# Improving the robustness of the Predictor-Corrector scheme of AROME models

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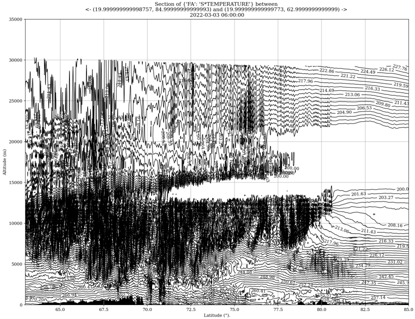
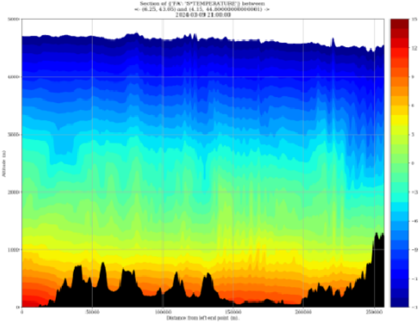
## Some Problematic runs with Predictor-Corrector (PC) scheme (NSITER=1)

AROME Svalbard (1.25 km, L90, 50s)




AROME MedAlp (500m, L120, 20s)



# Plan

## Constant-coefficient Iterative-Centered-Implicit (ICI) scheme

$$\frac{\partial \mathcal{X}}{\partial t} = \mathcal{M}(\mathcal{X}) = \underbrace{(\mathcal{M} - \mathcal{L}^*)(\mathcal{X})}_{\textit{NL-residual}} + \underbrace{\mathcal{L}^*.\mathcal{X}}_{\textit{Linear}}$$

- Iterative centred implicit resolution, for $i \in [0, N_{\text{siter}} - 1]$ :

$$\frac{\mathcal{X}^{+(i+1)} - \mathcal{X}^0}{\delta t} = \frac{(\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^{+(i)}\right) + (\mathcal{M} - \mathcal{L}^*)\left(\mathcal{X}^0\right)}{2} + \frac{\mathcal{L}^*.\mathcal{X}^{+(i+1)} + \mathcal{L}^*.\mathcal{X}^0}{2},$$

- **Quasi-Newton-Raphson approach** : $[\mathcal{I} - (\delta t/2)\mathcal{L}^*]$ plays as a preconditioner $\Rightarrow \mathcal{L}^*$ is typically chosen as a linear counterpart of $\mathcal{M}$ around a reference-state $\mathcal{X}^*$.
- **Constant-coefficient assumption :** The coefficients of $\mathcal{L}$ are taken constant in time and along horizontal directions $\Rightarrow$ Make easier the inversion of $[\mathcal{I} - (\delta t/2)\mathcal{L}^*]$.

# Constant-coefficient Iterative-Centered-Implicit (ICI) scheme

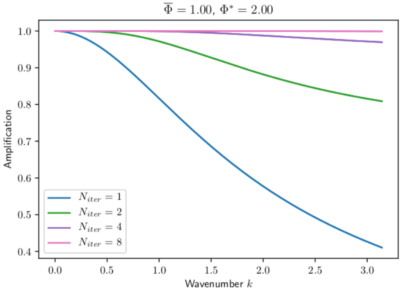
## Stability & Convergence :

They are closely related to the magnitude of

$$\mu = \frac{\delta t}{2} \frac{\|\mathcal{L}_{\bar{X}} - \mathcal{L}^*\|}{\|I - (\delta t/2)\mathcal{L}^*\|}$$

where $\mathcal{L}_{\bar{X}}$ denotes the jacobian of $\mathcal{M}$ at a given state $\bar{X}$. Trying to minimize $\mu$ two strategies can be adopted :

1. Exaggerate $\mathcal{L}^* \rightarrow$ increase NL residual magnitude but also the increase damping effect of SI correction at denominator.
2. Try to reduce NL residuals by maintaining $\mathcal{L}^*$ in a reasonable range around $\mathcal{M}$.




*Courtesy of Daan Degrauwe*

# NL residuals in mass-based EE systems :

- Stability and convergence strongly depend on the magnitudes of the non-linear residual terms ($\mathcal{M} - \mathcal{L}^*$). Such issues even are more significant in fully-compressible models [Bénard *et al.* (2003,2004,2005)].

1. **Thermal NL residuals** due to discrepancy between the actual temperature and the constant temperature used in the SI linear system $\Rightarrow$ Instability issue in presence too cold temperature in the top of the atmosphere.
2. **Orographic NL residuals** due to the presence in of horizontally-varying terrain-following metric terms involving the orography slope that are not taken into account in the SI linear model $\Rightarrow$ Steep slopes stability issue.
3. **Baric NL residuals** (specific to mass-based system) due to the discrepancy between actual hydrostatic surface pressure and the constant hydrostatic pressure entering in the definition of the vertical operartors of the SI linear system $\Rightarrow$ Stability issue in presence of high orography plateau (e.g, Tibetan plateau region).

## Current Constant-Coefficient SI linear of mass-based EE system : $\mathcal{L}^*$

$$\frac{\partial D}{\partial t} = -R\nabla^2 \left[ \left( \mathcal{G}^* (T) + T_r^* q_s \right) + T_r^* \left( \hat{q} - \mathcal{G}^* (\hat{q}) \right) \right]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{R T_a^*} \mathcal{L}^* (\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{R T_r^*}{C_v} [D + d]$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v} [D - (1 - \kappa) \mathcal{S}^* (D) + d]$$

$$\frac{\partial \text{l}_s}{\partial t} = -\mathcal{N}^* (D)$$

### Current SI stability parameters :

- **SITR** : Warm constant SI temperature $T_r^*$ $\Rightarrow$ impact on Thermal NL residuals related to gravity modes and horizontal elastic term (Lamd's mode).
- **SITRA** : Cold constant acoustic SI temperature $T_a^*$ $\Rightarrow$ impact on Thermal NL residuals due to vertical elastic term.
- **SIPR**: Constant SI hydrostatic surface pressure $\pi_{sr}^*$ $\Rightarrow$ Impact on Baric NL residuals.

- (**SITR**, **SITRA**, **SIPR**) are set once for all at the beginning of the Model integration. $\Rightarrow$ Need to be **tuned** over long period of tests trying to prevent that the Model blows up or exhibits noisy solution **for each domain**.

# Plan

- Some useful definitions :

$$T^{\ell}_{max}(t) = \max_{x \in \mathcal{D}_h} T(x, \ell, t)$$

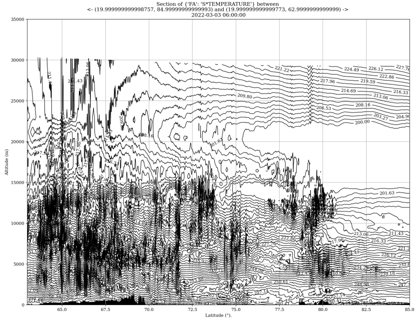
$$T_{max}(t) = \max_{\ell \in [1,L]} T^{\ell}_{max}(t)$$

and

$$T^{\ell}_{min}(t) = \min_{x \in \mathcal{D}_h} T(x, \ell, t)$$

$$T_{min}(t) = \min_{\ell \in [1,L]} T^{\ell}_{min}(t)$$

**SITR**$=300\,K$ and **SITRA**$=100,\,K$




- In Svalbard case, $T_{max}(t)$ never exceed $290\,K$ and $\min[T^{\ell}_{max}(t)]$ reaches cold value around $190\,K$ $\Rightarrow$ Current value **SITR** $= 350\,K$ exaggerates too much the thermal NL residuals $\Rightarrow$ generating noises that only one ICI iteration (PC) can not damp.

## Proposed Solution based on SI strategy n°2

- Reduce Thermal NL residual by defining vertically varying and time-dependent SI parameters from the grid-point norms min/max of temperature at each level as

$$T_r^\ell(t) = T_{max}^\ell(t) + \min\left[\left(T_r^* - T_{max}(t)\right), \beta_r\left(T_{max}^\ell(t) - T_{min}^\ell(t)\right)\right]$$

$$T_a^\ell(t) = \frac{T_{min}^\ell(t)}{2\beta_a + 1}, \qquad \text{for} \ell \in [1, L]$$

- where $\beta_r$ and $\beta_a$ are arbitrarily chosen control parameter lying in $[0, 1]$.

## Towards a PC scheme with updated SI parameters

**Supplementary details :**

1. The suggested formulation for $T_r^\ell(t)$ and $T_r^\ell(t)$ results from theoretical SI analyses (not presented here) but also from empirical knowledges regarding the behaviour of Constant-coefficient SI schemes.

2. SI parameters can be updated at each timestep or possibly at each SI iterations (more costly).

3. For the time being we are still imposing $T_r^s(t) = T_r^*$ and $\pi_{sr}(t) = \pi_{sr}^*$ from the SI_CST approach.

$$\frac{\partial D}{\partial t} = -R\nabla^2 \left[ (\mathcal{G}^*(T) + T_r^s q_s) + \left( T_r^\ell \hat{q} - \mathcal{G}^*(T_r^\ell \hat{q}) \right) \right]$$

$$\frac{\partial d}{\partial t} = -\frac{g^2}{R T_a^\ell} \mathcal{L}^* (\hat{q})$$

$$\frac{\partial T}{\partial t} = -\frac{R T_r^\ell}{C_v} [D + d]$$

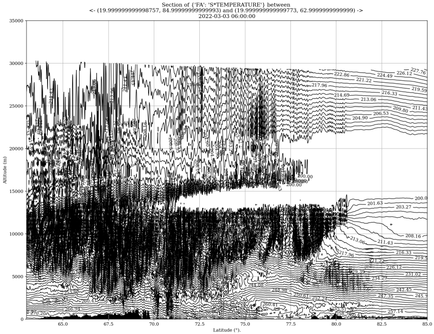
$$\frac{\partial \hat{q}}{\partial t} = -\frac{C_p}{C_v} [D - (1 - \kappa)\mathcal{S}^*(D) + d]$$

$$\frac{\partial \mathrm{l}_s}{\partial t} = -\mathcal{N}^*(D)$$

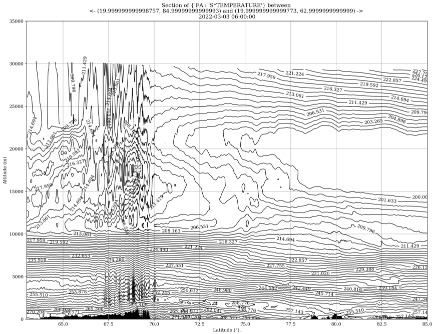
## Svalbard problematic run with PC scheme

AROME (1250m, L90, 50s) Temperature after 30H forecast :

PC + SI_CST




PC + SI_UPDATE

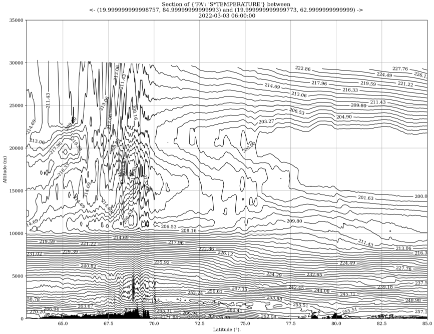



- Updating SI vertical parameters at each timestep with $\beta_r = 0.5$ and $\beta_a = 0.7$.

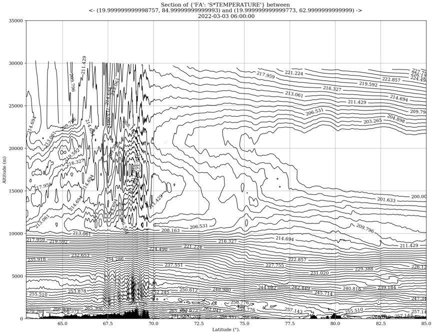
# Svalbard problematic run with PC scheme

AROME (1250m, L90, 50s) Temperature after 30H forecast :

NSITER=2




PC + SI_UPDATE




- PC + SI_UPDATE and (NSITER=2) solutions are comparable.

# Plan

# A constant SI parameter for orography Slope

## Basic underlying idea:

• Take into account the effect of slope in the vertical Laplacian-like operator definition $\mathcal{L}^*$ :

$$\mathcal{L}^*(\hat{q}) = \frac{\pi_r^*}{\partial_\eta \pi_r^*} \partial \left[ \left\{ \frac{\partial_\eta \left( \pi_r^* \hat{q} \right)}{\partial_\eta \pi_r^*} \right\} + \overline{\frac{\nabla \phi^{*2}}{g^2} \overline{\left\{ \frac{\partial_\eta \left( \pi_r^* \hat{q} \right)}{\partial_\eta \pi_r^*} \right\}}^{v}}^{w} \right]$$

with

$$\nabla \phi^* = f \left( \pi_{sr}^*, T_{max}^{\ell}, T_{min}^{L}, \Lambda_s^* \right)$$

• where $\Lambda_s^*$ (**SISLP**) is an additional constant SI parameter chosen as the maximum of orography slope over the domain ⇒

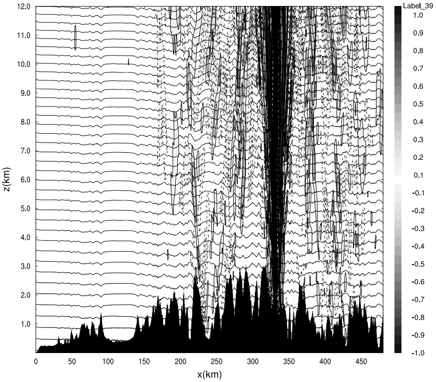
$$\Lambda_s^* = \max_{x \in \mathcal{D}_h} \left[ \sqrt{(\partial_x z_s)^2 + (\partial_y z_s)^2} \right]$$

• Interpolating vertical operators $\overline{(\,)}^{w}$ and $\overline{(\,)}^{v}$ are build in such a way that $\mathcal{L}^*$ akin to a negative definite vertical Laplacian operator.

## No-Flow test over the Alps

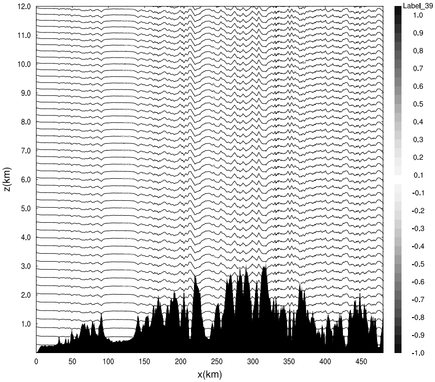
AROME (500m, L90, 20s) vertical velocity $w$, and $iso - \theta$ contours solutions after 24H forecast :

SI_CST (NO SISLP)




NSITER = 2

SI_CST + SISLP

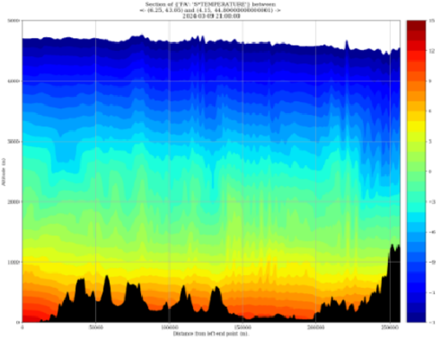



NSITER = 1 (PC)

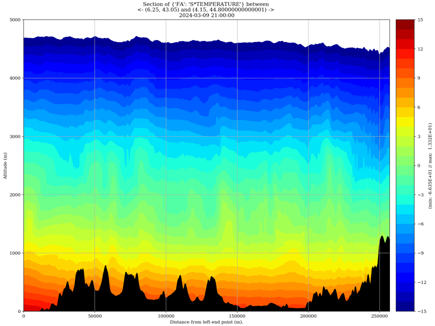
# MedAlp problematic run with PC

AROME (500m, L120, 20s) Temperature after 21H forecast :

PC+SI_CST



PC+SI_UPDATE+SISLP




- **SISLP** = 1.6 corresponding to a maximum slope of 58°.

# Plan

- SI_UPDATE + SISLP strategy offer the possibility to better control the impact of the NL residual terms on stability (at least for the thermal and Orographic NL residuals) and therefore to **enhance the robustness of PC scheme** at quite reasonably cost.
- Such a strategy needs also to be investigated for Baric NL residuals.

- There is still a real risk that such zigzag patterns (noisy solution) appear more frequently as we shift to very higher resolution ($<$500m), mainly due to the absence of horizontal variations in the implicit part of AROME Model. $\Rightarrow$ So **what to do next** if current SI_CST or SI_UPDATE fails ?

### Quo Vadis ?

Two avenues can be considered (in parallel) :

1. **Develop from the existing code an alterrnate non-constant coefficient mass-based fully compressible Implicit Solver** $\Rightarrow$ strongly impose to moving away from spectral method to adopt a more local horizontal discretisation as suggested by [Bénard and Glinton (2019a) (2019b)]
2. **Replace our Dyncore by a brand new one** $\Rightarrow$ FVM-LAM developped by C. Kuhnlein (ECMWF) in collaboration with ETH (Swiss) seems to be a good candidate.

**Thanks for listening !!!**