Local SI scheme & Sweep interpolation for SL

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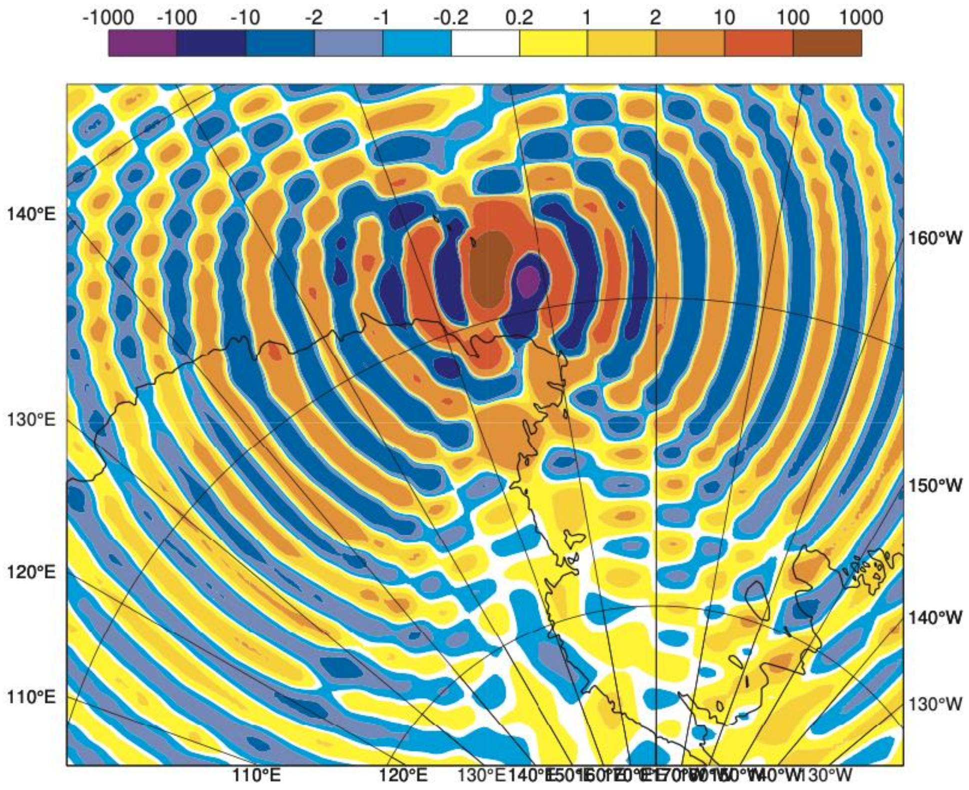
Semi-implicit scheme

- Introduced in 1969 by A. Robert
- Allows typically 5 times longer timestep compared to explicit
- Implicit treatment of selected linearised terms (those giving rise to high frequency waves)
- Remaining part assumed to be reasonably small (=residual) remains explicit
- Formally, it means to solve (in SISL):

$$\frac{X_A^+ - X_D^0}{\Delta t} = \frac{1}{2}\left[\mathcal{L}X_D^0 + \mathcal{L}X_A^+\right] + (\mathcal{M} - \mathcal{L})X_M^{\frac{1}{2}}$$

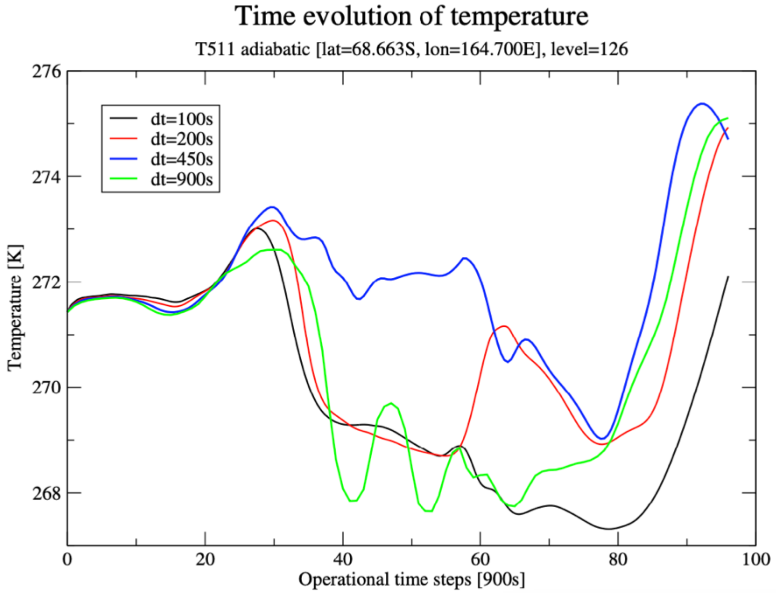
- Leads to Helmholtz equation problem
- Abundant (global) solvers available
- Spectral method 3-4 times more efficient but implies additional constraints

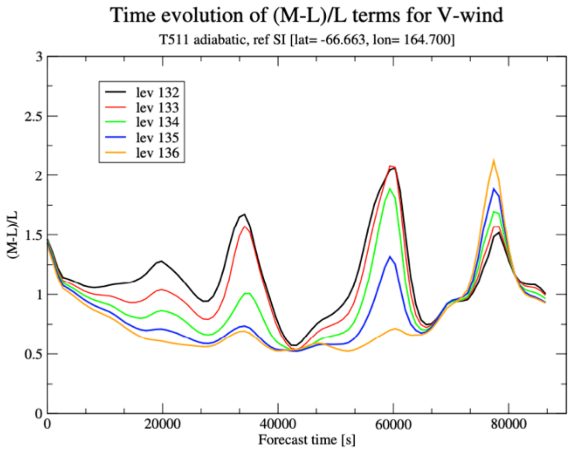
Known issues of SI

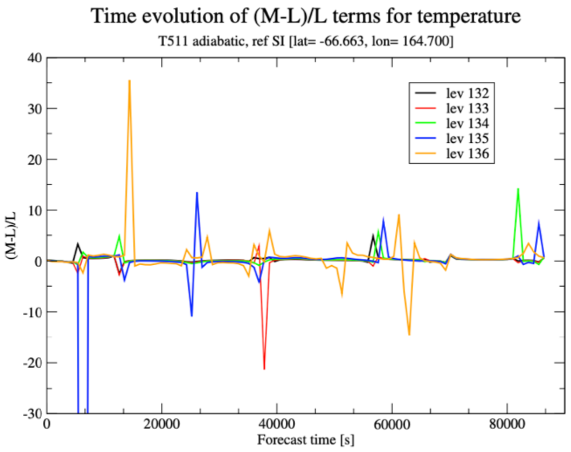


Temperature increment from TL model (T511 adiabatic)

⟹ Apparently, we do see points where: *M-L ≫ L*







Proposed upgrade for SI

- We wish to stick with SI method, just give it a boost
- To ensure M-$L \ll L \implies$ more realistic implicit model is required
- Extrapolation of highly non-linear residual can be problematic $\implies$ prefer iterative averaging
- Extreme approach: Take the best linear approximation of M based on TL approximation:

$$\mathcal{L}(X) = \mathcal{M}(X^0) + \mathbf{M}'(X^0)(X - X^0)$$

- Replace SETTLS method by non-extrapolating 2TL scheme of Diamantakis (2014):

$$\frac{X^+ - X^0}{\Delta t} = \frac{1}{2}\left[\mathcal{M}(X^0)\right]_D + \frac{1}{2}\left[\mathcal{M}(X^0) + M'(X^*)(X^+ - X^0)\right]_A$$

- No longer suited for Helmholtz method inversion $\implies$ matrix-free solver

Shallow water model

Governing equations:

$$\frac{dh}{dt} = -h\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \boxed{-\bar{H}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)} + (\bar{H} - h)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

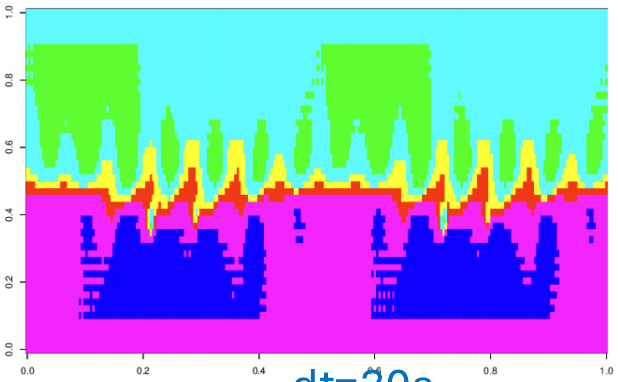
$$\frac{du}{dt} = \boxed{-g\frac{\partial h}{\partial x}} + fv - g\frac{\partial H_s}{\partial x} - \nu u,$$

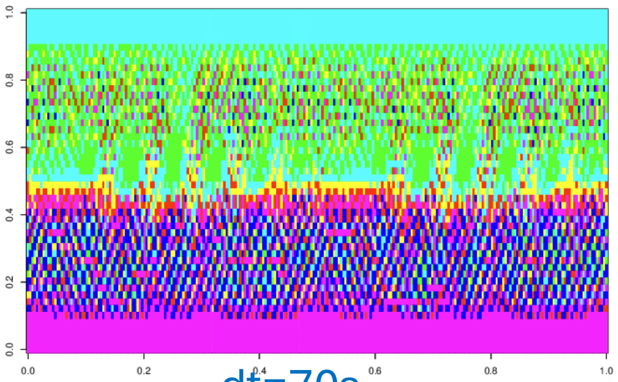
$$\frac{dv}{dt} = \boxed{-g\frac{\partial h}{\partial y}} - fu - g\frac{\partial H_s}{\partial y} - \nu v,$$

implying:

$$\mathbf{M}'(X^*)(X - X^0) = \begin{pmatrix} -\left(\frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y}\right)(h - h^0) - h^*\left(\frac{\partial u}{\partial x} - \frac{\partial u^0}{\partial x} + \frac{\partial v}{\partial y} - \frac{\partial v^0}{\partial y}\right) \\ f(v - v^0) - g(\frac{\partial h}{\partial x} - \frac{\partial h^0}{\partial x}) - \nu(u - u^0) \\ -f(u - u^0) - g(\frac{\partial h}{\partial y} - \frac{\partial h^0}{\partial y}) - \nu(v - v^0) \end{pmatrix}$$

Shallow water model results

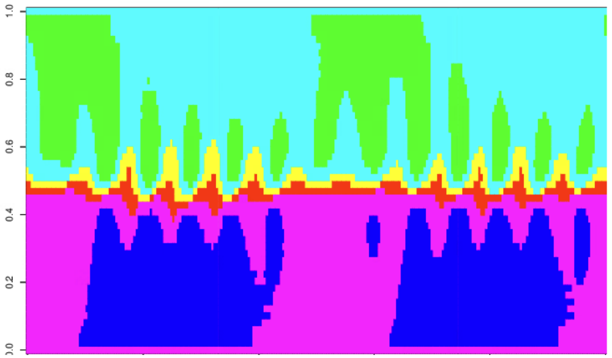


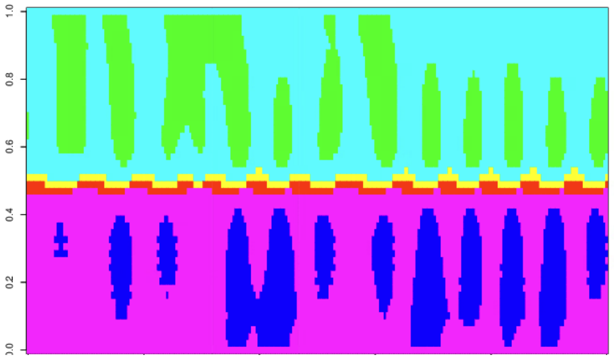


Explicit scheme

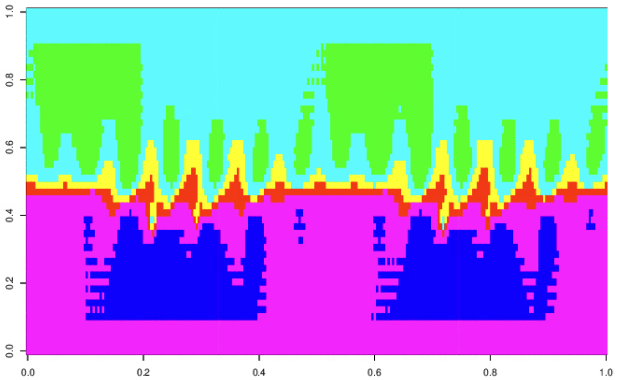
dt=30s   dt=70s   dt=300s

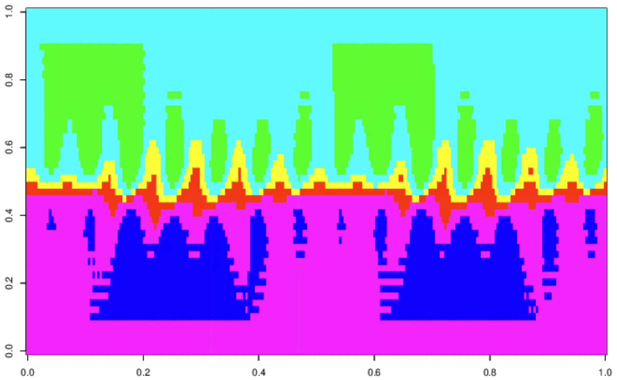
Semi-implicit scheme





New SI scheme





Solver method suitable for 3D model

- Solving linear system Ax=b with A being generally a rectangular matrix
- Preconditioned by square matrix B derived from A by assuming horizontal homogeneity (HEVI)
- The solution is iterated only inverting B by CG method

IFS implementation:

- Evaluate b and filter it.
- Three nested loops:
  1. Derivatives update in x (set to 5 iterations)
  2. Iterate $x_{(n+1)} = \mathbf{B}^{-1}[\mathbf{B}x_{(n)} - \mathbf{A}x_{(n)} + b]$ (2-3 iterations)
  3. Invert $\mathbf{B}$ by CG (2-4 iterations)
- Evaluate $X^{+} = X^{0} + x$

Solver method is strictly local

B definition in shallow water

$$\mathbf{A}x =$$

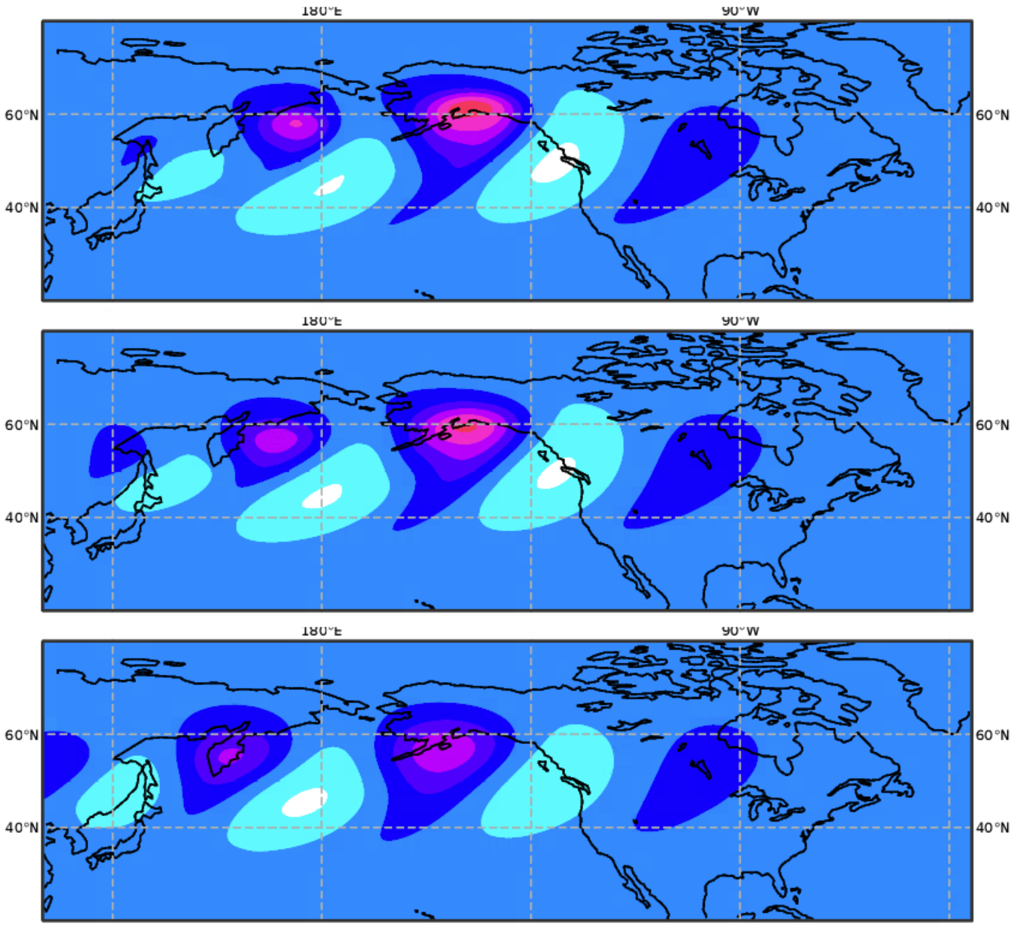
$$\begin{pmatrix} -\frac{\Delta t}{2}\left(\frac{\partial u^0}{\partial x}+\frac{\partial v^0}{\partial y}\right)-1 & 0 & 0 & 0 & 0 & -\frac{\Delta t}{2}h^0 & -\frac{\Delta t}{2}h^0 \\ 0 & -\frac{\Delta t}{2}\nu-1 & \frac{\Delta t}{2}f & -\frac{\Delta t}{2}g & 0 & 0 & 0 \\ 0 & -\frac{\Delta t}{2}f & -\frac{\Delta t}{2}\nu-1 & 0 & -\frac{\Delta t}{2}g & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta h \\ \delta u \\ \delta v \\ \delta\frac{\partial h}{\partial x} \\ \delta\frac{\partial h}{\partial y} \\ \delta\frac{\partial u}{\partial x} \\ \delta\frac{\partial v}{\partial y} \end{pmatrix}$$

$$\mathbf{B}x =$$

$$\begin{pmatrix} -\frac{\Delta t}{2}\left(\frac{\partial u^0}{\partial x}+\frac{\partial v^0}{\partial y}\right)-1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\Delta t}{2}\nu-1 & \frac{\Delta t}{2}f & 0 & 0 & 0 & 0 \\ 0 & -\frac{\Delta t}{2}f & -\frac{\Delta t}{2}\nu-1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \delta h \\ \delta u \\ \delta v \\ \delta\frac{\partial h}{\partial x} \\ \delta\frac{\partial h}{\partial y} \\ \delta\frac{\partial u}{\partial x} \\ \delta\frac{\partial v}{\partial y} \end{pmatrix}$$

Baroclinic instability test

Jablonowski & Williamson (2006), surface pressure at day 9



IFS ref, Δt = 3600s

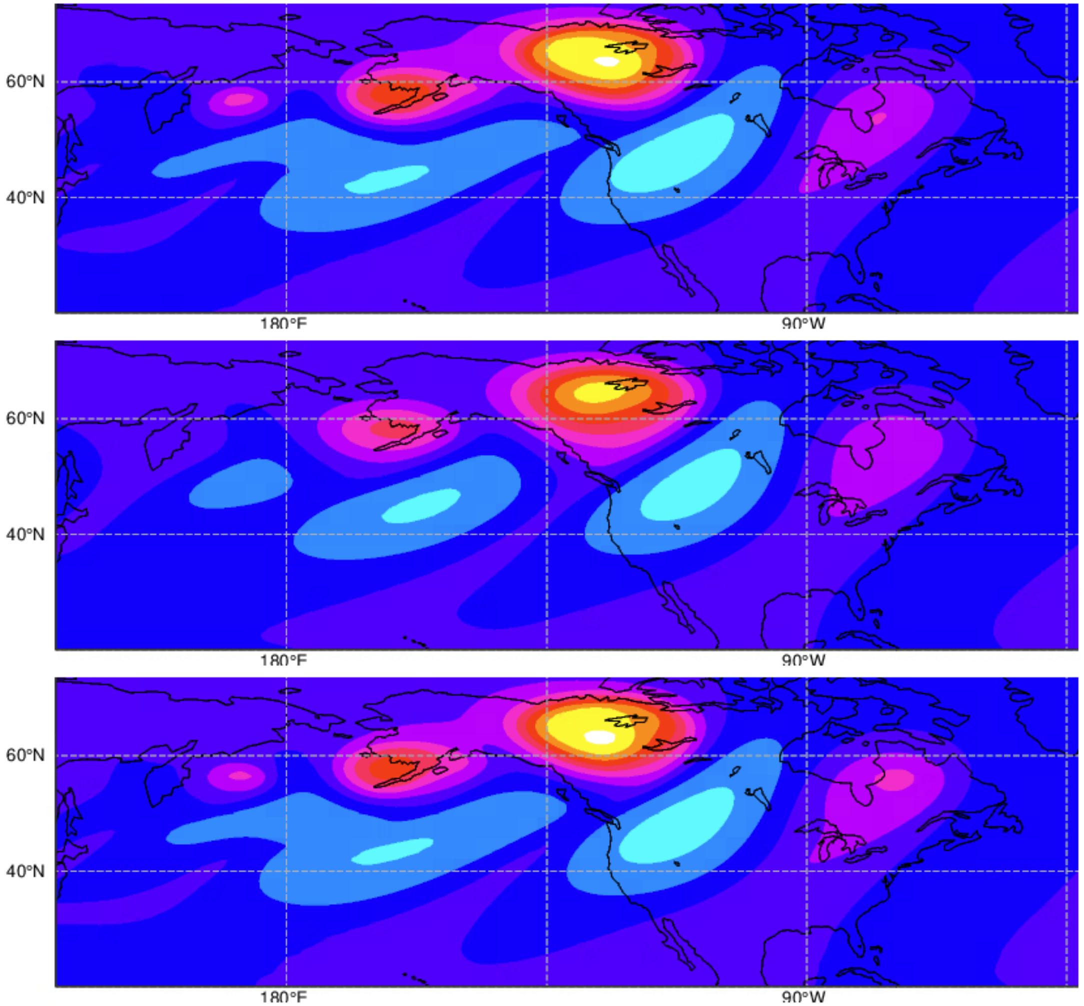
SIGP, Δt = 1800s

SIGP, Δt = 3600s

Linear spectral diffusion

Baroclinic instability test

Jablonowski & Williamson (2006), surface pressure at day 10



IFS ref

SIGP

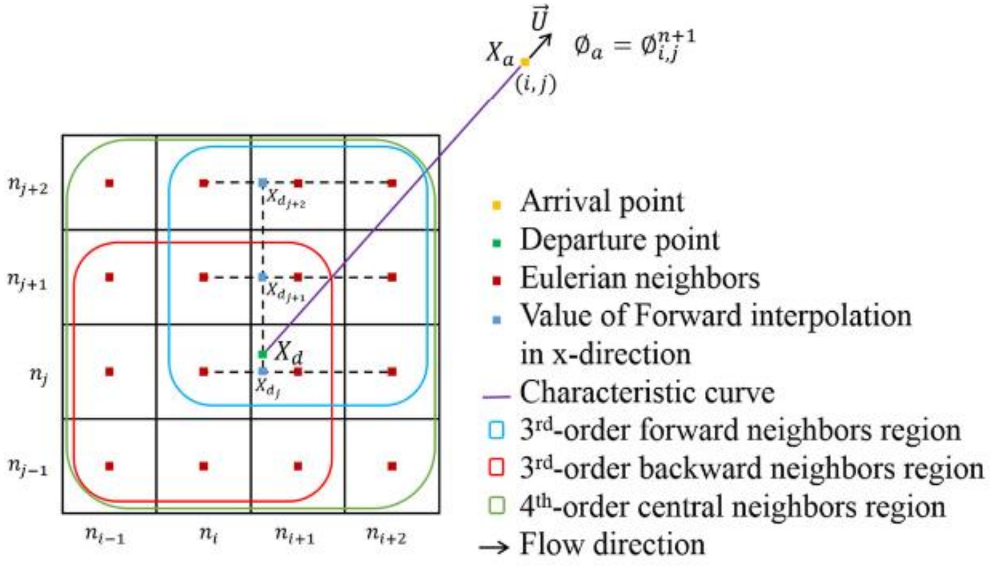
SIGP, non-linear diff

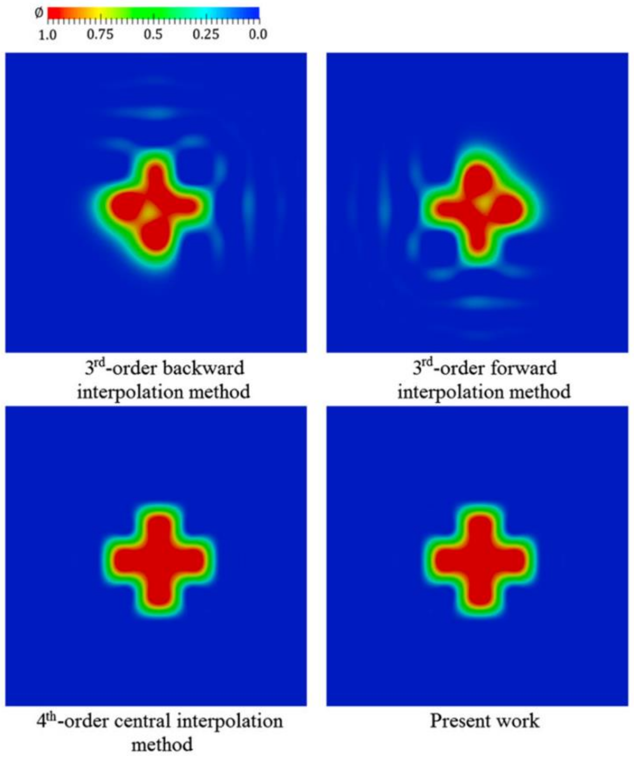
GP-SI scheme summary

- Proven to work in the context of IFS (evaluation is ongoing)
- Local method of the solver needs to be complemented by filtering/de-noising and derivatives evaluation
- Derivatives already maintained spectral and local (based on FV), local filtering method (based on conservative remapping) yet to come
- Plan to involve physics into the implicit model
- Long-term plan is to consider TL/AD counterpart for DA (implying coding TL of the TL model)

Sweep interpolation

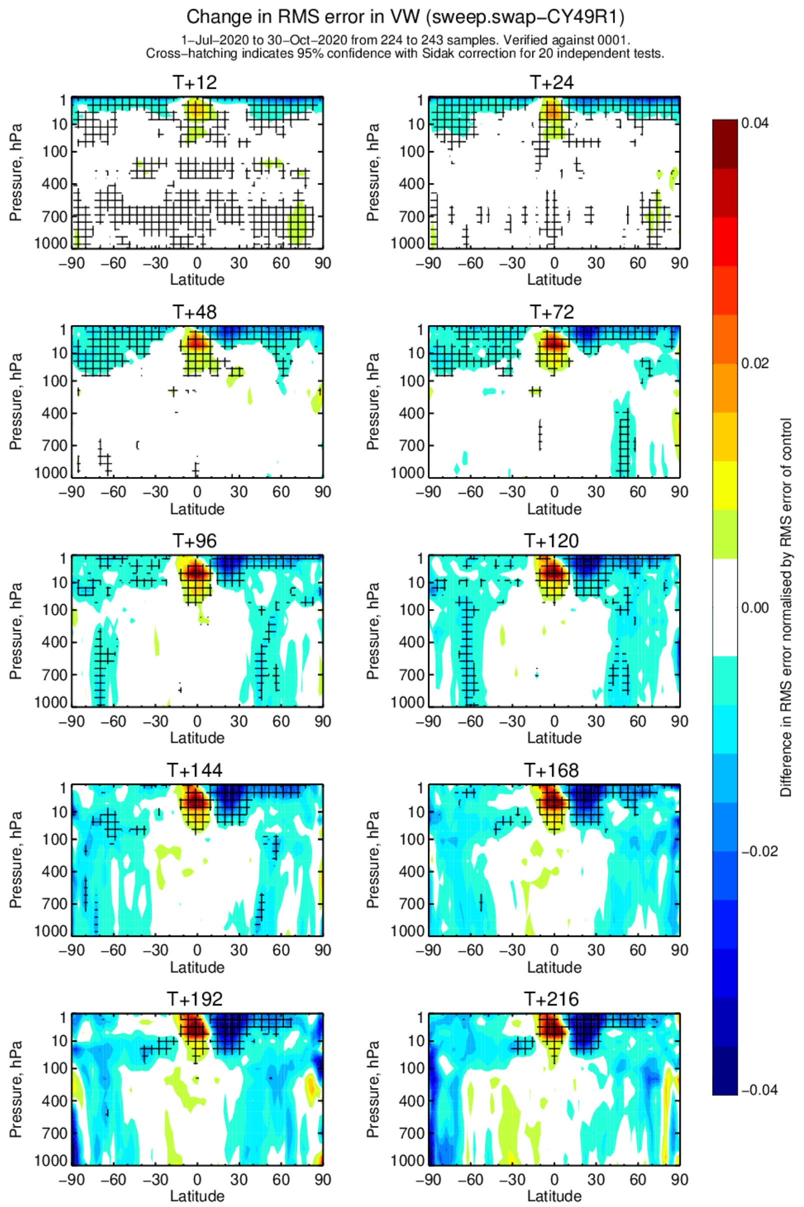
- Interesting idea from ECCC to reduce cost of tracer's interpolation
- 4th order interpolation is approximated by 3rd order method on two altering reduced stencils
- The two reduced stencil have similar error of opposite sign cancelling each other





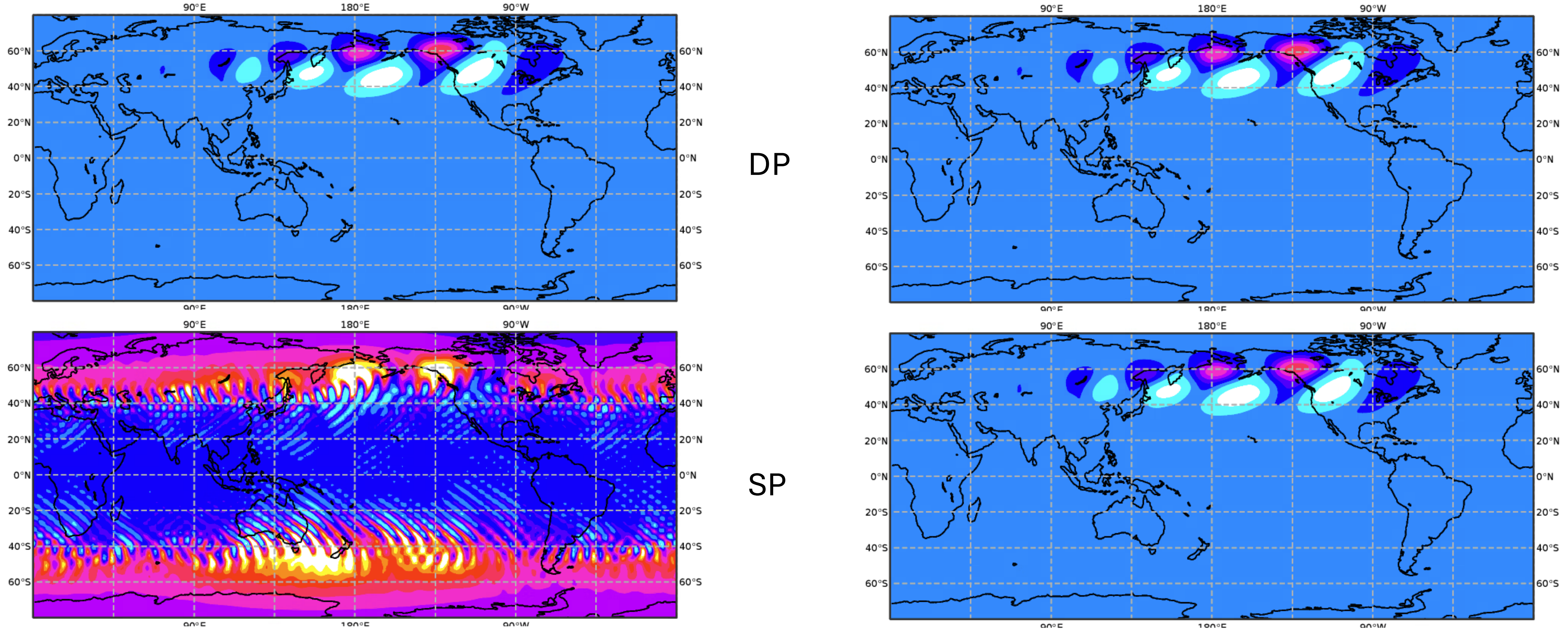
Sweep interpolation in IFS

- Tested in the IFS where it was implemented to all model variables
- 32-point (isotropic) stencil is reduced to 20-point stencil with 7 quadratic (was cubic) and 5 (was 10) linear interpolation
- No negative effect to tracers and cloud variables
- Rather positive effect to scores
- Some residual effect from mass fixer
- Cost of interpolation reduced by 33% (NEC SX-Aurora)
- Full implementation (including 4dvar) planned for CY51



Extra slides follow

Sensitivity to numerical precision



Reference

De-aliased