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Improving the solver efficiency in the LFRic LAM

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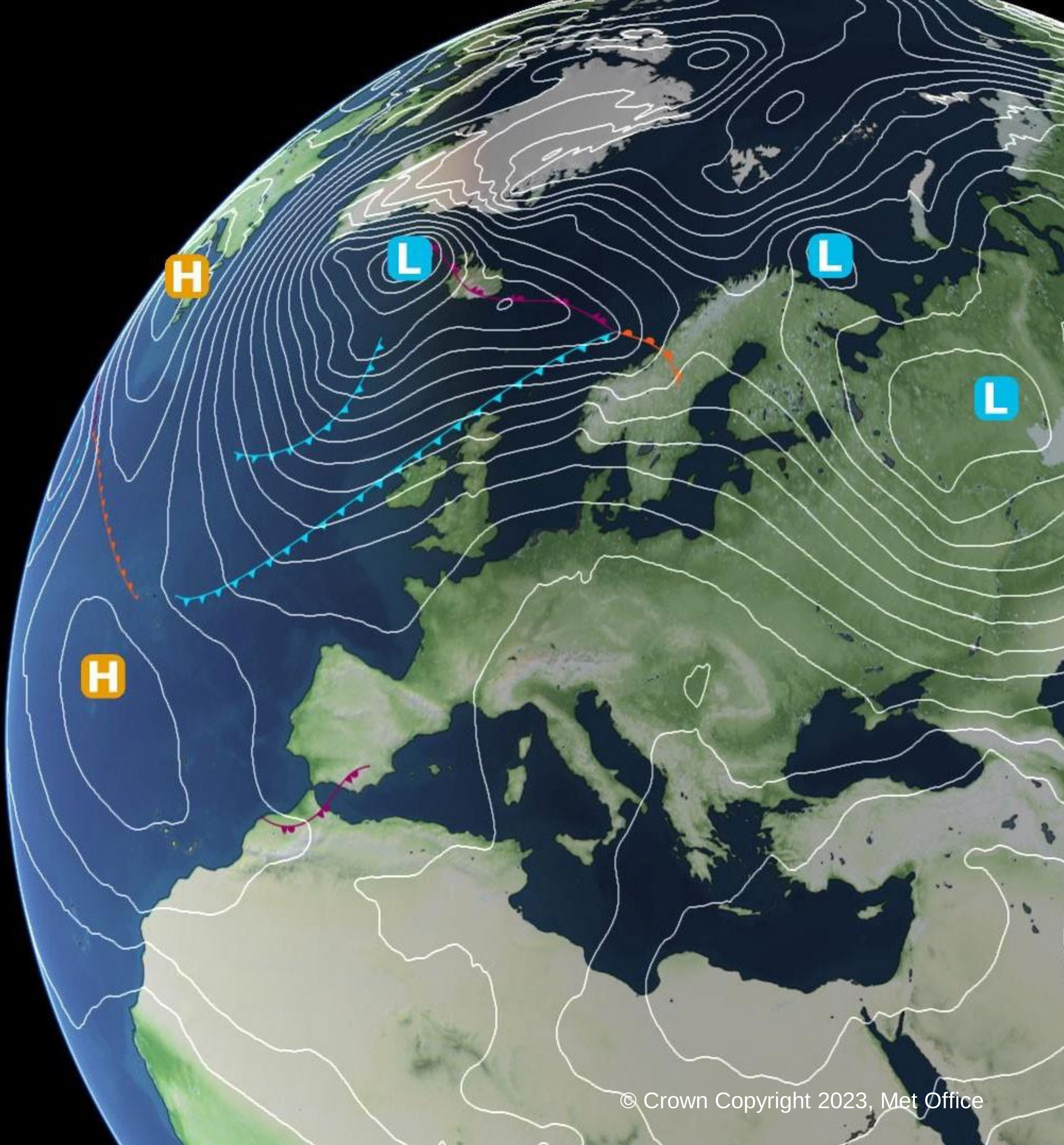
With thanks to the dynamics research team (Ben Shipway, Tom Melvin, Ian Boutle and Benjamin Buchenau).



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What is the LFRic solver? Part 1

The governing equations can be written as

$$R(x^{n+1}) = 0 \quad \text{where } x = (u, \rho, \theta, \Pi).$$

This includes dynamics forcings with semi-implicit timestepping, fast and slow physics and transport.

Let the state at time t^{n+1} and iteration $k+1$ be

$$x_{k+1}^{n+1} = x_k^{n+1} + x'_k, \qquad x_0^{n+1} = x^n.$$

and solve using a **quasi-Newton method**:

$$L(x^*)(x'_k) = -R\left(x_k^{n+1}\right)$$

where L is an approximation to the Jacobian, using the basic state $x^* = x^n$.

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What is the LFRic solver? Part 2

Solve a mixed solve equation using an iterative Krylov solve method and use the associated approximate Helmholtz equation as the preconditioner.

Mixed solve

$$\begin{pmatrix} M_u & G \\ D & M_p \end{pmatrix} \begin{pmatrix} u' \\ p' \end{pmatrix} = \begin{pmatrix} -R_u \\ -R_p \end{pmatrix}$$

Reduce to one equation by substituting for u' and a diagonal M_u

Helmholtz equation

$$Hp' = \hat{R}$$

There are **4 Quasi-Newton iterations** $x_{k+1}^{n+1} = x_k^{n+1} + x'_k$ on every timestep.

So mixed solve is solved 4 times.
+ about 5 GCR iterations for each mixed solve
+ about 50 BiCGstab iterations of Helmholtz for each GCR iteration.

How can we make the solver faster for limited-area regional models?

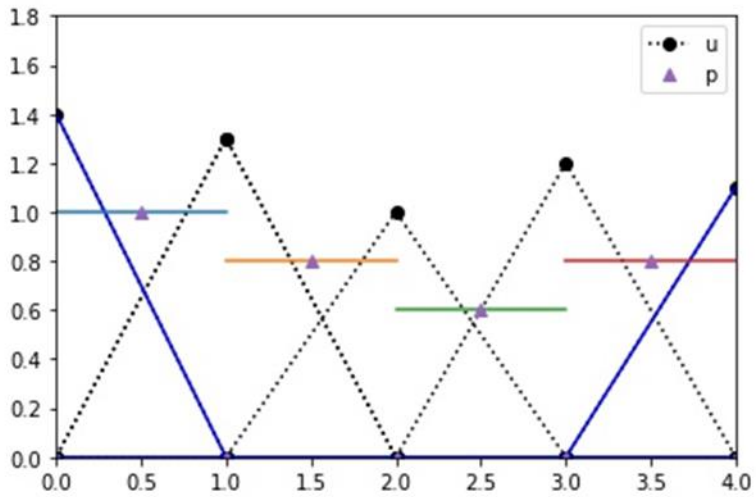
1. **Orthogonal Mesh:** LAMs have a (rotated-pole) latitude-longitude mesh. i.e. it is orthogonal. This removes the off-diagonal terms of M_u so maybe we don’t need the mixed solve.

2. **Orography:** But what about the impact orography? – a terrain following vertical coordinate isn’t orthogonal.

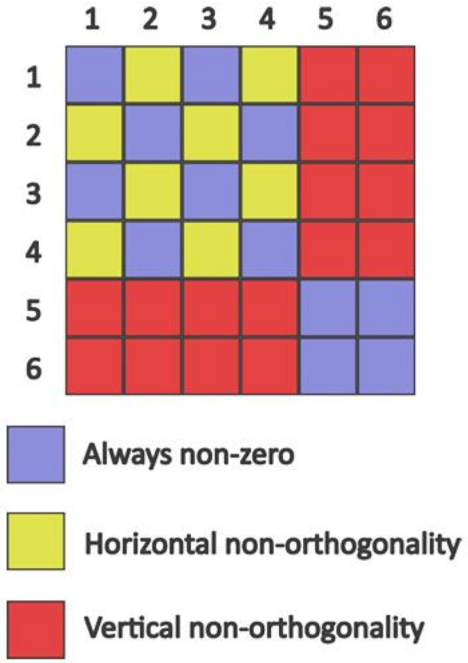
3. **Multigrid:** The global LFRic model uses a multigrid preconditioner, so can we make use of that in the regional LFRic model?

Non-Orthogonality

Cubed-sphere mesh: a non-orthogonal mesh



Finite-element spatial discretization allows us to deal with non-orthogonality in LFRic – but we need to run the mixed solve as well as the Helmholtz.



This gives a dense mass-matrix M_u - which contains the correlations of the finite element for each face of the cell.

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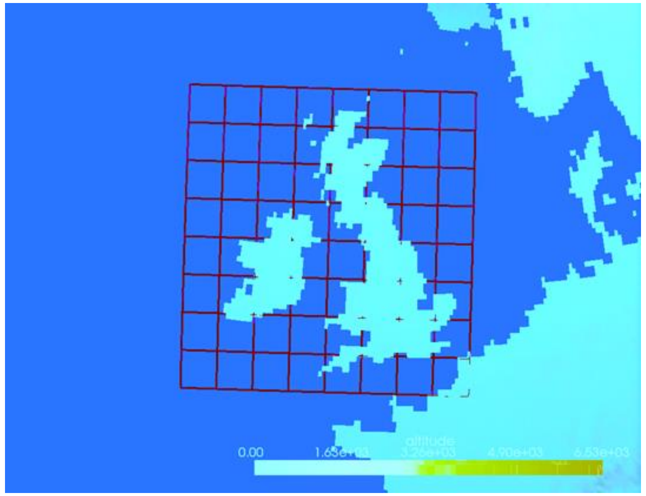
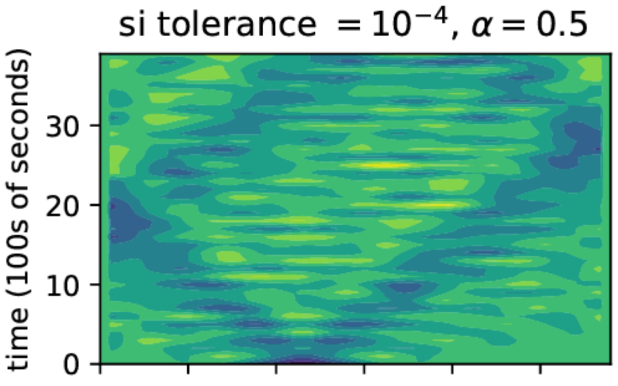
Exploiting orthogonality of the lat-lon mesh

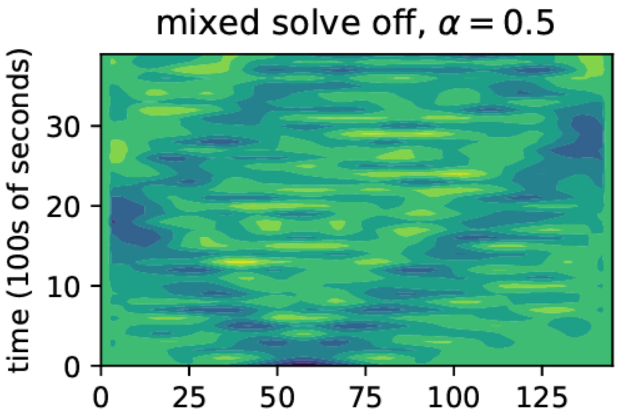


An orthogonal mesh means it is possible to run the model without the mixed solve.

Hovmöller plots from the evolution of a gravity wave on a flat, cartesian mesh.

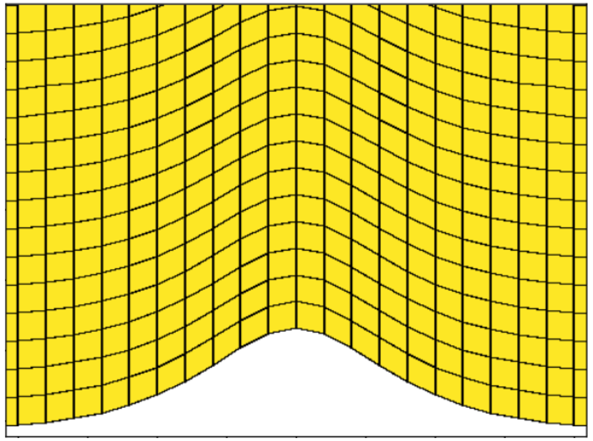


With mixed solve, time taken=65s



Helmholtz-only, time taken=10s

Orography

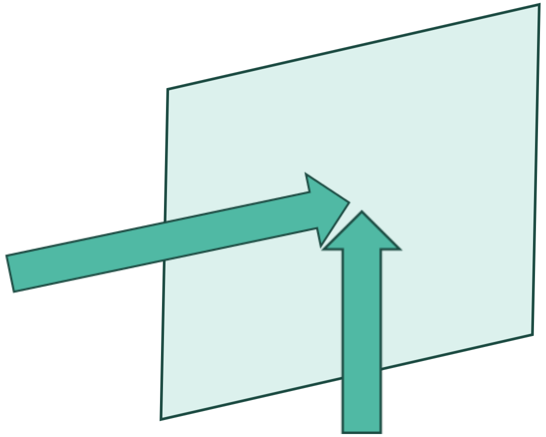


The terrain-following coordinate gives a non-orthogonal mesh.

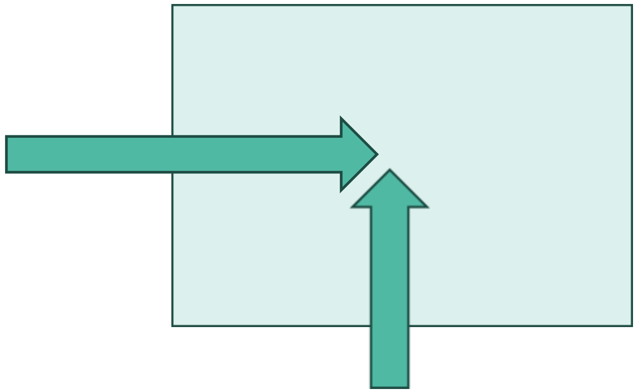
- This causes problems with convergence of the mixed solve. For our real 1.5km UK domain, extra smoothing needed to be applied to the orography for the model to run.
- It is not possible to run with Helmholtz only.

- The UM lags the ‘bendy terms’ associated with the orography, so maybe we can use a similar approach.

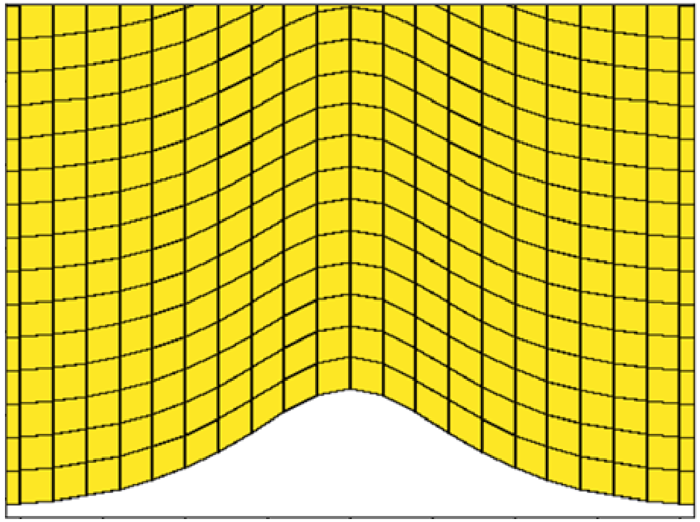
What is the lagged orography approach?



Winds are non-orthogonal for cells going over mountains.



Winds are orthogonal for flat terrain.



Terrain-following vertical coordinates.

Remove the terms (in the finite-element mass matrix for wind) corresponding to vertical-horizontal non-orthogonality. Only in the LHS of the solver. They are still there in the mass matrix on the RHS.

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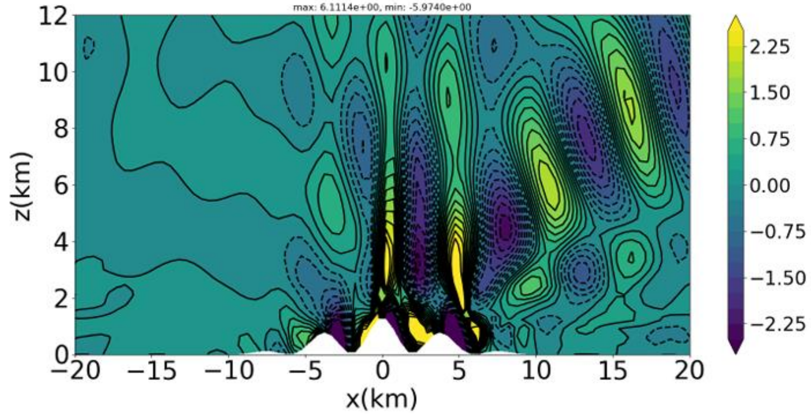
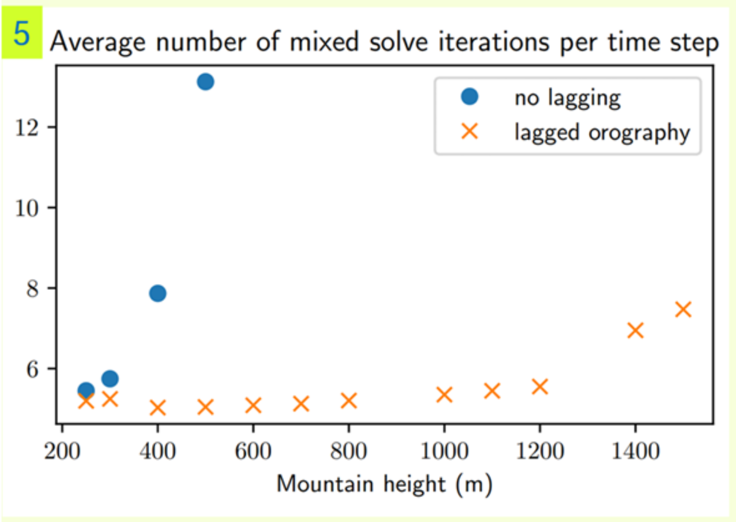
Lagged orography approach

Lagged orography approach: Separate the mass matrix M_u into two: $M_u = M_{uu} + M_{uw}$ and lag the terms associated with M_{uw} at the previous iteration. This is equivalent to replacing M_u with M_{uu} in the LHS of the mixed solve. i.e. remove M_{uw}.



Schar hill test case, with 1500m hill height & Helmholtz-only.



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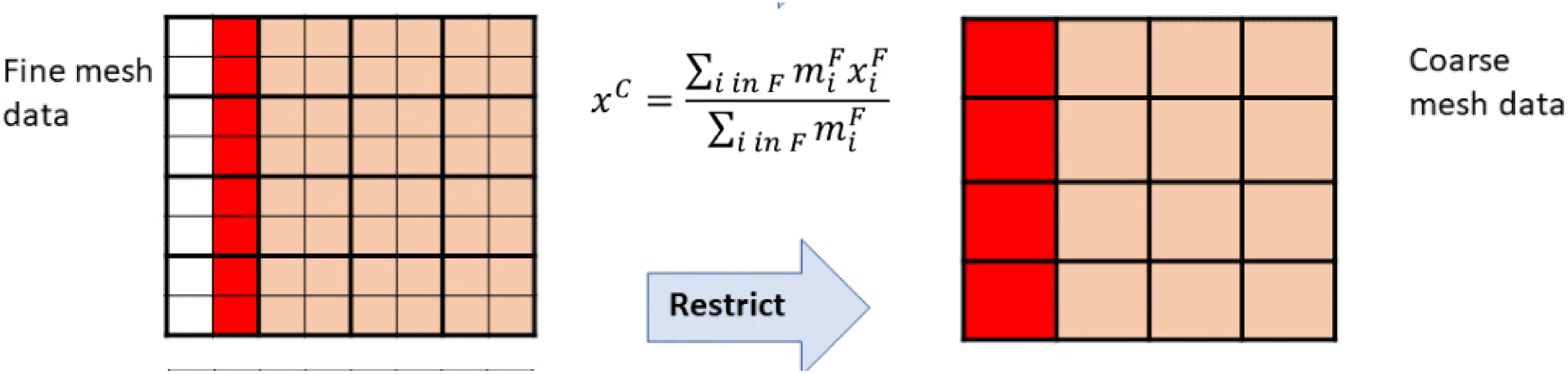
Multigrid

- Use the existing multigrid method that is used in the global LFRic model.
- This is used to solve the Helmholtz equation, instead of BiCGStab
- How to deal with the lateral boundary conditions?



$$x^C = \frac{\sum_{i\ in\ F} m_i^F x_i^F}{\sum_{i\ in\ F} m_i^F}$$

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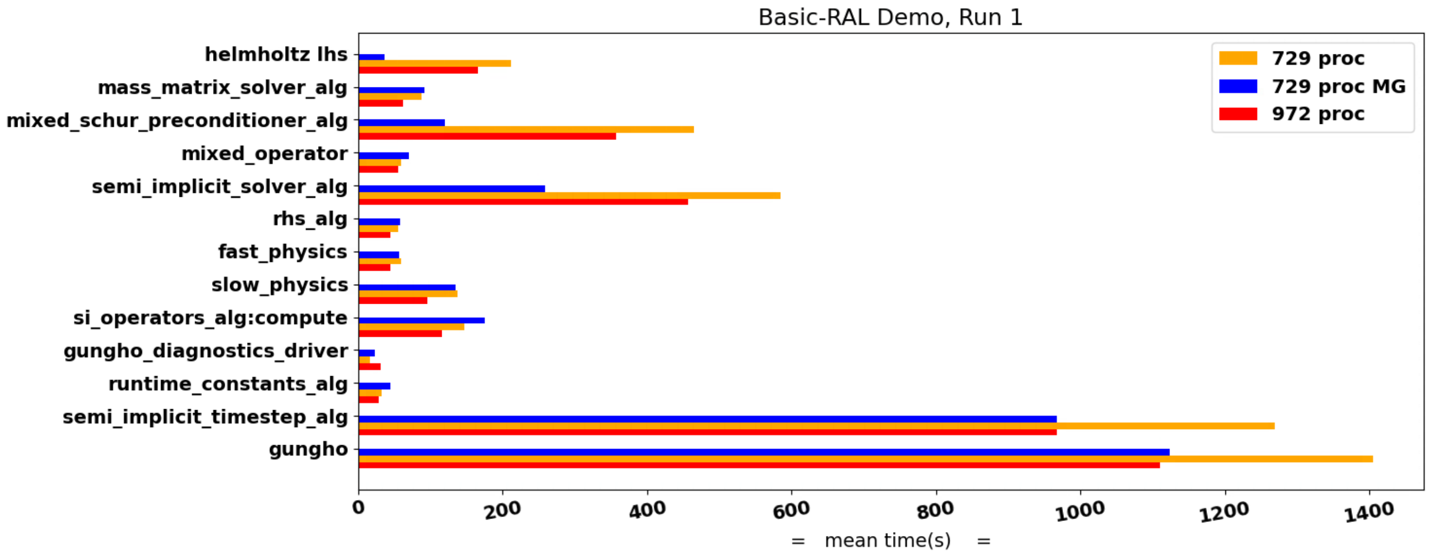
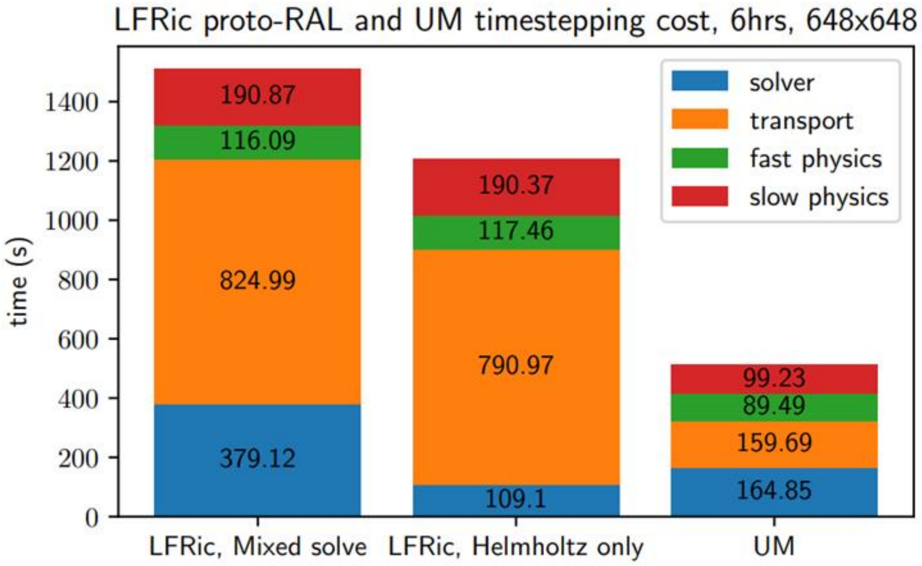
Overall impact on efficiency





Solver cost for **multigrid** runs is 50% that of Krylov solve. (Flat 'aquaplanet' domain).

Using **lagged orography** means that we can run UK domain without smoothing orography.

Plus **Helmholtz only** gives further reduction (30% of cost) and lower than the UM.

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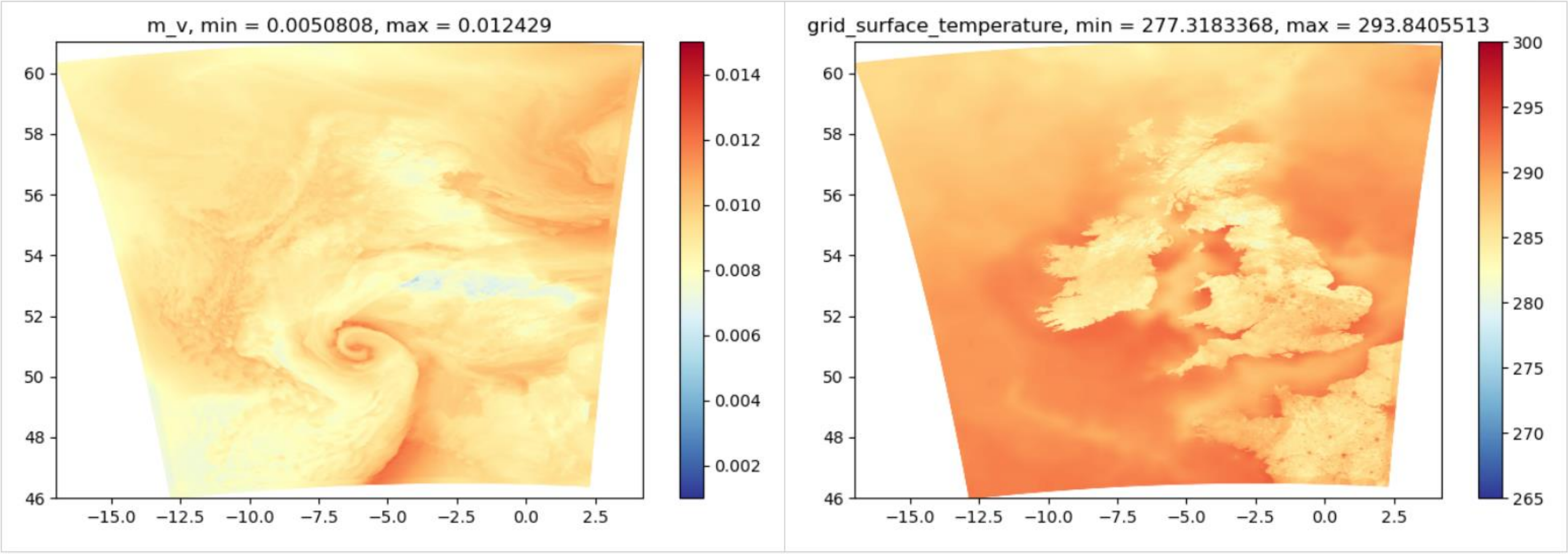
Full model results



768 x 1024 domain, 70 levels, 24 hour run

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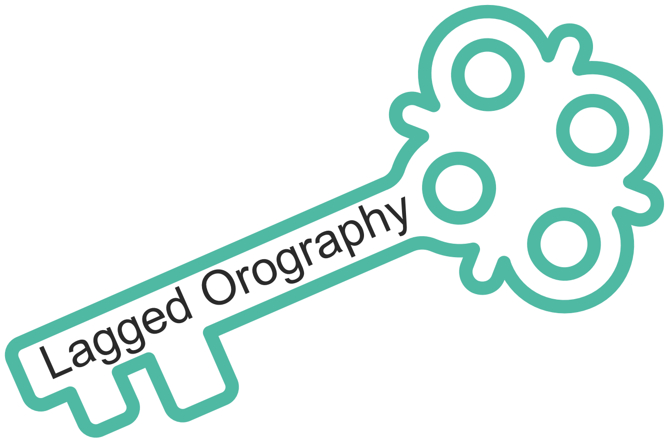
Conclusions

The lagged orography approach has been the key to unlocking the ability to make the solver more efficient. This is removing the impacts of non-orthogonality in the LHS of the solver but doesn’t degrade the quality of the result – and has allowed us to use less orography smoothing.



- Introducing lagged orography means the mixed solve requires less iterations – we are letting the quasi-Newton iterations do the work instead.

- Introducing lagged orography also allows the solver to work in **Helmholtz-only** mode and allows **multigrid** to work. These both give further improvements in the efficiency.