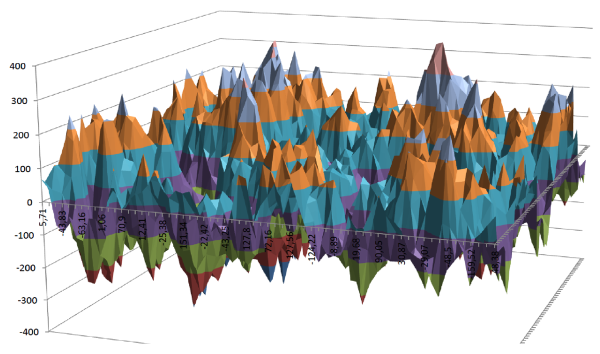

A Spatio-Temporal Limited Area Stochastic Pattern Generator (with an application in COSMO)

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Motivation

- Model errors are an important source of uncertainty in NWP and thus need to be simulated in ensemble prediction and ensemble data assimilation.
- In the stochastic approach, model errors are postulated to satisfy a stochastic spatio-temporal model error model (which can be additive, multiplicative, or other).
- The model error model requires a spatio-temporal random field as a stochastic input.
- The generation of pseudo-random 3-D and 2-D spatio-temporal fields with realistic and tunable structure is the purpose of the SPG.

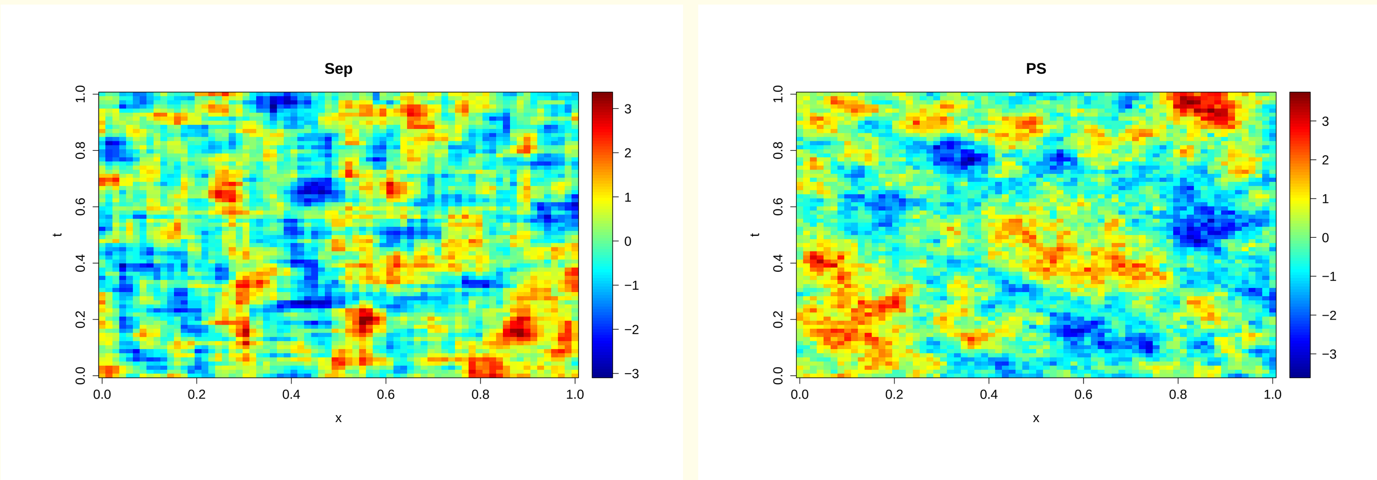
Separability of spatio-temporal correlations

The existing approaches all produce *separable* space-time correlations:

$$C(t,\mathbf{s}) = C_t(t) \cdot C_{\mathbf{s}}(\mathbf{s})$$

But: no space-time interactions.
In reality, longer spatial scales 'live longer' than shorter spatial scales, which 'die out' quicker. This "proportionality of scales" is widespread in geophysical fields (Tsyroulnikov QJRMS 2001) and other media.

Separable *vs.* non-separable correlation models



Approach: Stochastic partial differential equations

1. Flexibility (local inhomogeneity, non-stationarity, non-Gaussianity).
2. Sparse matrices ⇒ fast computations.

Formulation

• Starting point:

$$\frac{\partial \xi(t,\mathbf{s})}{\partial t} + A\,\xi(t,\mathbf{s}) = \alpha(t,\mathbf{s})$$

= t is time, s is the spatial vector
= α is the white driving noise
= A is the spatial operator.
= ξ is the output random field

•To achieve spatial isotropy, we specify $A = \mu(1-\lambda^2\Delta)^q$, where Δ is the spatial Laplacian and q the spatial order.

• Impose the "proportionality of scales": for $k \to \infty$, the temporal scale τ_k is proportional to the spatial scale k^{-1}. This leads to the model

$$\left(\frac{\partial}{\partial t} + \mu\sqrt{1-\lambda^2\Delta}\right)^3 \xi(t,\mathbf{s}) = \sigma\,\alpha(t,\mathbf{s})$$

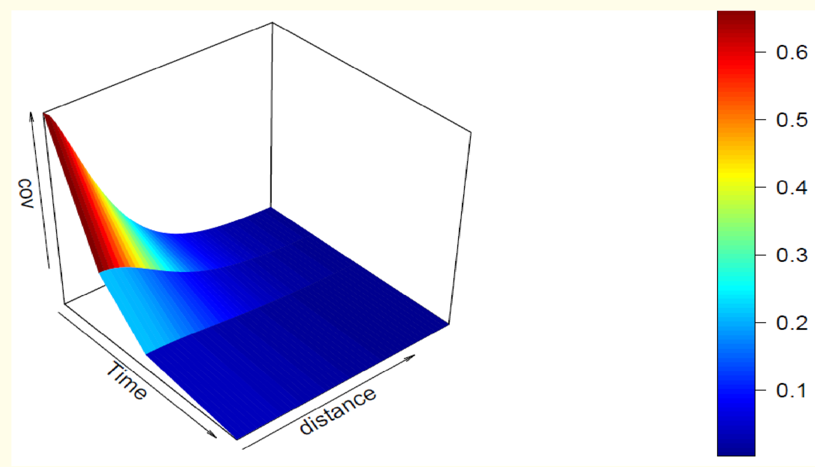
σ controls the variance
λ controls the spatial scale
μ controls the temporal scale

Numerical scheme

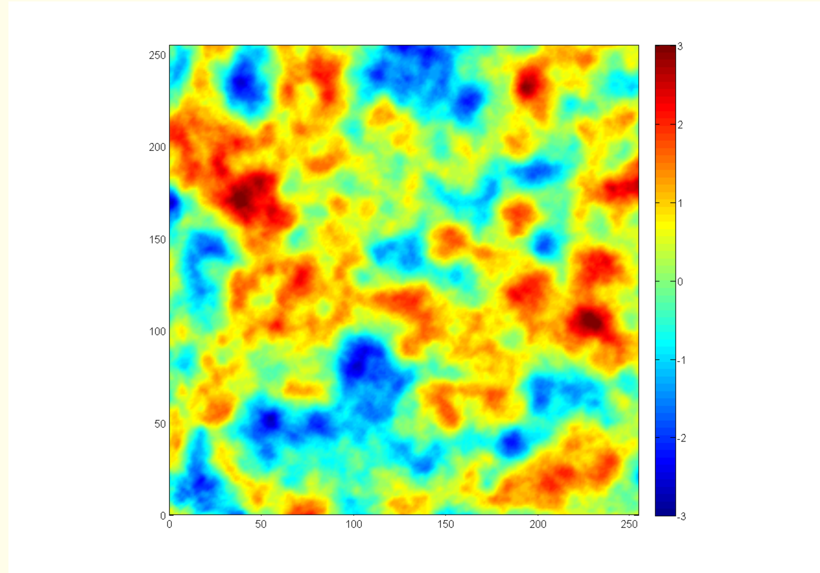
= A spectral (Fourier) in space and finite-difference in time solver.
= An implicit finite difference scheme in time.
= The time step depends on the wavenumber.
= To speed up the computations, the time integration is accomplished for wavevectors on a *coarse* grid in spectral space.

The 2D-in-space SPG takes 1 sec on one CPU to perform the spectral-space integration (the inverse FFT not accounted for) for the 300 × 300 spatial grid and 100 hours of lead time. The 3D SPG (on the 300 × 300 × 64 grid): 60 sec.

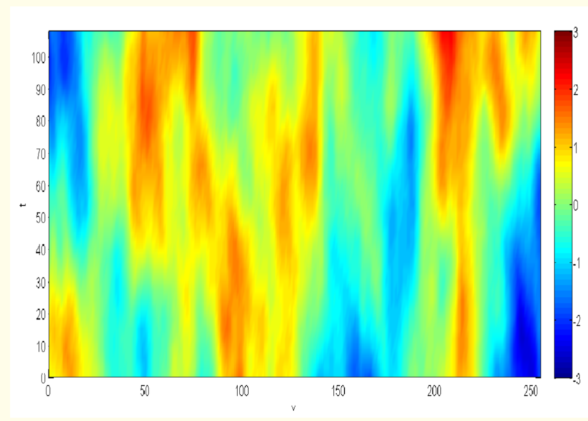
Spatio-temporal covariances



Spatial field (xy)

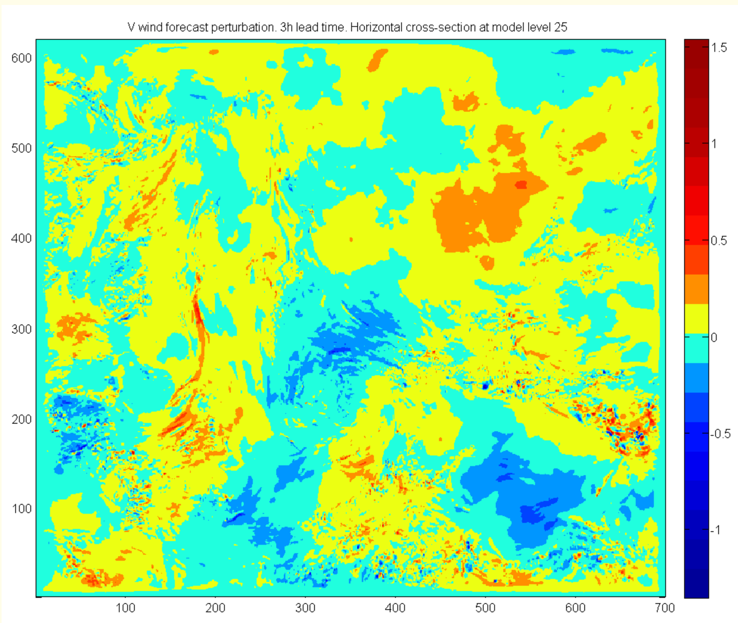


Space-time plot (xt)



Application with the COSMO model

The forecast V perturbation at at $t = 3$ h.
Independent additive 3-D SPG generated perturbations to the T, u, v fields (and the hydrostatically balanced pressure perturbation) were added every 15 min of the COSMO model time integration



Conclusions

The SPG produces 2-D and 3-D Gaussian pseudo-random fields on a limited area domain with non-separable spatio-temporal correlations.

1. Advantages of the SPG
   - Realistic space-time interactions, proportionality of scales.
   - Easily tunable spatial and temporal length scales.
   - Fast computations.
2. Application areas
   - Model error perturbations.
   - Initial and boundary-condition perturbations.
   - Soil perturbations.

= The *Fortran code* of the standalone SPG is freely available from **github.com/gayfulin/SPG**.

= Tsyrulnikov M. and Gayfulin D. A Stochastic Pattern Generator for ensemble applications. – **COSMO Tech. Rep. N29**, 2016 (available from the COSMO web site or from ResearchGate).

= Tsyrulnikov M. and Gayfulin D. A limited-area spatio-temporal stochastic pattern generator for ensemble prediction and ensemble data assimilation. – **Meteorol. Zeitschrift**, 2016 (under review).