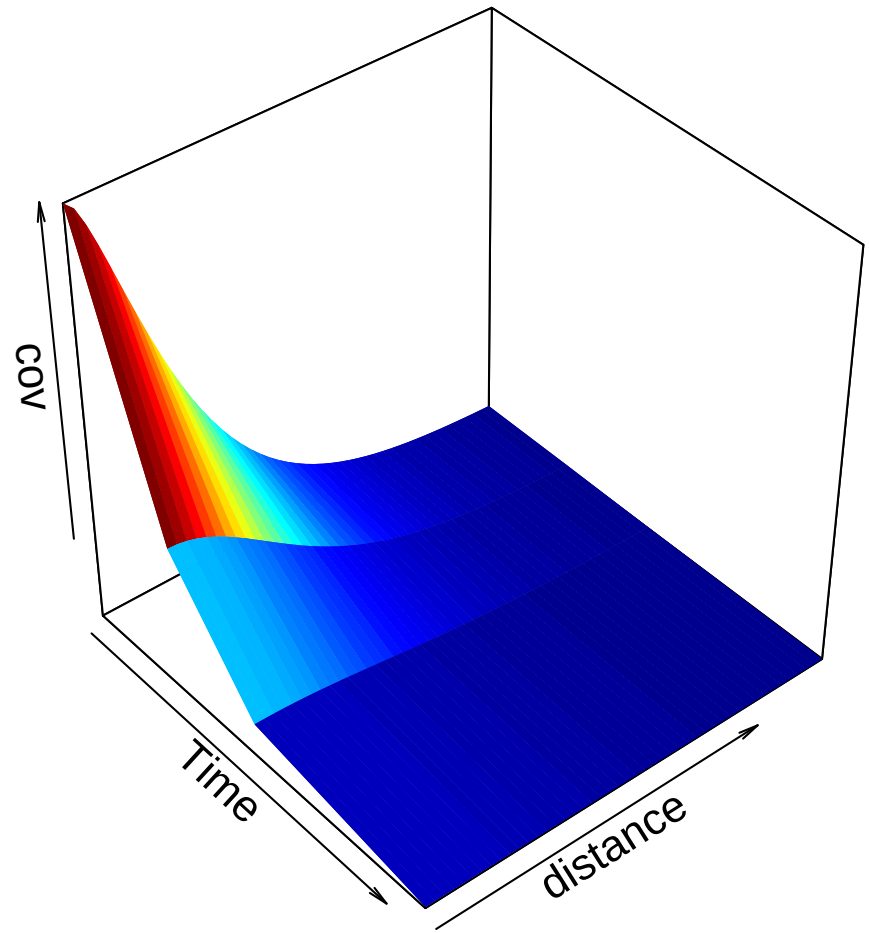
$$\left(\frac{\partial}{\partial t} + \frac{U}{\lambda}\sqrt{1-\lambda^2\Delta}\right)^3 \xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})$$

– t is time, $\mathbf{s}$ is the spatial vector
– α is the white driving noise
– ξ is the output random field

- σ controls the variance
- λ controls the spatial scale
- U controls the temporal scale

The numerical scheme is spectral in spatial coordinates and finite-difference in time.

Spatio−temporal covariances



Ranges: t=0 .. 12 h, r=0 .. 750 km

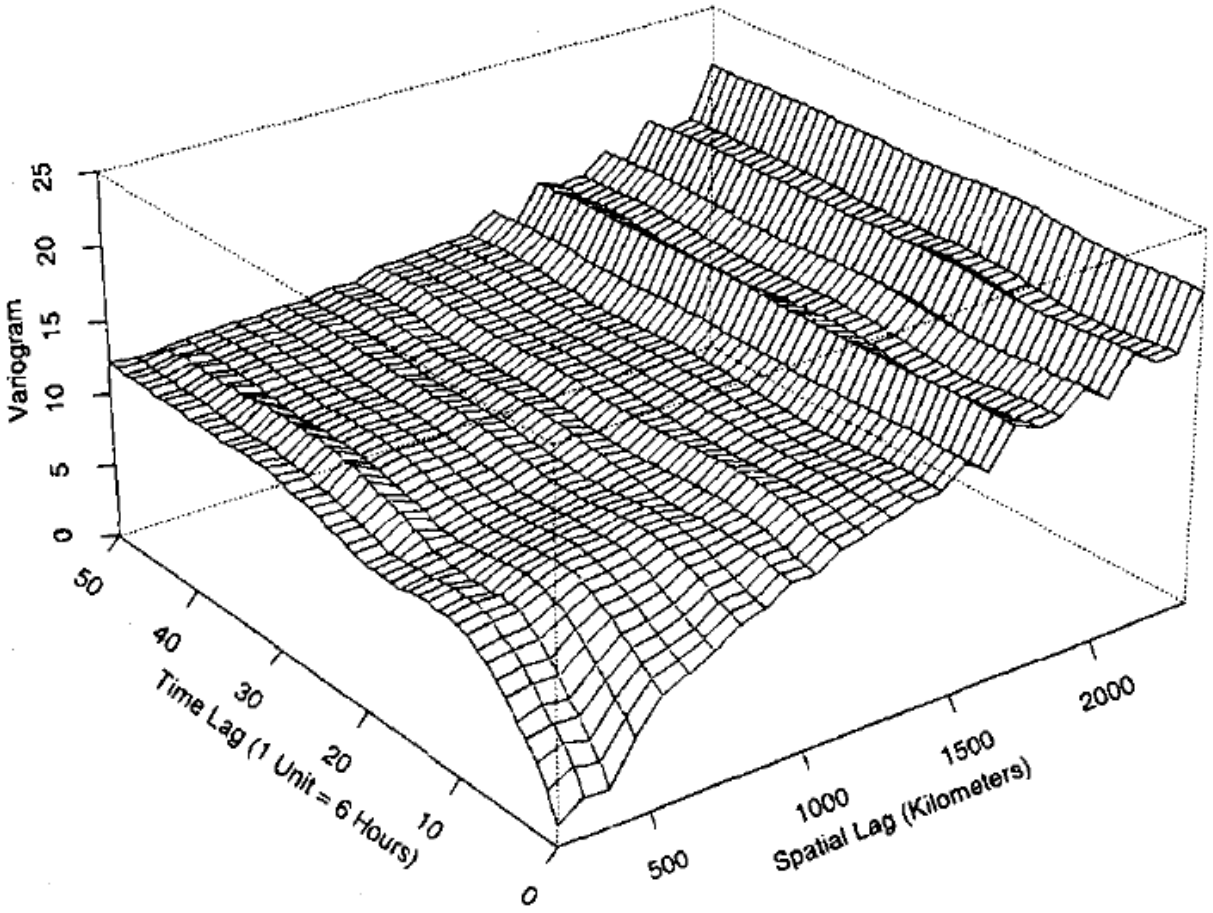
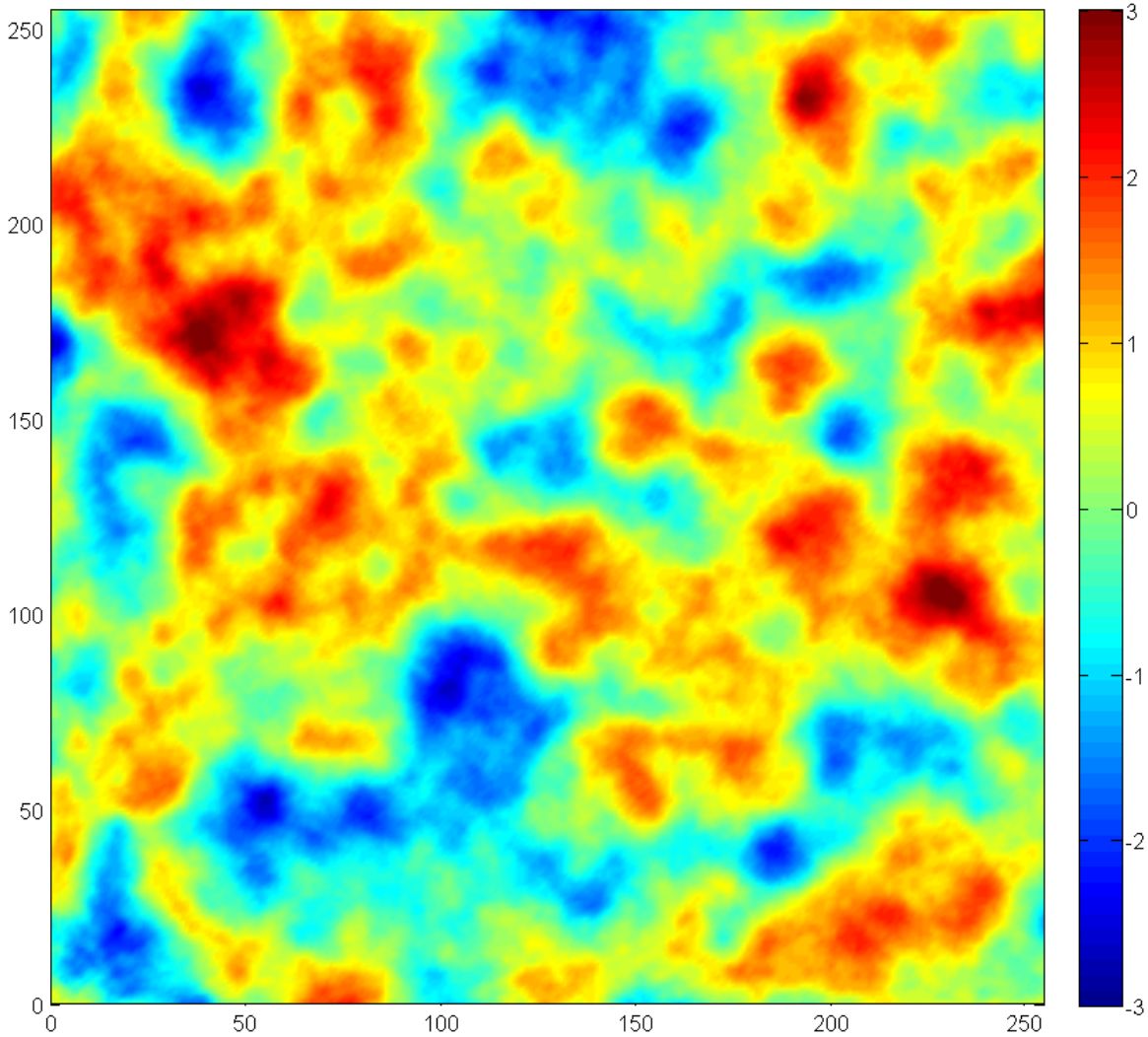


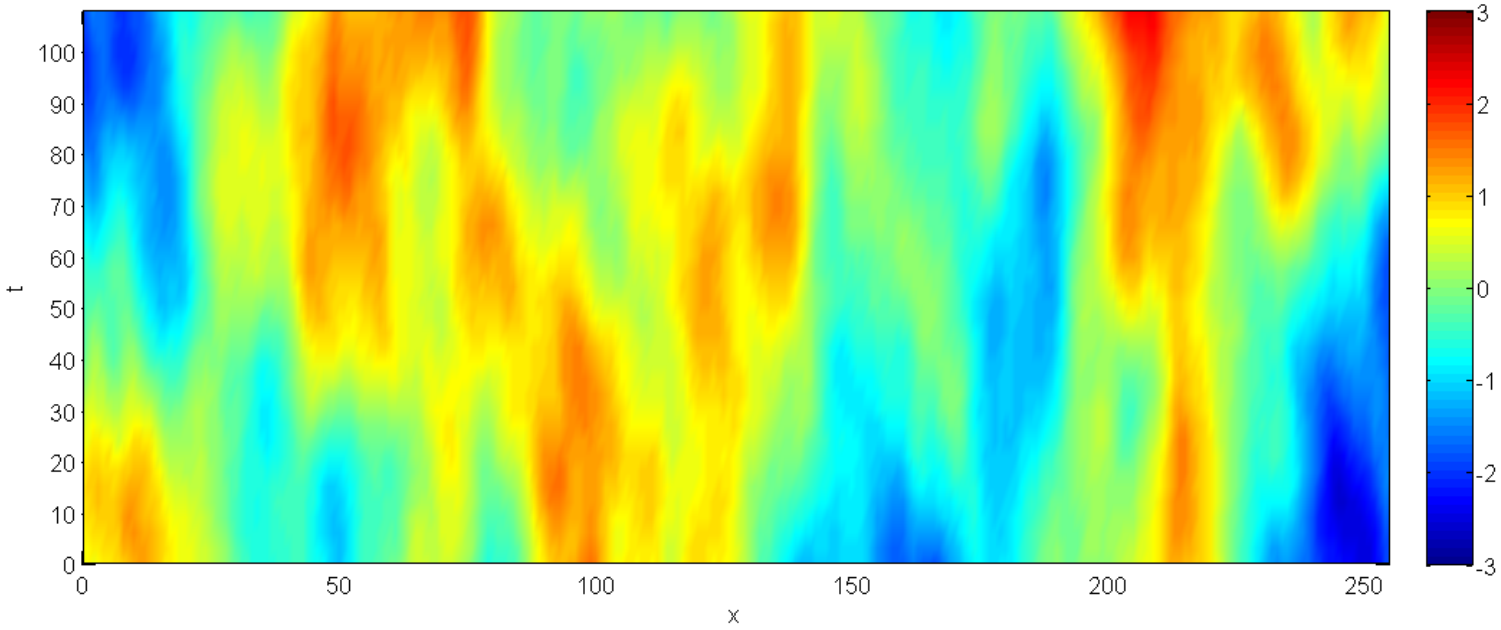
Figure 8. Empirical Spatio-Temporal Variogram Evaluated at Spatio-Temporal Lags $\{\mathbf{h}(1), \ldots, \mathbf{h}(27)\} \times \{0, 1, \ldots, 50\}$.

From (Cressie and Huang 1999): empirical spatio-temporal wind-speed variogram.

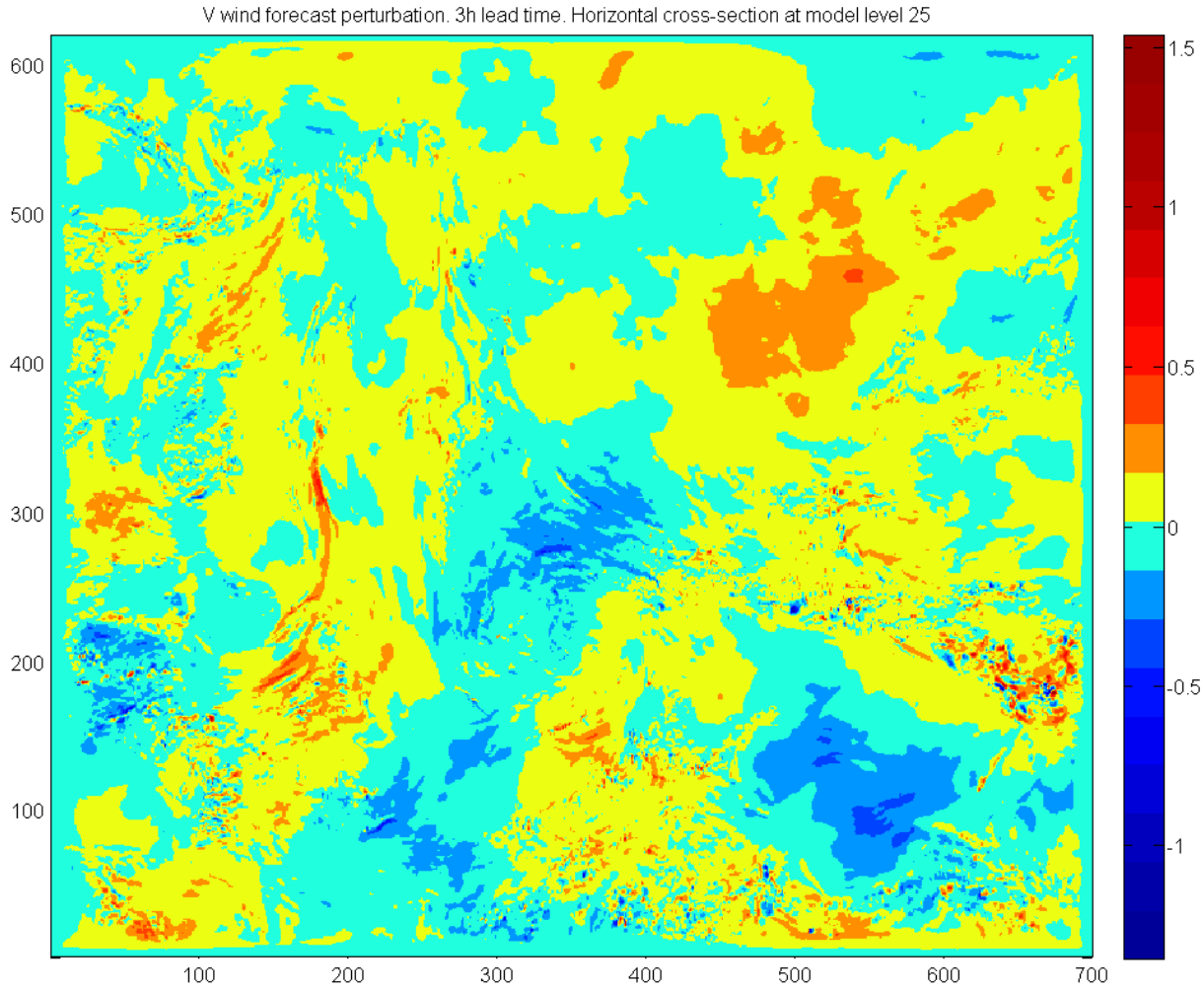
Spatial field (xy)



Space-time plot (xt)



Application with the COSMO model.
The forecast V perturbation at at $t = 3$ h.
(T, p, u, v fields were perturbed every 15 min)



Conclusions

The SPG produces 2-D and 3-D Gaussian pseudo-random fields on a limited area domain with non-separable spatio-temporal correlations.

1. Advantages of the SPG
   - Realistic space-time covariances, proportionality of scales.
   - Easily tunable spatial and temporal length scales.
   - Quite fast computations.
2. Application areas
   - Model error perturbations.
   - Initial and boundary-condition perturbations.
   - Soil perturbations.

References

= The **Fortran code** of the standalone SPG is **freely available** from github.com/gayfulin/SPG.

= Tsyrulnikov M. and Gayfulin D. A Stochastic Pattern Generator for ensemble applications. – COSMO Tech. Rep. N29, 2016 (available from the COSMO web site or from ResearchGate).

= Tsyrulnikov M. and Gayfulin D. A limited-area spatio-temporal stochastic pattern generator for ensemble prediction and ensemble data assimilation. – Meteorol. Zeitschrift, 2016 (under review).