Regional Cooperation for
Limited Area Modeling in Central Europe

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ACCORD
A Consortium for COnvection-scale modelling
Research and Development

Dynamics for ACCORD model higher numerical stability

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Outline

- Dynamical core in ACCORD
- SI time scheme
- Orographic terms in linear model (based on ideas of Fabrice Voitus and Jozef Vivoda)
- Idealised test cases
- Real simulations @200m

Basic equations

- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \hat{q} = \ln(\frac{p}{\pi})$

Discretization

- spectral method for horizontal direction
- hybrid vertical coordinate η based on hydrostatic pressure $\pi(\eta) = A(\eta) + B(\eta)\pi_s$; $A(top) = B(top) = 0, A(bottom) = 0, B(bottom) = 1$
- finite differences or finite elements for vertical direction discretization
- semi-implicit or iterative centred implicit scheme for time discretization
- semi-Lagrangian advection

Semi-Implicit time scheme

System evolution

$$\frac{dX}{dt} = \mathcal{M}X$$

Linearization

$$X = \mathbf{X}^* + \mathbf{X}', \quad \mathcal{M} \longrightarrow \mathcal{L}^*$$

Using linear model $\mathcal{L}^*$ we divide

$$\frac{dX}{dt} = \mathcal{L}^*\overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$

and discretize in time to obtain

Semi-implicit scheme

$$\frac{\mathbf{X}^+ - \mathbf{X}^0}{\Delta t} = \mathcal{L}^*\left(\frac{\mathbf{X}^+ + \mathbf{X}^0}{2}\right) + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+\frac{1}{2}}$$

or

Iterative centered implicit scheme

$$\frac{\mathbf{X}^{+(n)} - \mathbf{X}^0}{\Delta t} = \frac{\mathcal{L}^*\mathbf{X}^{+(n)} + \mathcal{L}^*\mathbf{X}^0}{2} + \frac{(\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+(n-1)} + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^0}{2}$$

We know that both can be second order accurate in time when some care is taken (averaging along semi-Lagrangian trajectory).

Full model

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Temperature

$$\frac{dT}{dt} = \frac{\kappa T}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{d\vec{v}}{dt} = -RT\frac{\nabla\pi}{\pi} - \nabla\phi - RT\nabla\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\nabla\phi$$

Vertical momentum

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

Pressure departure

$$\frac{d\hat{q}}{dt} = \frac{1}{\kappa - 1}(D + d) - \frac{1}{\pi}\frac{d\pi}{dt}$$

Surface pressure

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s}\int_0^1 \nabla\cdot(m\vec{v})d\eta$$

Diagnostic relations

$$\frac{d\pi}{dt} = \vec{v}\cdot\nabla\pi - \int_0^\eta \nabla\cdot(m\vec{v})d\eta'$$

$$\phi = \phi_s - \int_\eta^1 \frac{mRT}{p}d\eta'$$

$$d = \frac{p}{mRT}\left(\nabla\phi\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)$$

Definitions

$$D = \nabla\cdot\vec{v}$$

$$\kappa = \frac{c_p}{R}$$

$$m = \frac{\partial\pi}{\partial\eta}$$

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Definitions

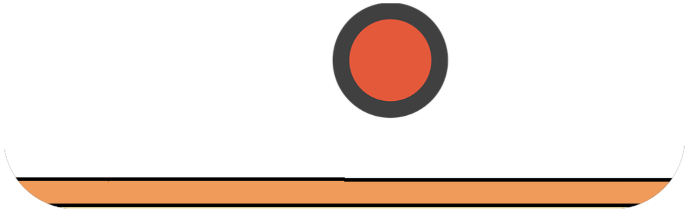
$$D = \nabla\cdot\vec{v}$$

$$\kappa = \frac{c_p}{R}$$

$$m = \frac{\partial\pi}{\partial\eta}$$

Current:

- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orography ($\nabla\phi^* = 0$)



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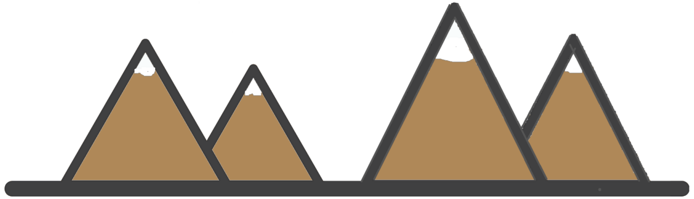
New:

- stationary
- resting
- hydrostatically balanced (π_s^*)
- dry
- isothermal (T^*)
- with constant orographic slope (in absolute value, $|\nabla\phi^*| \neq 0$)



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Linear model

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Temperature

$$\frac{\partial T}{\partial t} = \frac{\kappa T^*}{\kappa - 1}(D + d)$$

Horizontal momentum

$$\frac{\partial \vec{v}}{\partial t} = -RT^* \frac{\nabla \pi}{\pi^*} - \nabla \phi - RT^* \nabla \hat{q} - \frac{1}{m^*} \frac{\partial \pi^* \hat{q}}{\partial \eta} \nabla \phi^*$$

Vertical momentum

$$\frac{\partial w}{\partial t} = \frac{g}{m^*} \frac{\partial \pi^* \hat{q}}{\partial \eta}$$

Pressure departure

$$\frac{\partial \hat{q}}{\partial t} = \frac{1}{\kappa - 1}(D + d) + \frac{1}{\pi^*} \int_0^\eta m^* D d\eta'$$

Surface pressure

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s^*} \int_0^1 m^* D d\eta$$

Diagnostic relations

$$\nabla \phi^* = \nabla \phi_s^* - \int_\eta^1 \nabla \left(\frac{m^* R T^*}{\pi^*} \right) d\eta'$$

$$\nabla \phi_s^* = g \Lambda^*$$

$$d = \frac{\pi^*}{m^* R T^*} \left(\nabla \phi^* \frac{\partial \vec{v}}{\partial \eta} - g \frac{\partial w}{\partial \eta} \right)$$

Definitions

New SI parameter

$$\Lambda^* = \frac{1}{g}\, max\, (||\nabla \phi_s||, \text{ over domain})$$

$$m^* = \frac{\partial \pi^*}{\partial \eta}$$

Helmholtz elimination

- formulate time evolution for the modified vertical divergence
- omit the first order terms in $\nabla\phi^*$, then $\frac{\partial\vec{v}}{\partial t}$ is unchanged and all operators of the RHS of $\frac{\partial d}{\partial t}$ apply on $\widehat{q}$
- formulate RHS of $\frac{\partial d}{\partial t}$ as new vertical Laplacian operator applied on $\widehat{q}$
- with no slopes present the new vertical Laplacian collapses to the old one
- new vertical Laplacian has only real and negative eigenvalues in "reasonable cases"
- discretize (we omit the details here)
- eliminate all variables up to horizontal divergence D
- solve the Helmholtz equation for D as usually

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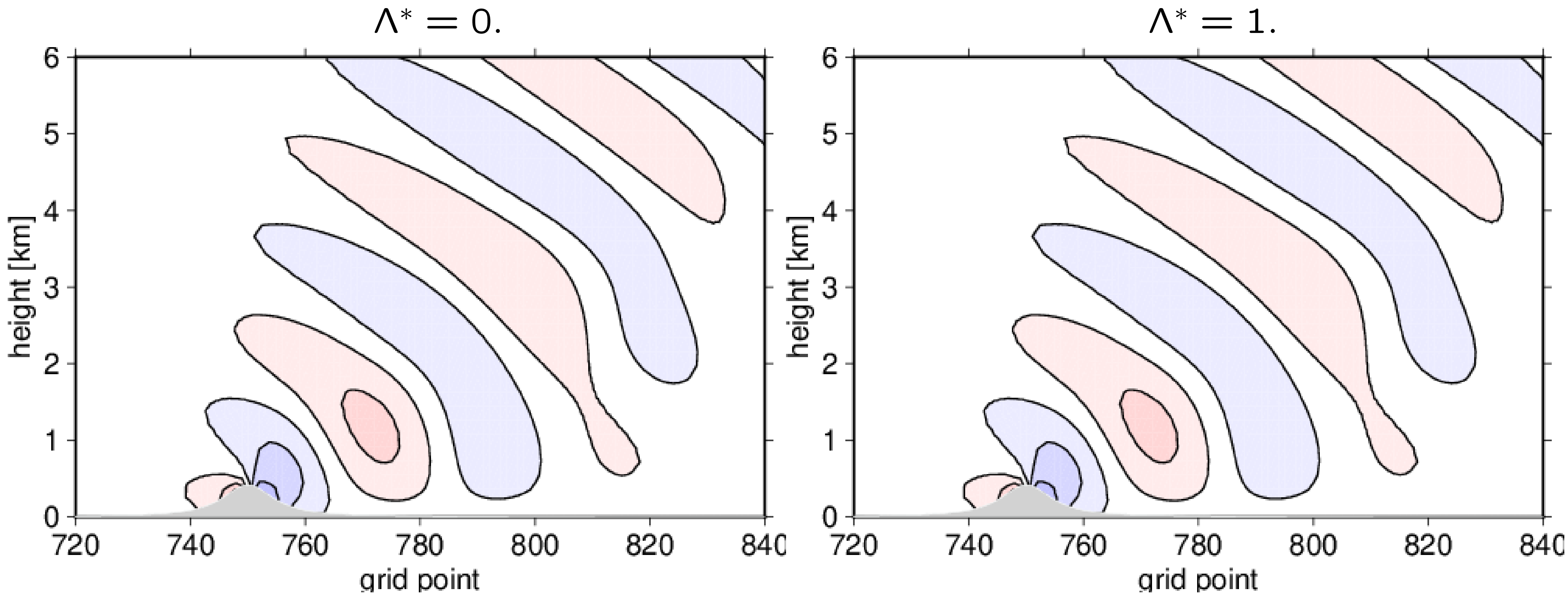
Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\,(||\nabla\phi_s||) = 0.6$



31°

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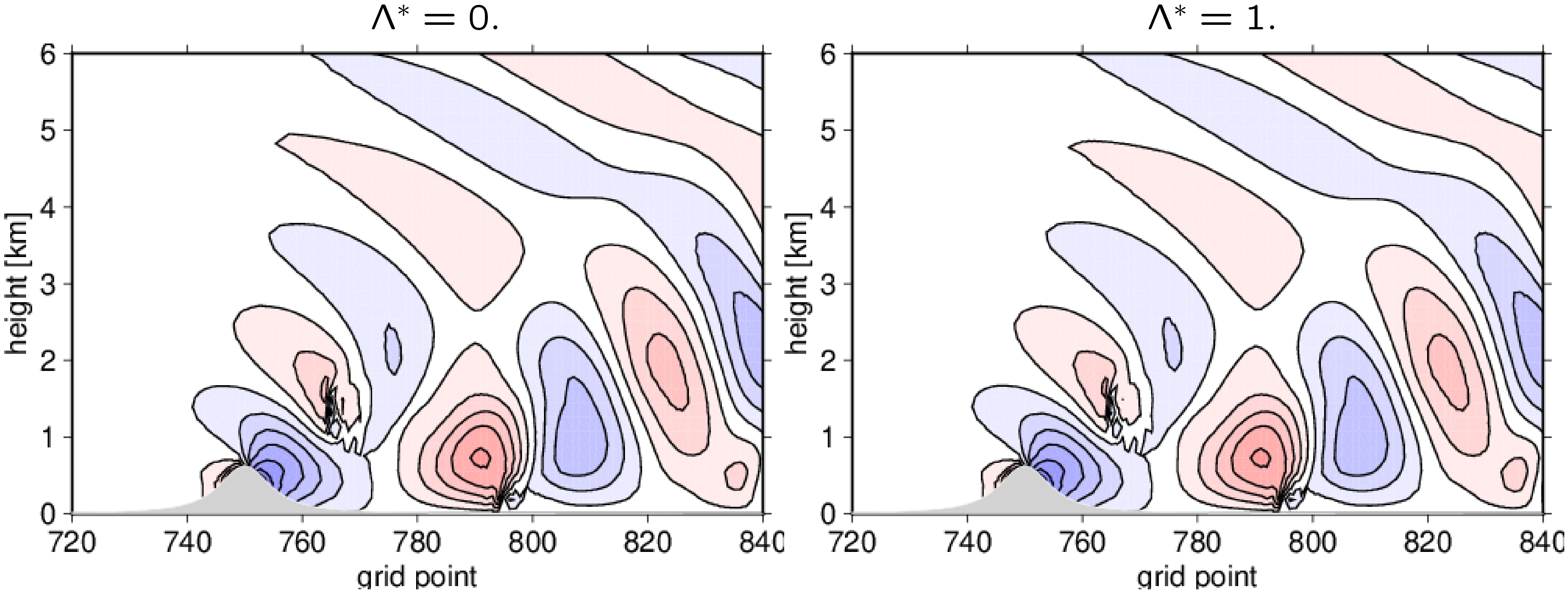
Idealized tests

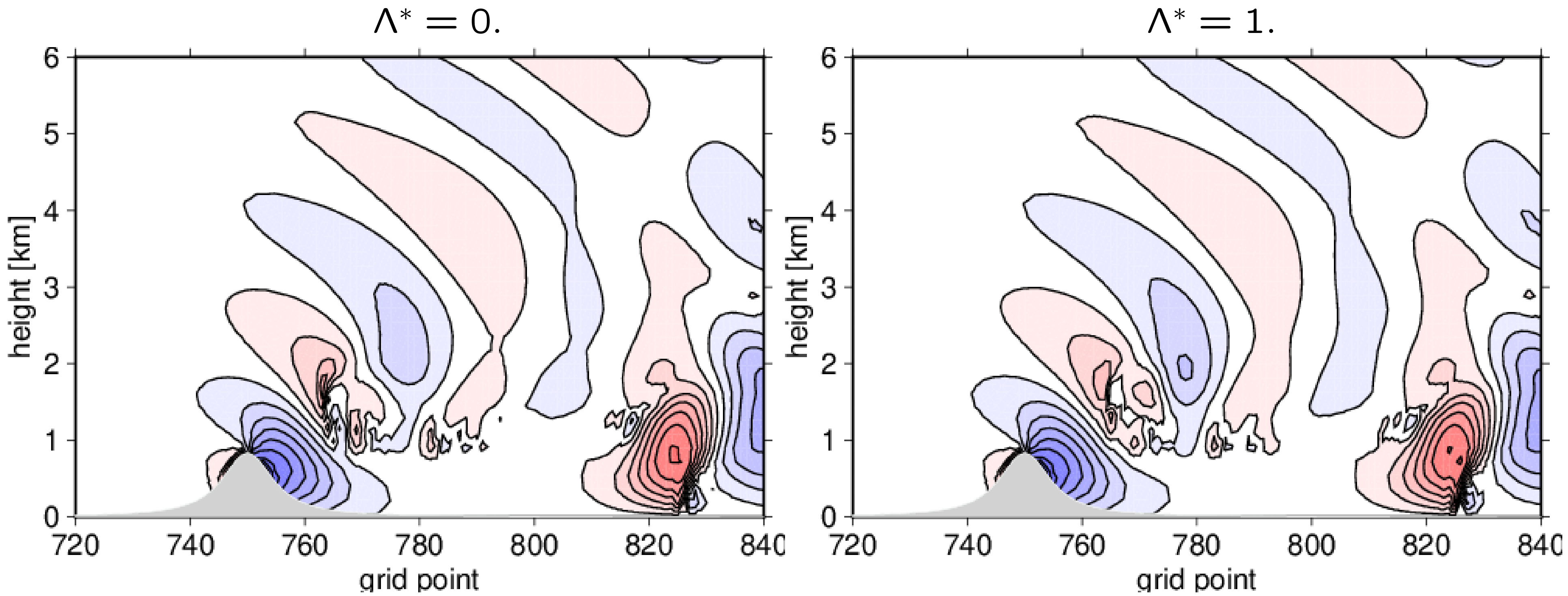
Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\,(||\nabla\phi_s||) = 0.9$



42°

Idealized tests

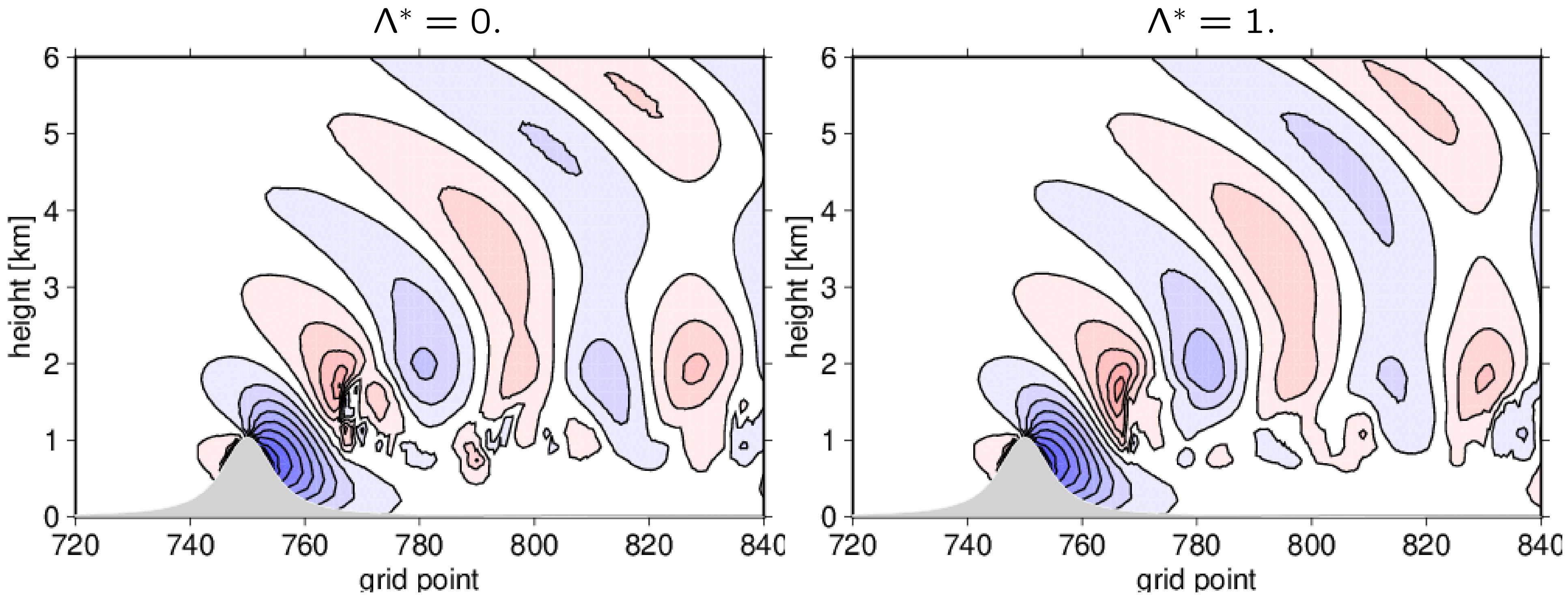
Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\left(||\nabla\phi_s||\right) = 1.2$



50°

Idealized tests

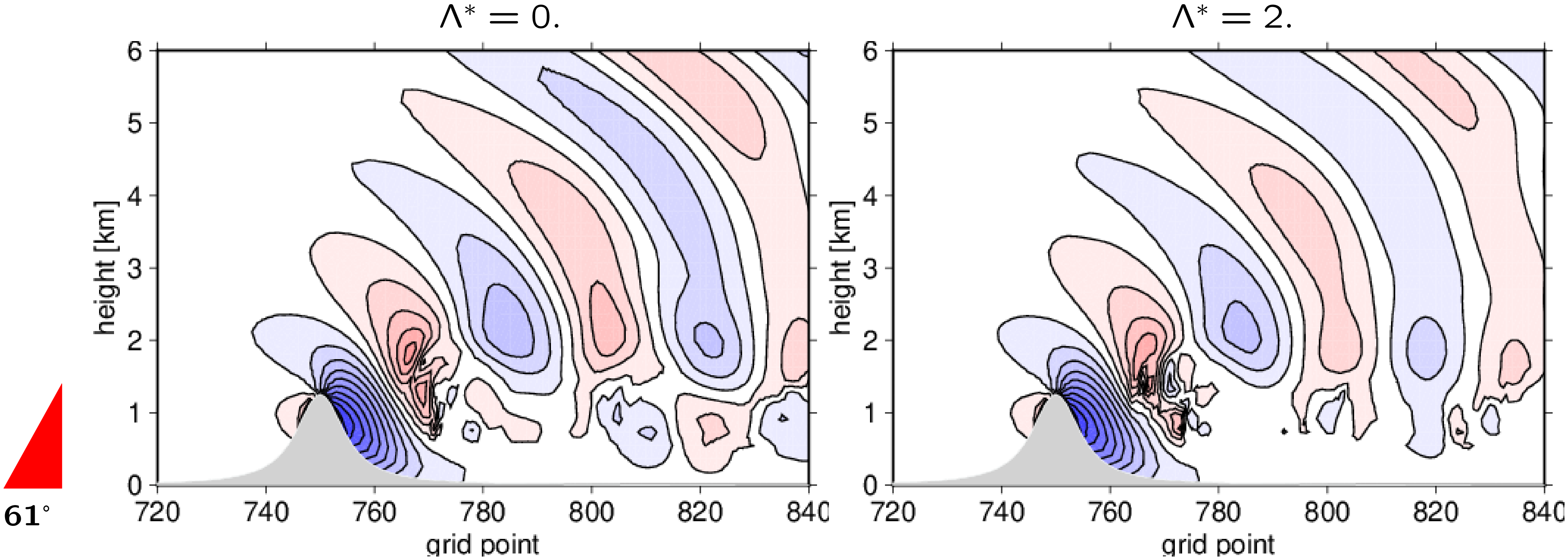
Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope,
$max\left(||\nabla\phi_s||\right) = 1.5$



56°

Idealized tests

Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\left(||\nabla\phi_s||\right) = 1.8$

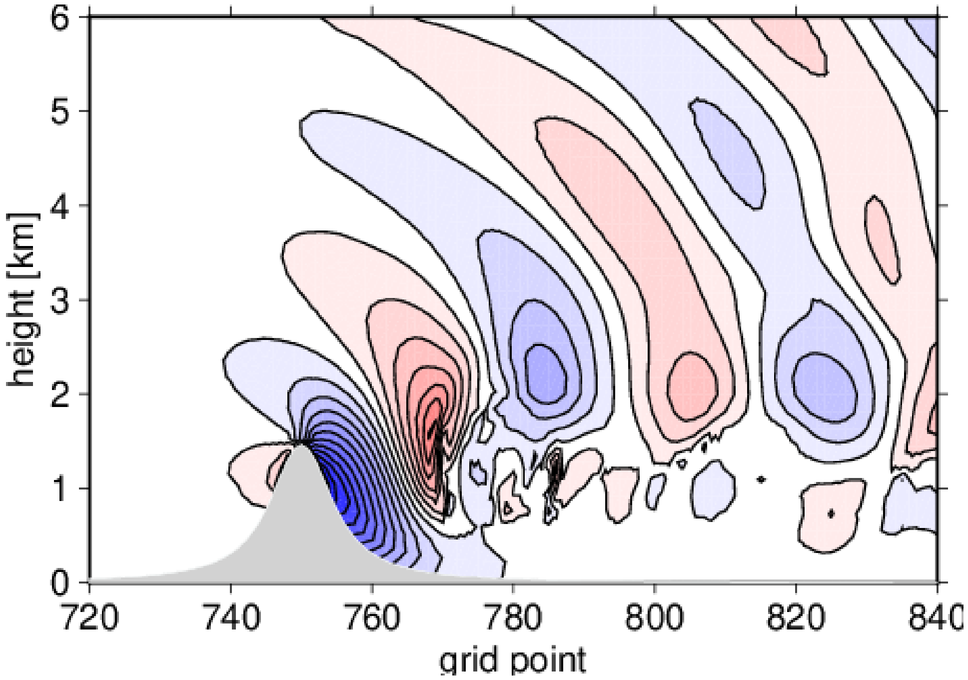


Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\,(||\nabla\phi_s||) = 2$

$\Lambda^* = 0.$

$\Lambda^* = 2.$

63°

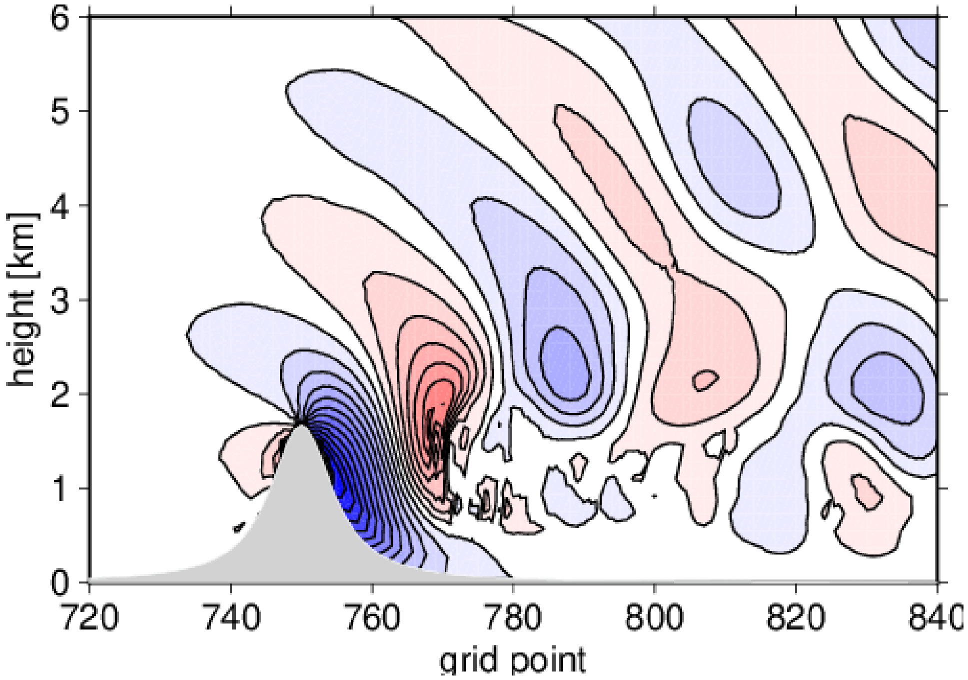


Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\,(||\nabla\phi_s||) = 2.4$

$\Lambda^* = 0.$

$\Lambda^* = 2.$

67°



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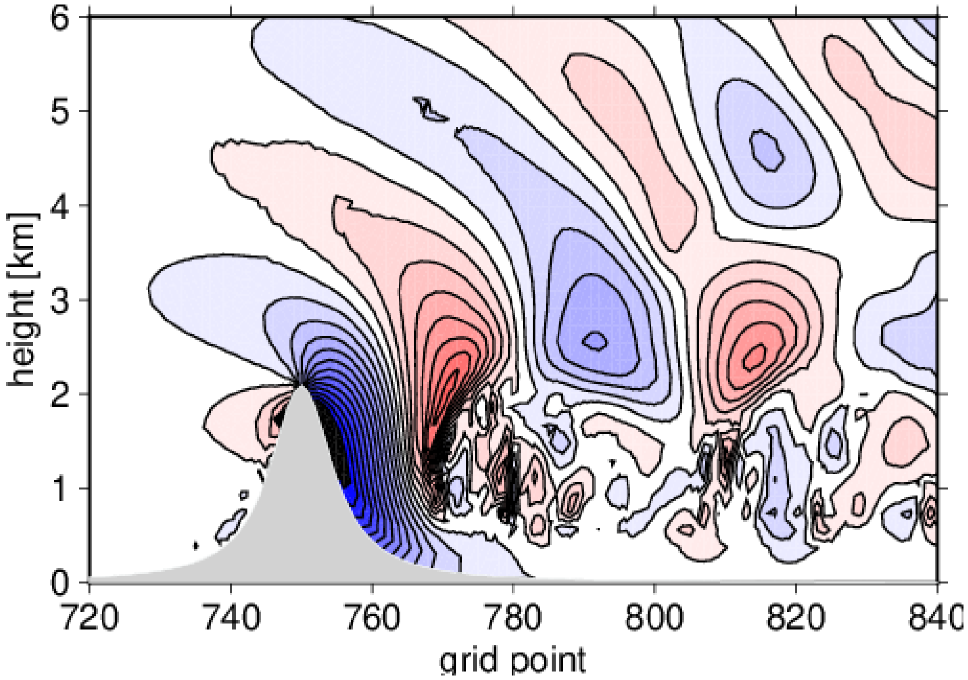
Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\left(||\nabla\phi_s||\right) = 3$

$\Lambda^* = 0.$

$\Lambda^* = 3.$

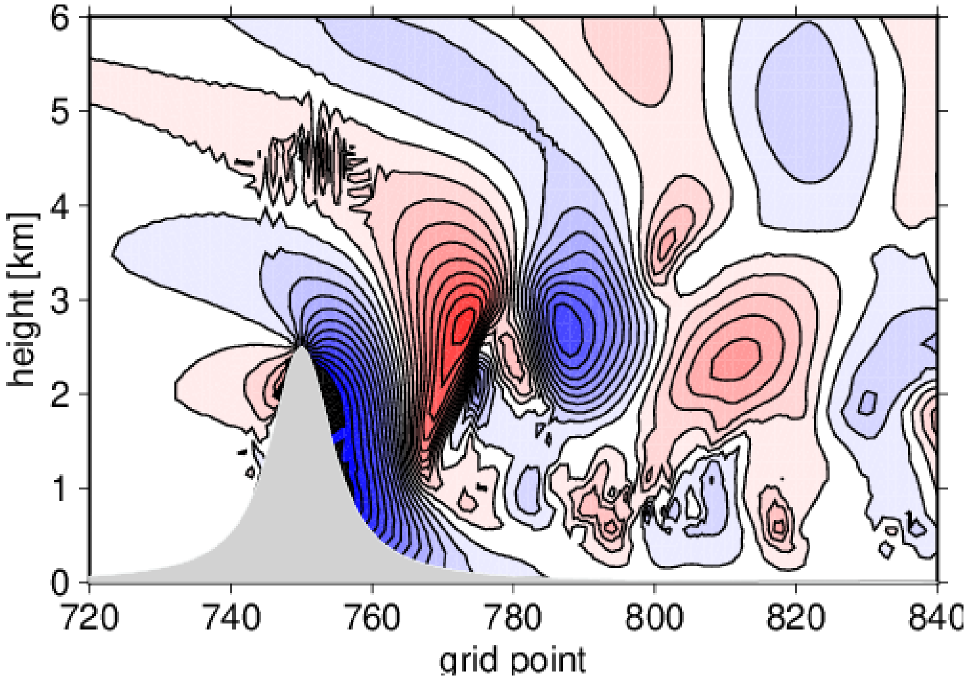


72°

Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\,(||\nabla\phi_s||) = 3.5$

$\Lambda^* = 0.$

$\Lambda^* = 4.$

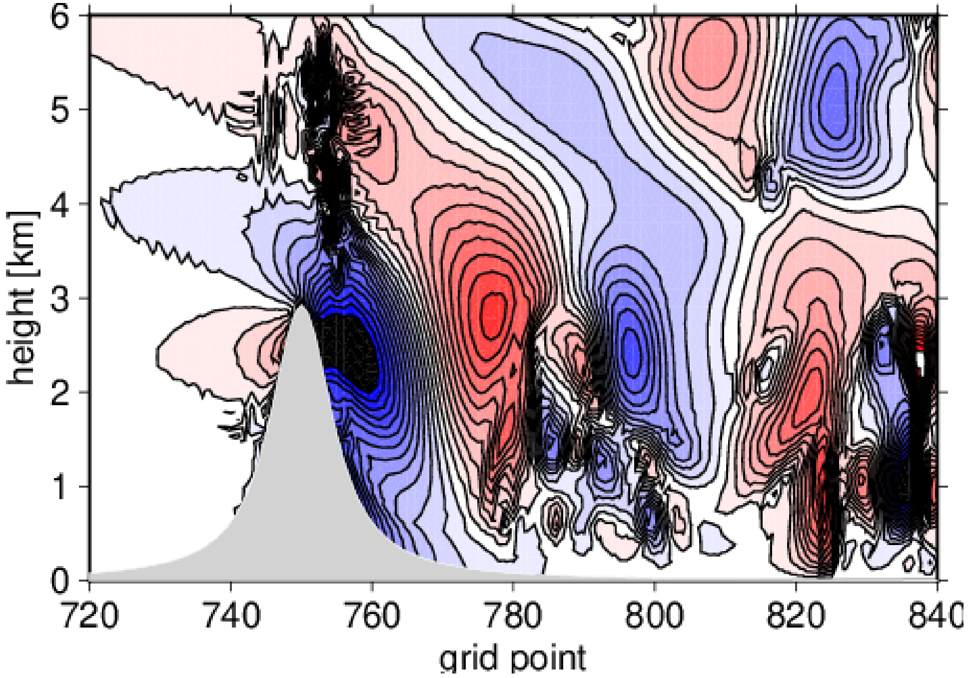


Vertical velocity for the uniform wind over an Agnesi shaped mountain with increasing slope, $max\left(||\nabla\phi_s||\right) = 4$

$\Lambda^* = 0.$

$\Lambda^* = 4.$

76°



Linear model

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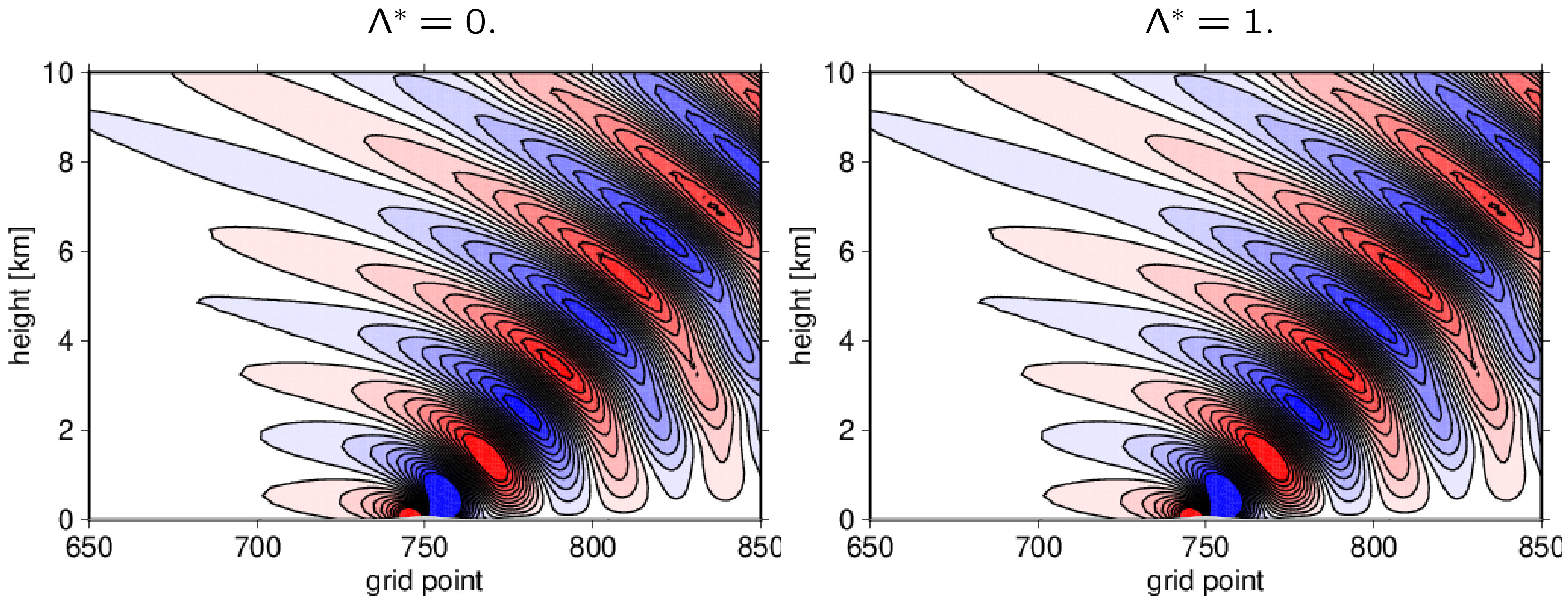
- The ICI scheme in ACCORD uses horizontally constant coefficients.
- In
$$\frac{dX}{dt} = \mathcal{L}^* \overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$
  the linear model $\mathcal{L}^*$ is just a matrix with numbers, constant during the whole integration.
- The Helmholtz matrix calculated from it is constant in time and applied in spectral space on spectral coefficients.
- It means that we chose just one value for each of the reference parameters in the linear model, being the same in the whole domain.
- With the added reference slope it is the same - one value is chosen for the whole domain.
- It follows that we must chose a value which will work well in flat terain too.

Idealized tests

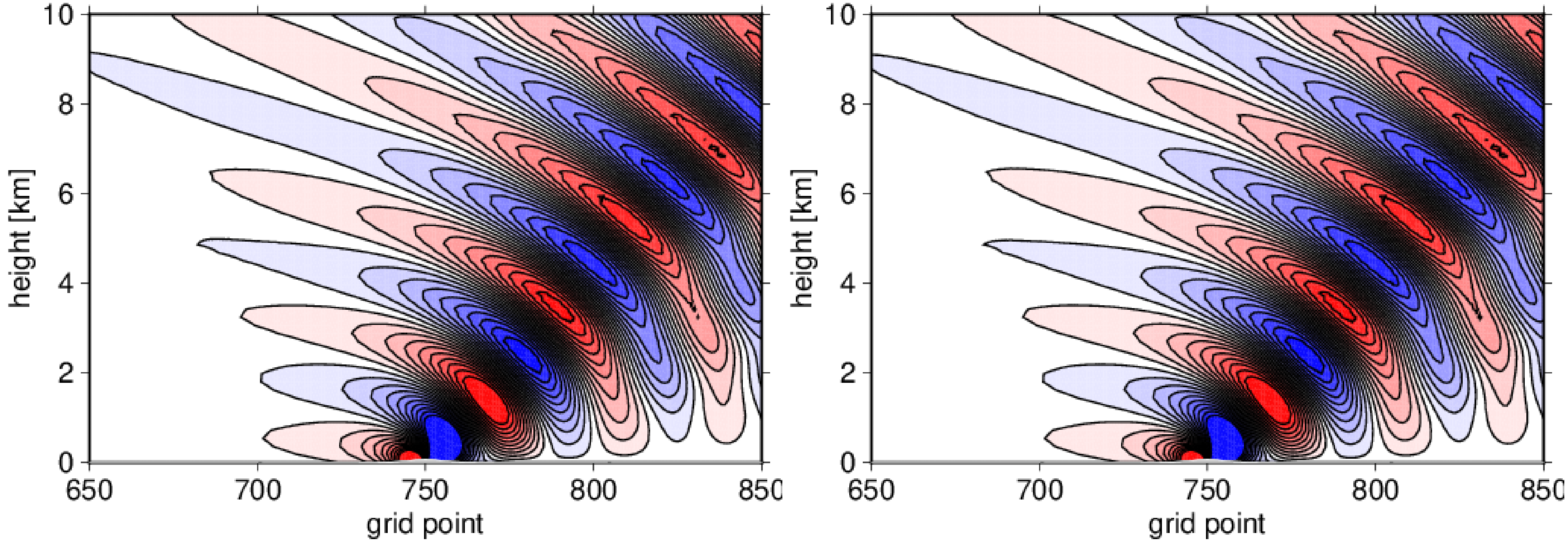
Vertical velocity for the uniform wind over an Agnesi shaped mountain with very small height, $max\left(||\nabla\phi_s||\right) = 0.1$



6°

Vertical velocity for the uniform wind over an Agnesi shaped mountain with very small height, $max\left(||\nabla\phi_s||\right) = 0.1$

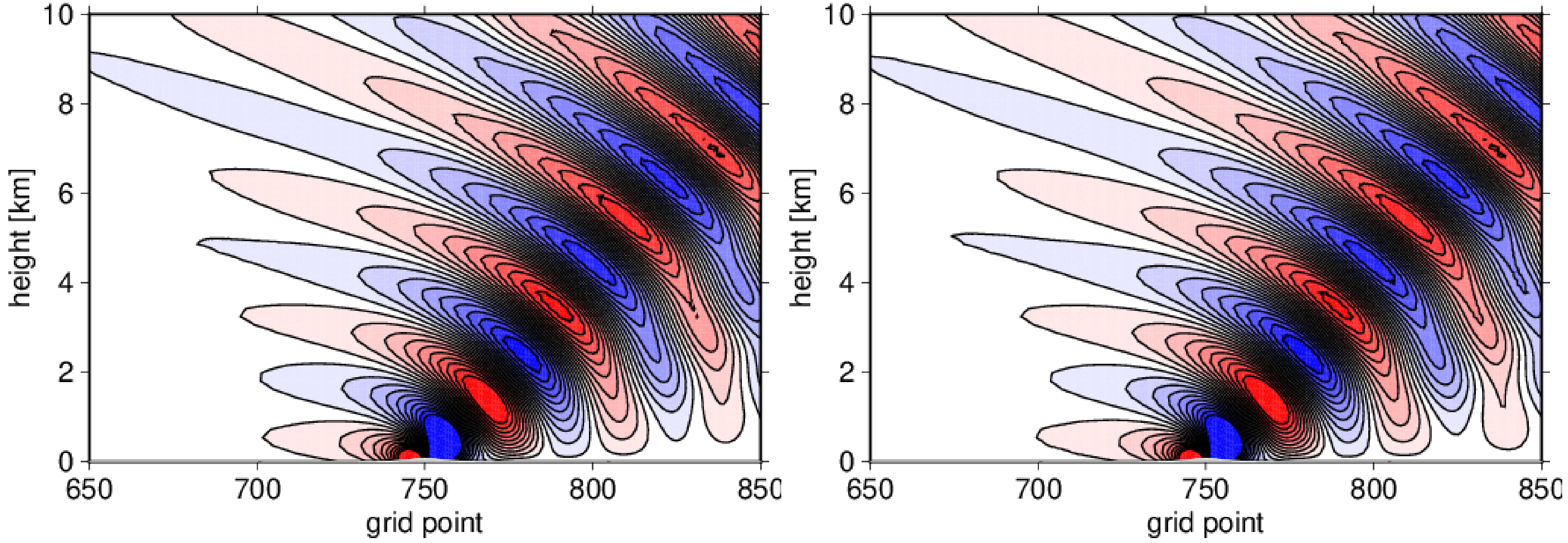
$\Lambda^* = 0.$ $\Lambda^* = 2.$



6°

Vertical velocity for the uniform wind over an Agnesi shaped mountain with very small height, $max\,(||\nabla\phi_s||) = 0.1$

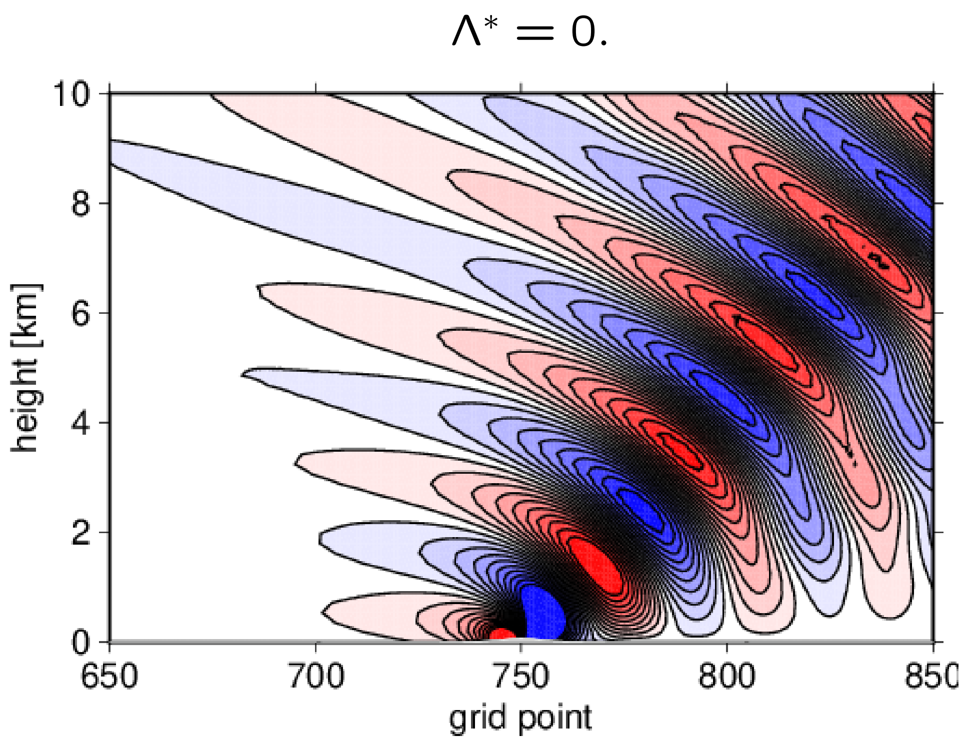
$\Lambda^* = 0.$ $\Lambda^* = 3.$

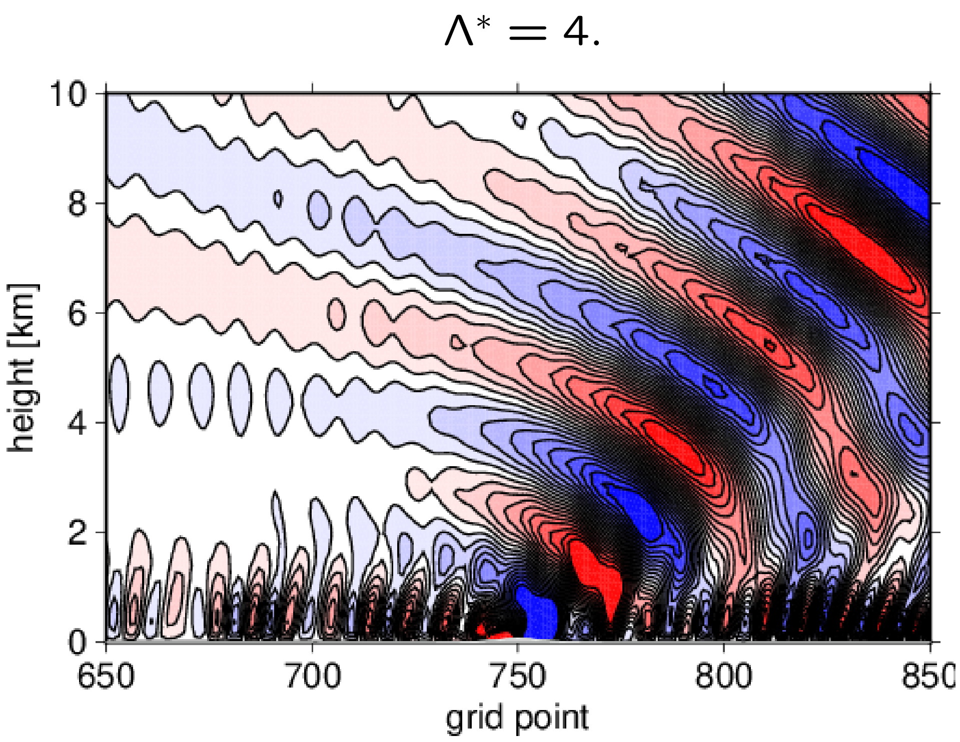


6°

Idealized tests

Vertical velocity for the uniform wind over an Agnesi shaped mountain with very small height, $max\,(||\nabla\phi_s||) = 0.1$

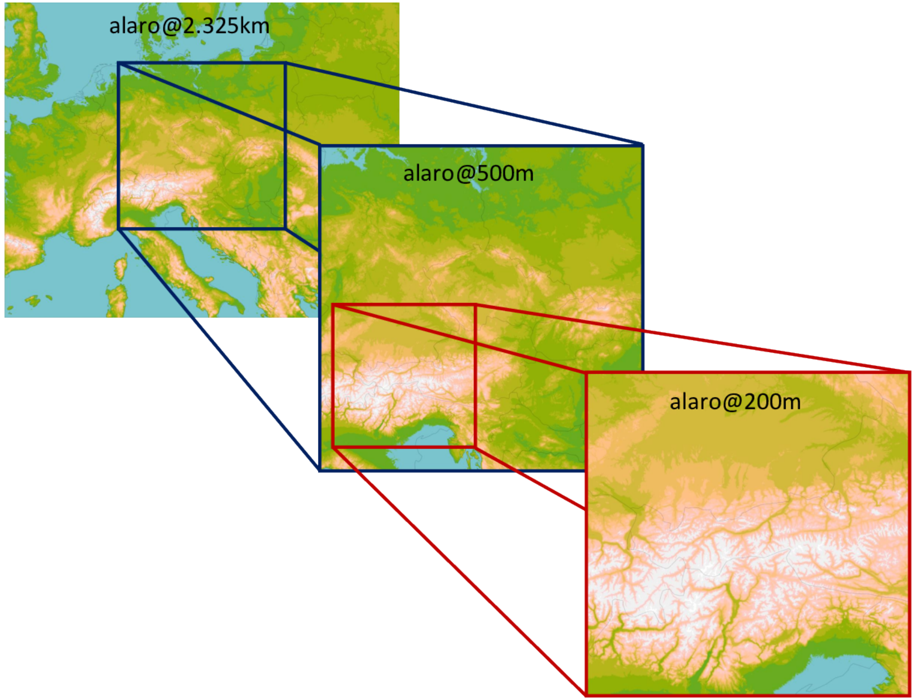




Real simulations @200m

The basic algorithmic choices for ALARO configurations are:

- semi-Lagrangian advection with 4 iterations for trajectory calculation
- PC time scheme with one iteration, cheap variant (SL trajectories are not recalculated in corrector)
- modified vertical divergence d4 for vertical motion, transformation to vertical velocity w in the non-linear model
- reference values of the linear model: SITR=300K, SITRA=100K, SIPR=900hPa
- no decentering
- semi-Lagrangian horizontal diffusion applied on all model variables + TKE,TTE, hydrometeors
- linear truncation for all spectral fields except orography; quadratic truncation of orography



Real simulations @200m

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ALARO physics

- radiation scheme ACRANEB2
- turbulence and shalow convection scheme TOUCANS, model 2
- scale aware deep convection and microphysics scheme 3MT

Initialization

- initialization with 3DVAR + surface DA (CANARI) for 2.325km run; dynamical adaptation + DFI for 500m and 200m runs

Particular choices for ALARO@200m:

- cubic truncation of orography
- SITRA=50K
- no 3MT (deep convection), only STRAPRO (stratiform precipitation)

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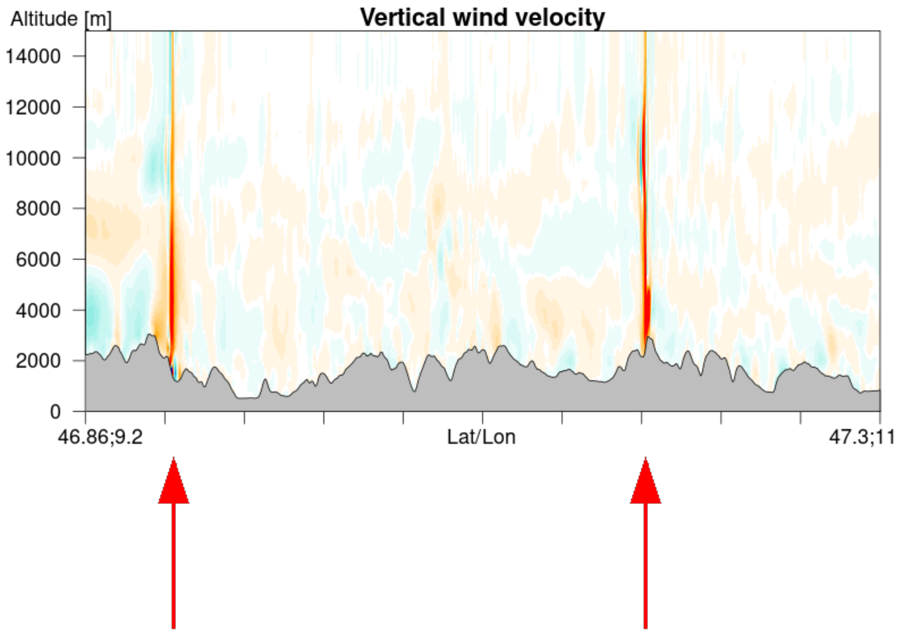
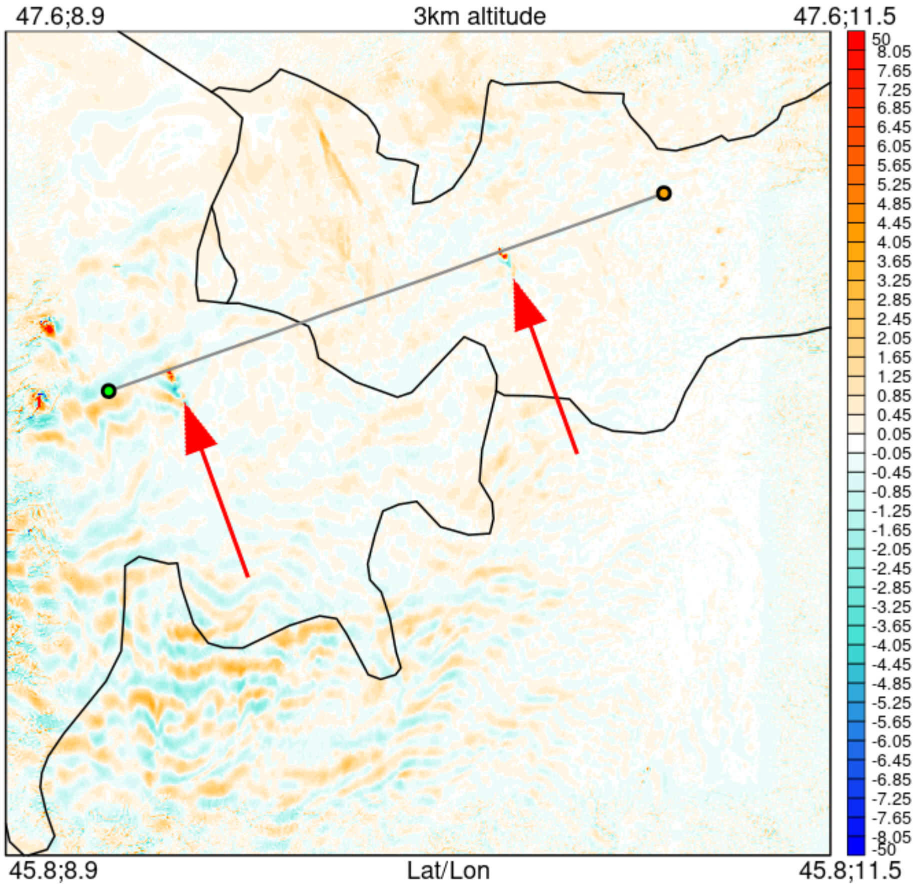
Real simulations

Vertical velocity for the alpine case 19 August 2022 0UTC + 24hours.





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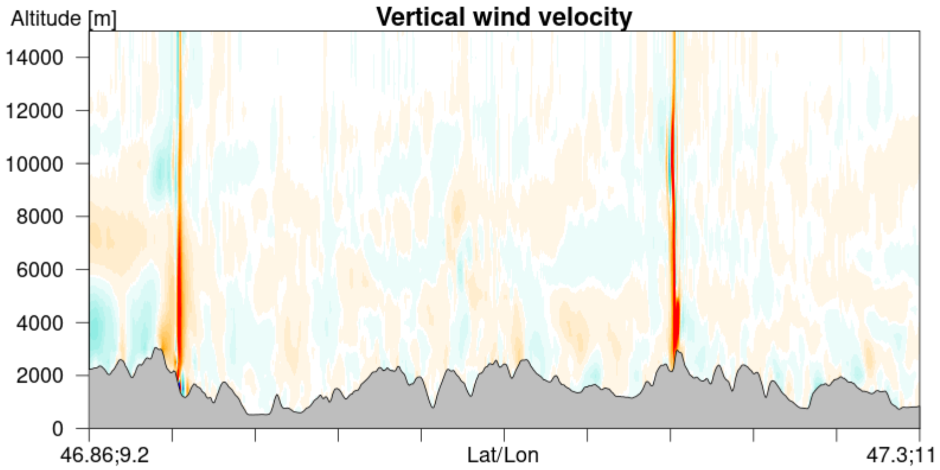
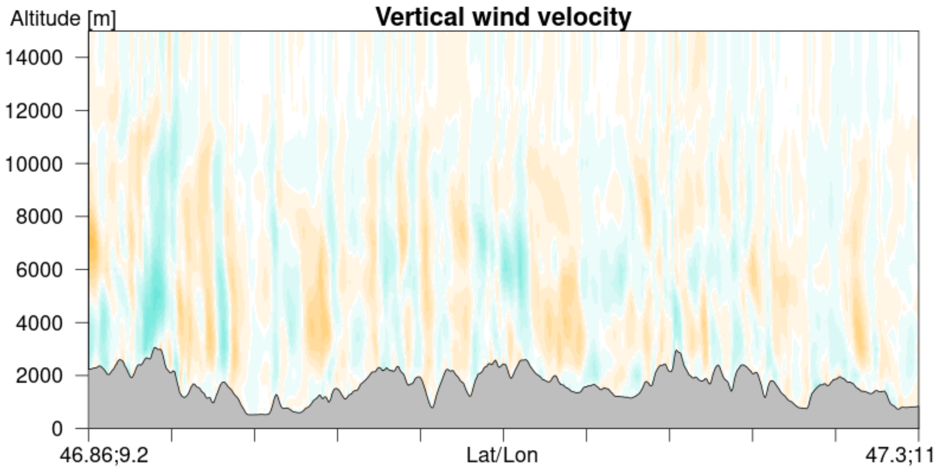
Vertical velocity for the alpine case 19 August 2022 0UTC + 24hours.

$\Lambda^* = 0$



$\Lambda^* = 1$



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Conclusions

- Including constant reference slope in the linear model of the ICI time scheme helps to improve stability.
- With moderate values of the reference slope the accuracy of results is not endangered.
- With very high values of the reference slope the results may be spoiled with noise.


Děkuji Vám za pozornost!