Improving the robustness of the Constant-Coefficient ICI scheme of IFS/ARPEGE and AROME models

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Plan

Constant-coefficient Iterative-Centered-Implicit (ICI) scheme

$$\frac{\partial \mathcal{X}}{\partial t} = \underbrace{(\mathcal{M} - \mathcal{L})(\mathcal{X})}_{\textit{NL-residual}} + \underbrace{\mathcal{L}\mathcal{X}}_{\textit{Linear}}$$

- For $i \in [0, N_{\text{siter}} - 1]$

$$\frac{\mathcal{X}^{+(i+1)} - \mathcal{X}^0}{\delta t} = \frac{(\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^{+(i)}\right) + (\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^0\right)}{2} + \frac{\mathcal{L}^*.\mathcal{X}^{+(i+1)} + \mathcal{L}^*.\mathcal{X}^0}{2},$$

1. **Quasi-Newton-Raphson approach** : $[I - (\delta t/2)\mathcal{L}^*]$ plays as a preconditioner $\Rightarrow \mathcal{L}^*$ is typically chosen as a linear counterpart of $\mathcal{M}$ around a reference-state $\mathcal{X}^*$.
2. **Constant-coefficient assumption :** The coefficients of $\mathcal{L}$ are taken constant in time and along horizontal directions $\Rightarrow$ Make easier the inversion of $[I - (\delta t/2)\mathcal{L}^*]$.

Major concerns about Constant-coefficient approach :

Stability strongly depends on the magnitudes of the non-linear residual terms $(\mathcal{M} - \mathcal{L}^*)$. Such issues are more significant in NH models [Bénard *et al.* (2003,2005)].

1. **Thermal NL residuals** due to discrepancy between the actual temperature and the constant temperature of the SI reference state T^*.
2. **Baric NL residuals** due to the discrepancy between actual hydrostatic surface pressure and the constant SI surface pressure π_s^*.
3. **Orographic NL residuals** due to the presence in of horizontally-varying terrain-following metric terms involving the orography slope, not taken into account in the SI reference state.

already proven and new stabilizing parameters in SI:

1. **SITRA** : An extra acoustic temperature T_a^* introduced by Bénard (2004) in the SI linear to deal with the detrimental impact of Thermal NL residual term due to elastic term of the NH system.
2. **SIPRA**: Introduction of an acoustic SI hydrostatic surface pressure $\pi_{s,a}^*$ in order to stabilize the model above very high orography (e.g, Himalaya, Andes,...) where the amplitude of the baric NL residual terms starts to be dramatically significant.
3. **SISLP**: Introduction of the maximum orography slope Λ_s^* over the domain in the formulation of the discrete vertical Laplacian-like operator in the SI linear model.

Motivations :

- Improve the stability of the Constant-Coefficient Implicit-Centred-Iterative (CC-ICI) time discretisation of the mass-based EE system in presence of very high mountains and steep slopes allowed by higher horizontal resolution.

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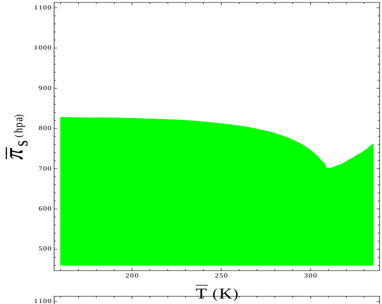
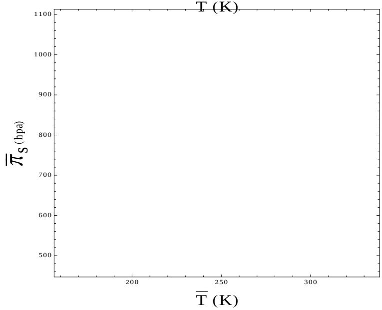
An acoustic SI hydrostatic surface pressure





Stability analyses outcomes :

- **Bad news** : Fully compressible with PC exhibits a wide region where Baric NL residuals are detrimental for stability (panel at top).
- Very good news : Hydrostatic with Predictor-Corrector does not suffer from the effect of Baric NL residuals (panel at bottom).

SIPRA :

- **Basic idea :** Introduction of an extra SI parameter for surface hydrostatic pressure in the NH part of $\mathcal{L}^*$ as :

$$\mathcal{L}^* = \mathcal{L}^*_{\mathrm{H}}[T^*, \pi_s^*] + \mathcal{L}^*_{\mathrm{NH}}[T^*, T_a^*, \pi_{a_s}^*]$$

- **How to perform that H/NH splitting ?**

- Current SI linear system :

$$\frac{\partial D}{\partial t} = -R\nabla^2 \left[\mathcal{G}^*(T - T^*\hat{q}) + T^*(\hat{q} + q_s)\right]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{R T_a^*}\mathcal{L}_v^*(\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{RT^*}{C_v}[D + d]$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v}[D - (1-\kappa)\mathcal{S}^*(D) + d]$$

$$\frac{\partial q_s}{\partial t} = -\mathcal{N}^*(D)$$

Introduction of the auxiliary variable $\widehat{T} = T - \kappa T^* \hat{q}$

Hydrostatic part :

$$\frac{\partial D}{\partial t} = -R\nabla^2\left[\mathcal{G}^*\left(\widehat{T}\right) + T^* q_s\right],$$

$$\frac{\partial \widehat{T}}{\partial t} = -\kappa T^* \mathcal{S}^*(D),$$

$$\frac{q_s}{\partial t} = -\mathcal{N}^*(D)$$

NH part :

$$\frac{\partial D}{\partial t} = -RT^*\nabla^2[\hat{q} - (1-\kappa)\mathcal{G}_a^*(\hat{q})]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_a^*}\mathcal{L}{v}_a^*(\hat{q})$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v}[D - (1-\kappa)\mathcal{S}_a^*(D) + d]$$

H/NH splitting

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NH part :

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$$\frac{\partial d}{\partial t} = -\frac{g^2}{RT_a^*}\mathcal{L}_{v\,a}^{\,*}(\hat{q})$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v}\left[D - (1-\kappa)\mathcal{S}_a^*(D) + d\right]$$

New SI linear system with SIPRA:

$$\frac{\partial D}{\partial t} = -R\nabla^2 \left[\mathcal{G}^* \left(T - \kappa T^* \hat{q}\right) T^* q_s\right] - RT^* \nabla^2 \left[\hat{q} - (1-\kappa)\mathcal{G}_a^*(\hat{q})\right]$$

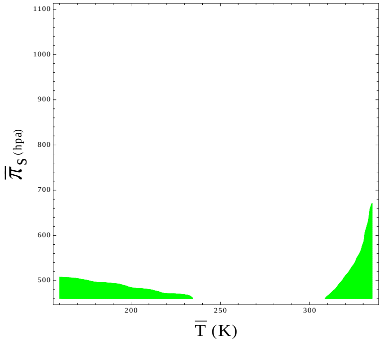
$$\frac{\partial d}{\partial t} = -\frac{g^2}{R T_a^*} \mathcal{L}_{v}{}_a^* (\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{RT^*}{C_v} [D + d] - \kappa T^* \left[\mathcal{S}^* - \mathcal{S}_a{}^*\right](D)$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v} \left[D - (1-\kappa)\mathcal{S}_a{}^*(D) + d\right]$$

$$\frac{\partial q_s}{\partial t} = -\mathcal{N}^*(D)$$

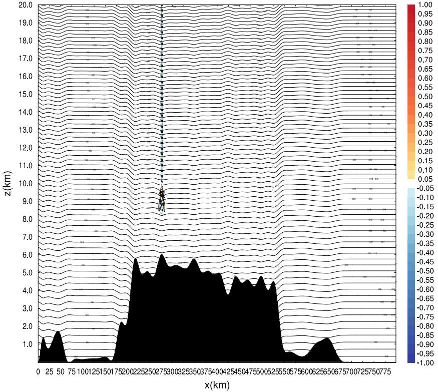
- Stability analyses with SIPRA :



3D No-Flow test over Himalaya plateau, $H_{max} = 6030m$**,** $S_{max} = 15^\circ$

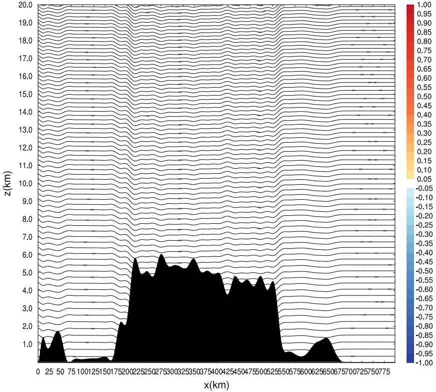
AROME (1km, L137, 30s, NSITER=1) vertical velocity w solution :

8H forecast



NO SIPRA

24H forecast

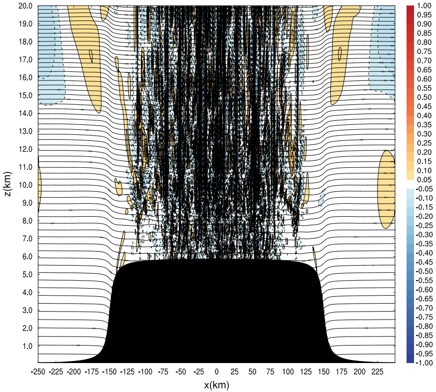


SIPRA = 450 hpa

3D No-Flow test over a prescribed high plateau, $H_{max} = 6000m$**,** $S_{max} = 19^\circ$

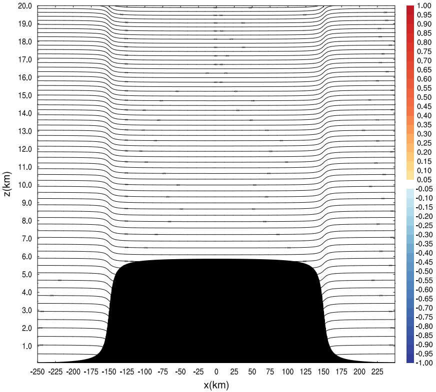
AROME (500m, L137, 15s, NSITER=1) vertical velocity w solution :

6H forecast



NO SIPRA

24h forecast



SIPRA = 450 hpa

Plan

• Extra coupling between $\hat{q}$ and $\mathbf{v}$ induced by orographic metric terms.

$$\frac{\partial \mathbf{v}}{\partial t} = ... - \nabla\phi \left\{ \frac{\partial \left[\pi \left(e^{\hat{q}} - 1\right)\right]}{\partial \pi} \right\}$$

$$\frac{\partial w}{\partial t} = g \left\{ \frac{\partial \left[\pi \left(e^{\hat{q}} - 1\right)\right]}{\partial \pi} \right\}$$

$$\frac{\partial \hat{q}}{\partial t} = ... - \frac{C_p}{C_v} \frac{g\, e^{\hat{q}}}{RT} \frac{\pi}{\partial \pi} \partial \left(w - \mathbf{v}.\frac{\nabla\phi}{g} \right)$$

Basic underlying idea:

• Take into account the effect of slope in the vertical Laplacian-like operator :

$$\mathcal{L}_{v}{}_{a}^{*}(\hat{q}) = \frac{\pi_a^* \partial}{\partial \pi_a^*} \left[color\,blue \frac{\partial_\eta \left(\pi_a^* \hat{q}\right)}{\partial \pi_a^*} + \Lambda_s^{*2} S(\eta)^2 \overline{\left\{ \overline{\frac{\partial_\eta \left(\pi_a^* \hat{q}\right)}{\partial \pi_a^*}}^{\mathrm{v}} \right\}}^{\mathrm{w}} \right]$$

• where Λ_s^* (**SISLP**) is a newly constant parameter that can be chosen as the maximum of slopes over the domain ⇒

$$\Lambda_s^{*2} = \max \left[(\nabla z_s)^2 \right]$$

• Interpolation operators $\overline{(\,)}^{\mathrm{w}}$ and $\overline{(\,)}^{\mathrm{v}}$ are determined so that $\mathcal{L}_v^*$ akin to a negative definite vertical Laplacian operator.

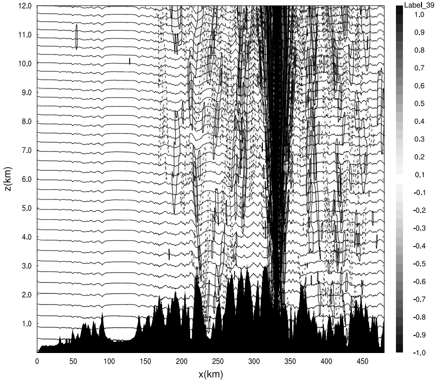
Table: AROME 3D (L90) no-flow 24 hours experiments over the Alps for different targeted resolutions $(\Delta x, \Delta t)$, for NSITER lying in $\{1, 2, 3\}$, with LPC_FULL, and no explicit 4-th order diffusion. Starting from an isothermal $T_0 = 288.15K$, hydrostatically balanced and resting atmosphere. SITR = 350 K, SITRA = 70 K, SIPR = 900 hpa, the calculations are deemed stable if the run terminates. $\circ$ denotes that the run is stable, $\times$ means that the run stops prematurely, and $\star$ indicates that the run terminates but the solution looks noisy.

	NSITER =1		NSITER =2		NSITER =3	
Target	Current	New-dev	Current	New-dev	Current	New-dev
$\Delta x = 500\,m$, $\Delta t = 15\,s$, $S_{max} = 58^\circ$	$\times$	$\circ$	$\star$	$\circ$	$\circ$	$\circ$
$\Delta x = 250\,m$, $\Delta t = 7.5\,s$, $S_{max} = 66^\circ$	$\times$	$\circ$	$\times$	$\circ$	$\circ$	$\circ$
$\Delta x = 100\,m$, $\Delta t = 3.75\,s$, $S_{max} = 77^\circ$	$\times$	$\star$	$\times$	$\circ$	$\times$	$\circ$

No-Flow test over the Alps

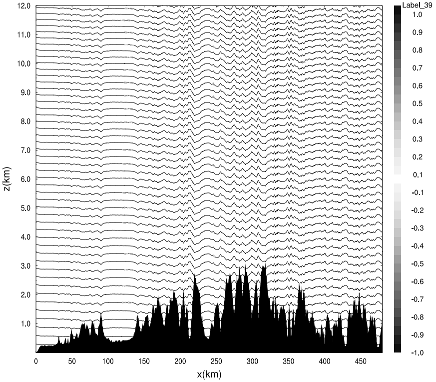
AROME (500m, L90, 15s) vertical velocity w, (reps. iso-θ), solutions after 24H forecast :

d_4



NSITER $= 2$

$d_4 + \text{SISLP}$



NSITER $= 1$

Plan

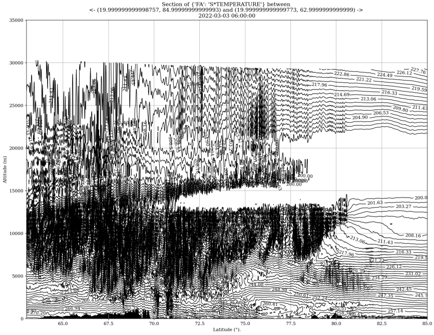
Summary and Perspective

- Two new SI constant parameters SIPRA and SISLP have been introduced in the SI linear model along the line of SITRA, in order to better control the magnitudes of the Baric and Orographic NL residual terms ⇒ They have clearly a positive impact on the robustness of the constant-coefficient ICI scheme.
- Assessment of the impact of these two new SI parameter on severe real cases where the stability of the model might be jeopardized ⇒ on going tests.

Svalbard problematic test-case with NSITER=1

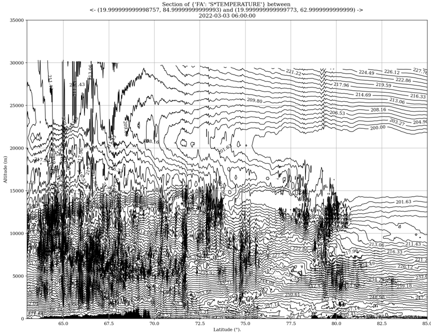
AROME (1250m, L90, 50s) (reps. iso-T), solutions after 30H forecast :

d_4



OPER + *VESL*

d_4 + SISLP



NEW + *NOVESL*

- Slightly encouraging but still a lot a tuning to do from the various SI parameter to find the optimal setting for this case.