Regional Cooperation for
Limited Area Modeling in Central Europe

LACE
nwp central europe

ACCORD
A Consortium for COnvection-scale modelling
Research and Development

Dynamics for ACCORD model in RC LACE operations and in DEODE project

Petra Smolíková (CHMI), Fabrice Voitus (MF), Nika Kastelec (ARSO),
Natalia Szopa (IMGW), Mario Hrastinski (DHMZ) and Jozef Vivoda (ECMWF)

GeoSphere Austria · DHMZ · Czech Hydrometeorological Institute · HungaroMet · IMGW · METEO ROMANIA · SHMU · ARSO METEO Slovenia

Outline

- Dynamical core in ACCORD
- SI time scheme
- Novelties in dynamics
- Blended approach for fields initialization
- Real simulations @500m and @200m
- Real simulations @2.2km
- Fast spectral transformations ?

Basic equations

- hydrostatic primitive equation system (HPE) or Euler equations (EE); recently implemented quasi elastic equation system (QE)
- prognostic variables $\vec{v}, T, q_s = \ln(\pi_s)$, in EE with $w, \hat{q} = \ln(\frac{p}{\pi})$

Discretization

- spectral method for horizontal direction
- hybrid vertical coordinate η based on hydrostatic pressure $\pi(\eta) = A(\eta) + B(\eta)\pi_s$; $A(top) = B(top) = 0, A(bottom) = 0, B(bottom) = 1$
- finite differences or finite elements for vertical direction discretization
- semi-implicit or iterative centred implicit scheme for time discretization
- semi-Lagrangian advection

Semi-Implicit time scheme

System evolution

$$\frac{dX}{dt} = \mathcal{M}X$$

Linearization

$$X = \mathbf{X}^* + \mathbf{X}', \quad \mathcal{M} \longrightarrow \mathcal{L}^*$$

Using linear model $\mathcal{L}^*$ we divide

$$\frac{dX}{dt} = \mathcal{L}^* \overline{[X]}^t + (\mathcal{M} - \mathcal{L}^*)X$$

and discretize in time to obtain

Semi-implicit scheme

$$\frac{\mathbf{X}^+ - \mathbf{X}^0}{\Delta t} = \mathcal{L}^* \left(\frac{\mathbf{X}^+ + \mathbf{X}^0}{2} \right) + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+\frac{1}{2}}$$

or

Iterative centered implicit scheme

$$\frac{\mathbf{X}^{+(n)} - \mathbf{X}^0}{\Delta t} = \frac{\mathcal{L}^*\mathbf{X}^{+(n)} + \mathcal{L}^*\mathbf{X}^0}{2} + \frac{(\mathcal{M} - \mathcal{L}^*)\mathbf{X}^{+(n-1)} + (\mathcal{M} - \mathcal{L}^*)\mathbf{X}^0}{2}$$

We know that both can be second order accurate in time when $\mathrm{X}^{+\frac{1}{2}}$ properly defined.

Full model

LACE
nwp central europe

Prognostic equations

$$\frac{dT}{dt} = -\frac{\kappa T}{1-\kappa}(D+d)$$

$$\frac{d\vec{v}}{dt} = -RT\frac{\vec{\nabla}\pi}{\pi} - \vec{\nabla}\phi - RT\vec{\nabla}\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\vec{\nabla}\phi$$

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa}(D+d) - \frac{\dot{\pi}}{\pi}$$

$$\frac{dq_s}{dt} = -\frac{1}{\pi_s}\int_0^1 \vec{\nabla}\cdot(m\vec{v})\ d\eta$$

Diagnostic relations and definitions

$$\dot{\pi} = \vec{v}\cdot\vec{\nabla}\pi - \int_0^\eta \vec{\nabla}\cdot(m\vec{v})\ d\eta'$$

$$\phi = \phi_s + \int_\eta^1 \frac{mRT}{p}\ d\eta'$$

$$D = \vec{\nabla}\cdot\vec{v}$$

$$d = \frac{p}{mRT}\left(\vec{\nabla}\phi\,\frac{\partial\vec{v}}{\partial\eta} - g\frac{\partial w}{\partial\eta}\right)$$

$$\kappa = \frac{c_p}{R}$$

$$m = \frac{\partial\pi}{\partial\eta}$$

GeoSphere Austria DHMZ Czech Hydrometeorological Institute HungaroMet IMGW METEO ROMANIA SHMU ARSO METEO Slovenia

Linear model

LACE nwp central europe

Prognostic equations

$$\frac{\partial T}{\partial t} = -\frac{\kappa^* T^*}{1-\kappa^*}(D+d)$$

$$\frac{\partial \vec{v}}{\partial t} = -RT^*\frac{\vec{\nabla}\pi}{\pi^*} - \vec{\nabla}\phi - RT^*\vec{\nabla}\hat{q}$$

$$\frac{\partial w}{\partial t} = \frac{g}{m^*}\frac{\partial \pi^* \hat{q}}{\partial \eta}$$

$$\frac{\partial \hat{q}}{\partial t} = -\frac{1}{1-\kappa^*}(D+d) + \frac{1}{\pi^*}\int_0^{\eta} m^* D \, d\eta'$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s^*}\int_0^1 m^* D \, d\eta$$

For stability reasons rather:

$$\longrightarrow \quad \frac{\partial D}{\partial t} = -RT^*\frac{\Delta\pi}{\pi^*} - \Delta\phi - RT^*\Delta\hat{q}$$

$$\frac{\partial d}{\partial t} = -g^2\frac{\pi^*}{m^* RT^*}\frac{\partial}{\partial \eta}\left(\frac{1}{m^*}\frac{\partial \pi^* \hat{q}}{\partial \eta}\right)$$

Diagnostic relations and definitions

$$\Delta\phi = -\int_{\eta}^{1} \Delta\left(\frac{m^* RT^*}{\pi^*}\right) d\eta'$$

$$\kappa^* = \left(\frac{c_p}{R}\right)_{dry}$$

$$m^* = \frac{\partial \pi^*}{\partial \eta}$$

Linear model

- to solve the linear system, we eliminate the system up to one model variable ψ
- we solve the Helmholtz equation of the shape $\left[\mathbb{I} - \delta t^2 \Delta \mathbb{B}^*_{\psi}\right] \psi = \psi^{\bullet\bullet}$ for this variable ψ
- we back substitute to get solution for all model variables
- the shape of the linear system and of the non-linear residual depends on the choices in definition of model variables and influences the numerical stability of the system time evolution and the accuracy of the solution
- we tend to stay close to the hydrostatic primitive equation system and it's solution

Novelties in dynamics

- available in DEODE workflow CY48t3 and CY49t2, and in ACCORD CY50T1

- Helmholtz elimination up to horizontal divergence D
- New vertical divergence d formulation
- New bottom boundary condition for vertical velocity w
- Consistent inclusion of moisture in vertical motion variables
- Diagnostic definition of the orographic X-term in vertical divergence d

Helmholtz elimination up to horizontal divergence

LACE
nwp central europe

Previous solution:

- elimination up to vertical divergence d in NHEE
- the vertical operators have to satisfy the necessary constraint
- space discretized solution does not corespond exactly the space continuous one ($\mathbb{T} \neq \mathbb{I}$)

LSI_NHEE=F

$$\left[\mathbb{I} - \frac{\Delta t}{2}\Delta\mathbb{B}_d^*\right] d = d^{\bullet\bullet}$$

$$\mathbb{B}_d^* = \frac{1}{1-\kappa^*}\mathbf{H}^{*-1}\left(RT^* + \kappa^* g^2 \frac{\Delta t}{2}\mathbb{T}\right)$$

Proposed solution:

- elimination up to horizontal divergence D in NHEE
- no constraint; similar to HPE elimination
- B-matrix has hydrostatic and non-hydrostatic part (allows for blended NH-HY approach)
- more suitable for VFE discretization

LSI_NHEE=T

$$\left[\mathbb{I} - \frac{\Delta t}{2}\Delta\mathbb{B}_D^*\right] D = D^{\bullet\bullet}$$

$$\mathbb{B}_D^* = \underbrace{RT^*\left[\kappa^*\mathbf{G}^*\mathbf{S}^* + \mathbf{N}^*\right]}_{\text{hydrostatic}} + \underbrace{RT^*\frac{1}{1-\kappa^*}\mathbf{G}_\kappa{}^*\mathbf{H}^{*-1}\mathbf{S}_\kappa^*}_{\text{nh increment}}$$

(courtesy of Fabrice Voitus)

Vertical motion variables

LACE
nwp central europe

LGWADV=T allows for the usage of vertical divergence $\hat{d}$ in the linear model and vertical velocity w in the non-linear model with the transformations between them

$$\hat{d} = -\frac{p}{mRT}\frac{\partial gw}{\partial \eta}$$
$$w = \frac{1}{g}\int_{\eta}^{1}\left(\frac{mRT}{p}\;\hat{d}\right) d\eta'$$

The advantages: $\hat{d}$ helps with stability of the implicit linear solver, while w with the accuracy of the non-linear residual.

Moreover, the vertical divergence variable can be modified three times

$$d = \hat{d} + \mathbf{X}$$
$$\mathbf{X} = \delta_S \mathbf{X}^S + \delta_d \mathbf{X}^d + \delta_w \mathbf{X}^w$$

where

δ_S is mastered with NVDVAR=4
δ_d is mastered with NVDVAR=5
δ_w is mastered with LBIGW=T

(developed by Fabrice Voitus)

with consequences on the shape of linear system and non-linear residual, on the numerical stability, accuracy and on the bottom boundary condition.

GeoSphere Austria · DHMZ · Czech Hydrometeorological Institute · HungaroMet · IMGW · METEO ROMANIA · SHMÚ · ARSO METEO Slovenia

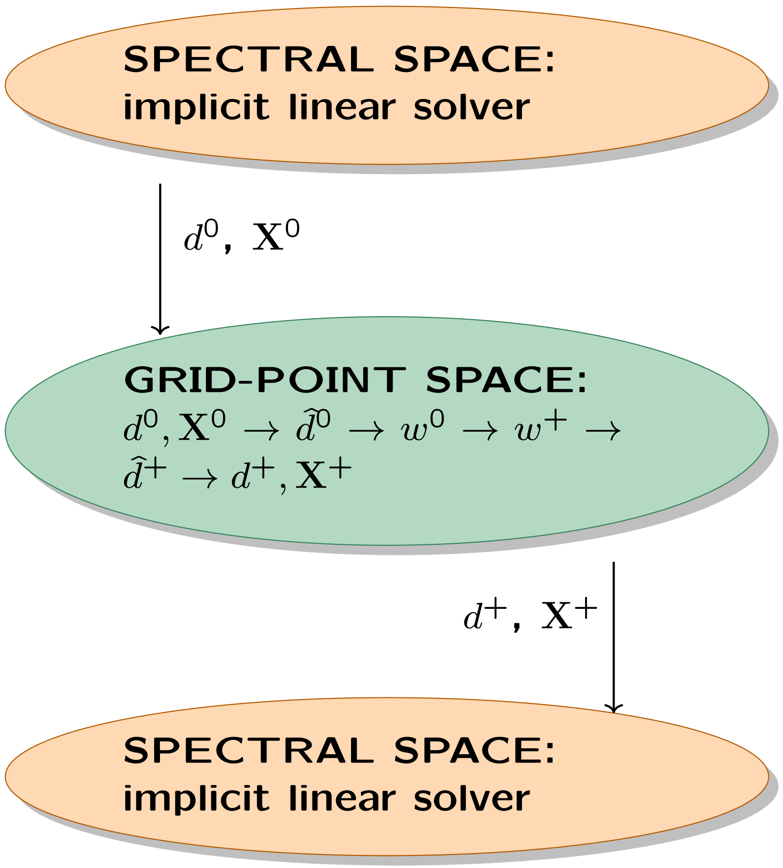
Vertical motion variables

At each time step or at each iteration of the ICI time scheme the following process is realized:

- the implicit part is realized in the SP space
- the vertical divergence d is transformed into the GP space, possibly together with the X-term (option LSPNHX=TRUE)
- the true vertical divergence $\hat{d}$ is being calculated as $\hat{d} = d - \mathrm{X}$ and w is calculated from $\hat{d}$
- this value is available as w^0 in predictor and as $w^{+(k-1)}$ in the k-th corrector
- the new value $w^{+(k)}$ comes from

$$\frac{w_F^{+(k)} - w^0_{O(k)}}{\Delta t} = \frac{(\mathcal{M} - \mathcal{L}^*)[w]^0_{O(k)} + (\mathcal{M} - \mathcal{L}^*)[w]_F^{+(k-1)}}{2}$$

- $w^{(k)}$ is transformed to $\hat{d}^+$ and d^+
- d^+ is transformed back into the SP space for implicit calculations for the next iteration or the next time step, possibly together with the newly calculated X-term (option LSPNHX=TRUE)



Vertical divergence

Previous: NVDVAR=4

$$d = \hat{d} + \mathbf{X}^S$$

$$\mathbf{X}^S = \frac{p}{mRT} \vec{\nabla} \phi \frac{\partial \vec{v}}{\partial \eta}$$

from NL part of T-equation

$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa}(D + d)$$
$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa}(D + d + \mathbf{X}^d) + \mathrm{S}D$$

Proposed: NVDVAR=5

$$d_5 = \hat{d} + \mathbf{X}^S + \mathbf{X}^d$$

$$\mathbf{X}^d = (1-\kappa)\left(\frac{\dot{\pi}}{\pi} + \mathrm{S}D\right)$$

from NL part of hydrostatic T-equation

$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa}(D + d_5 - \mathbf{X}^d)$$
$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa}(D + d_5) + \mathrm{S}D$$

In linear model

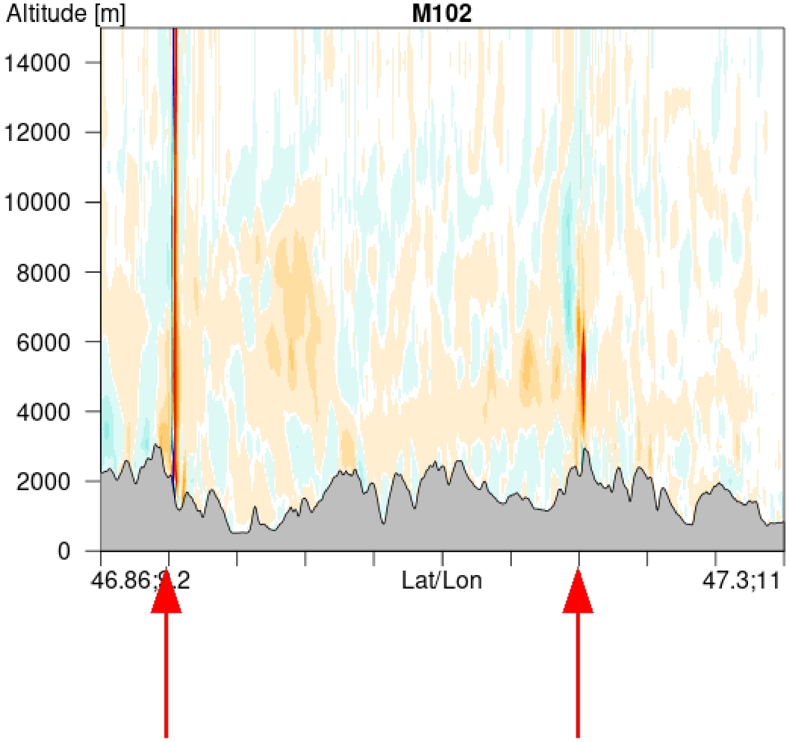
$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa}(D + d_i)$$
$$\frac{d\hat{q}}{dt} = -\frac{1}{1-\kappa}(D + d_i) + \mathrm{S}D$$

The non-linear term X^d is hidden inside the vertical divergence variable in $\hat{q}$-equation and temperature equation stays closer to the HPE solution

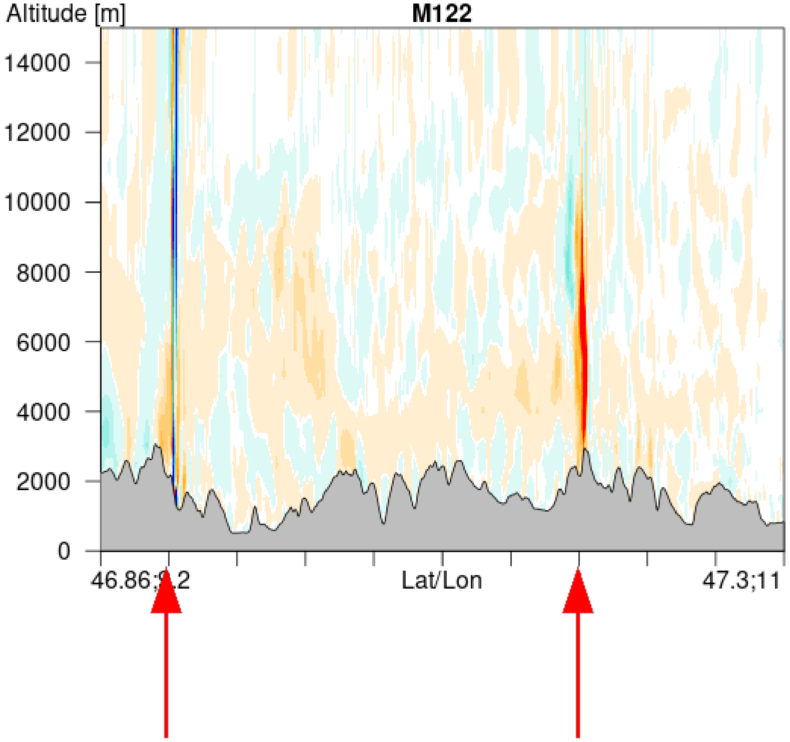
$$\frac{dT}{dt} = -\kappa T \frac{1}{1-\kappa}\left((1-\kappa)\,\mathrm{S}D - \mathbf{X}^d\right)$$

Vertical divergence

NVDVAR=4



NVDVAR=5

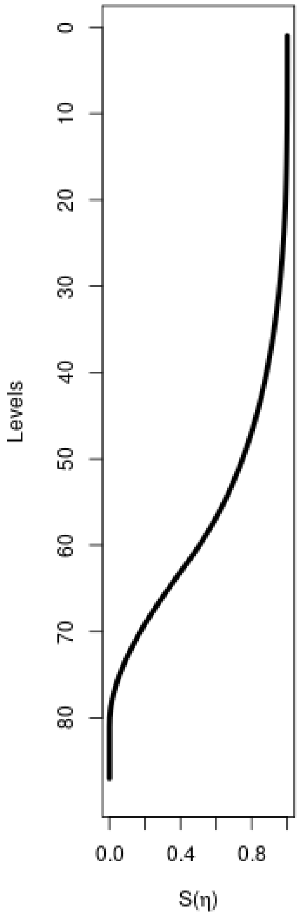


- more consistent
- the chimney like patterns still persistent

LBIGW=.TRUE.

New vertical velocity $\quad g\mathbf{W} = g\mathbf{w} - S(\eta)\ \vec{v} \cdot \vec{\nabla}\phi_s$

where $\quad S(\eta) = \dfrac{B(\eta)\pi_{ref}}{A(\eta) + B(\eta)\pi_{ref}}$



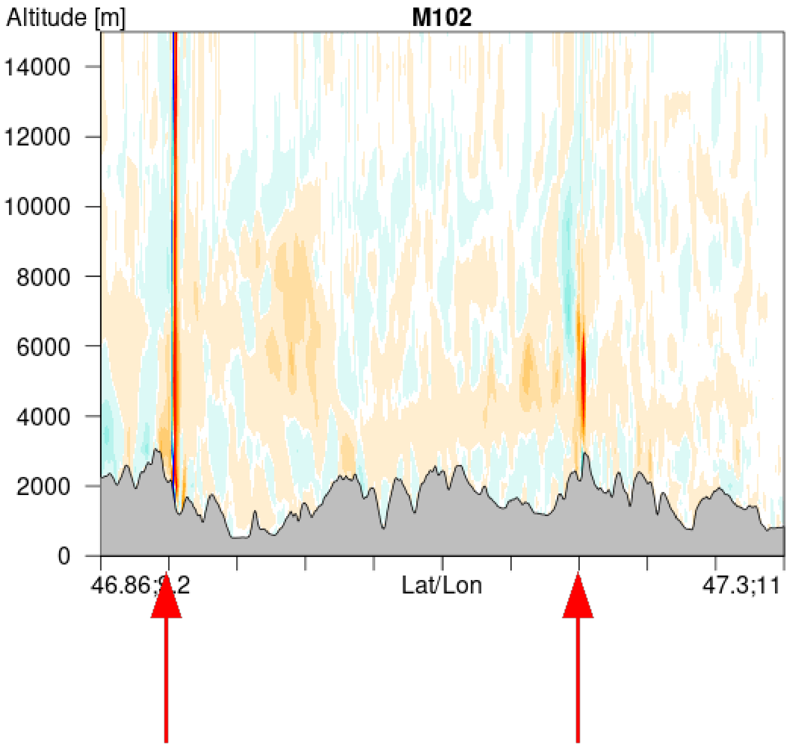
- $S = S(\eta)$ is a prescribed monotonic vertical function satisfying $S(0) = 0$ at the top, and $S(1) = 1$ at the bottom
- $S(\eta)\nabla\Phi_S$ fits $\nabla\Phi$ for a stationary isothermal hydrostatic atmosphere.
- W behaves as w at the top and as $\dot{\eta}$ at the bottom.
- Complicated rigid BBC for w reads $gw_S = \vec{v}_S \cdot \vec{\nabla}\phi_s,\ \ g\dot{w}_S = \dot{\vec{v}_S} \cdot \vec{\nabla}\phi_s + J_S$.
- Rigid BBC reads $\mathbf{W}_S = 0, \dot{\mathbf{W}}_S = 0$.
- It can be seen as a third X–term part added to d

$$\mathbf{X}^w = -\frac{p}{mRT}\frac{\partial}{\partial\eta}(g\mathbf{W} - g\mathbf{w})$$

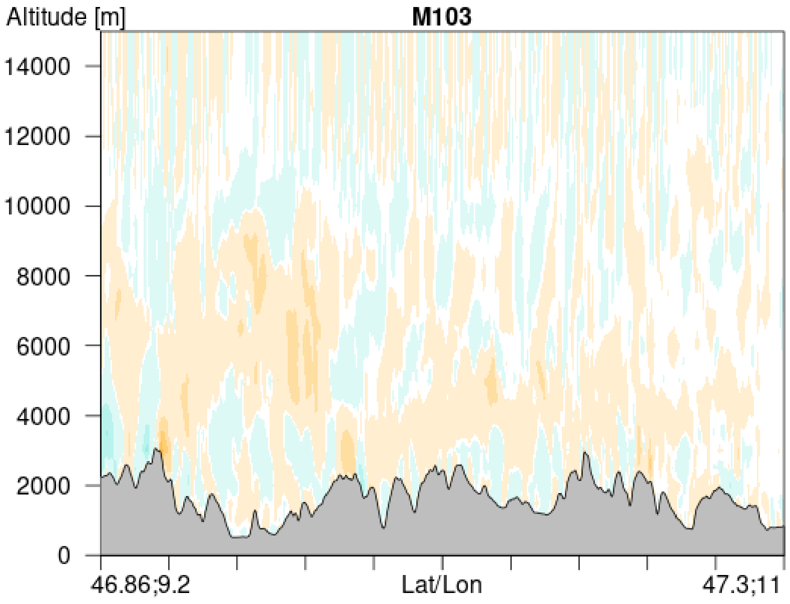
(developed by Fabrice Voitus)

Vertical velocity

LBIGW=.FALSE.



LBIGW=.TRUE.



- simple BBC
- the chimney like patterns disappear

Consistent inclusion of moisture in $\hat{d}$ and X

The definition of the vertical divergence variable is made consistently everywhere in the code with the dry variant of the gaz constant R_d.

L_RDRY_VD=T

$$\hat{d} = -\frac{p}{m(\delta_v R_d + (1-\delta_v)R)T}\frac{\partial gw}{\partial \eta}$$

The definition of X-term is treated independently depending on the key L_RDRY_NHX.

The definition of the X-term is made consistently everywhere in the code with the dry variant of the gaz constant R_d.

L_RDRY_NHX=T

$$\mathrm{X} = \frac{p}{m(\delta_X R_d + (1-\delta_X)R)T}\vec{\nabla}\phi\frac{\partial \vec{v}}{\partial \eta}$$

The true 3D-divergence is always calculated with the moist $\hat{d}$.

(developed by Fabrice Voitus)

Diagnostic treatment of the X-term

Previous solution:
prognostic treatment of X

ND4SYS=1 and ND4SYS=2

$$\hat{d}^{+} = RHS\left[\hat{d}\right]$$
$$d = \hat{d} + \mathbf{X}$$
$$d^{+} = \hat{d}^{+} + \frac{dX}{dt} \cdot \Delta t$$

complicated treatment of $\frac{d\mathbf{X}}{dt}$

Proposed solution:
diagnostic treatment of X

ND4SYS=0

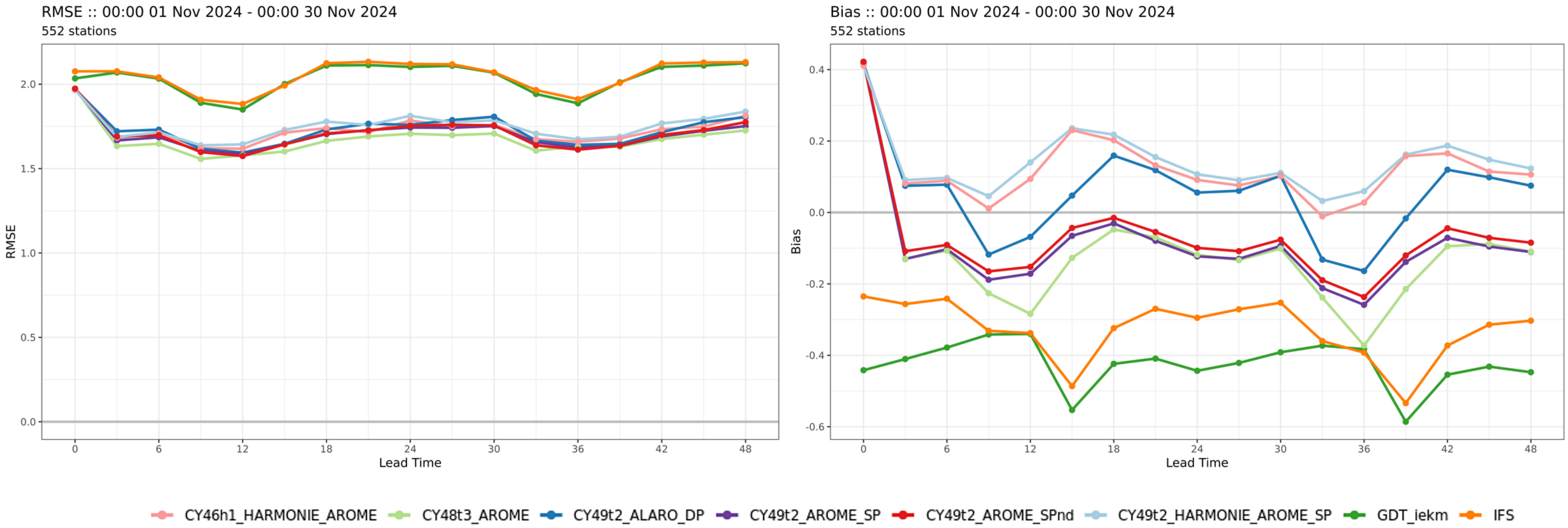
$$\hat{d}^{+} = RHS\left[\hat{d}\right]$$
$$d^{+} = \hat{d}^{+} + \mathbf{X}^{+}$$

easy calculation of X^{+}

The X-term may be calculated at the beginning and at the end of the grid-point calculations. The transformation of the X-term to and from spectral space may be avoided resulting in the reduced usage of the CPU time ($\approx 6-8\%$ with LSPNHX=.TRUE.).

Comparison of reference and new dynamics

- in DEODE workflow @ 500m resolution, Mediterranean domain, one month verifications (November 2024), one run per day 0UTC+48 hours
- AROME physics, PC scheme, spectral diffusion, cubic truncation, 90 vertical levels
- RMSE and BIAS for 10m wind speed; reference dynamics, new dynamics



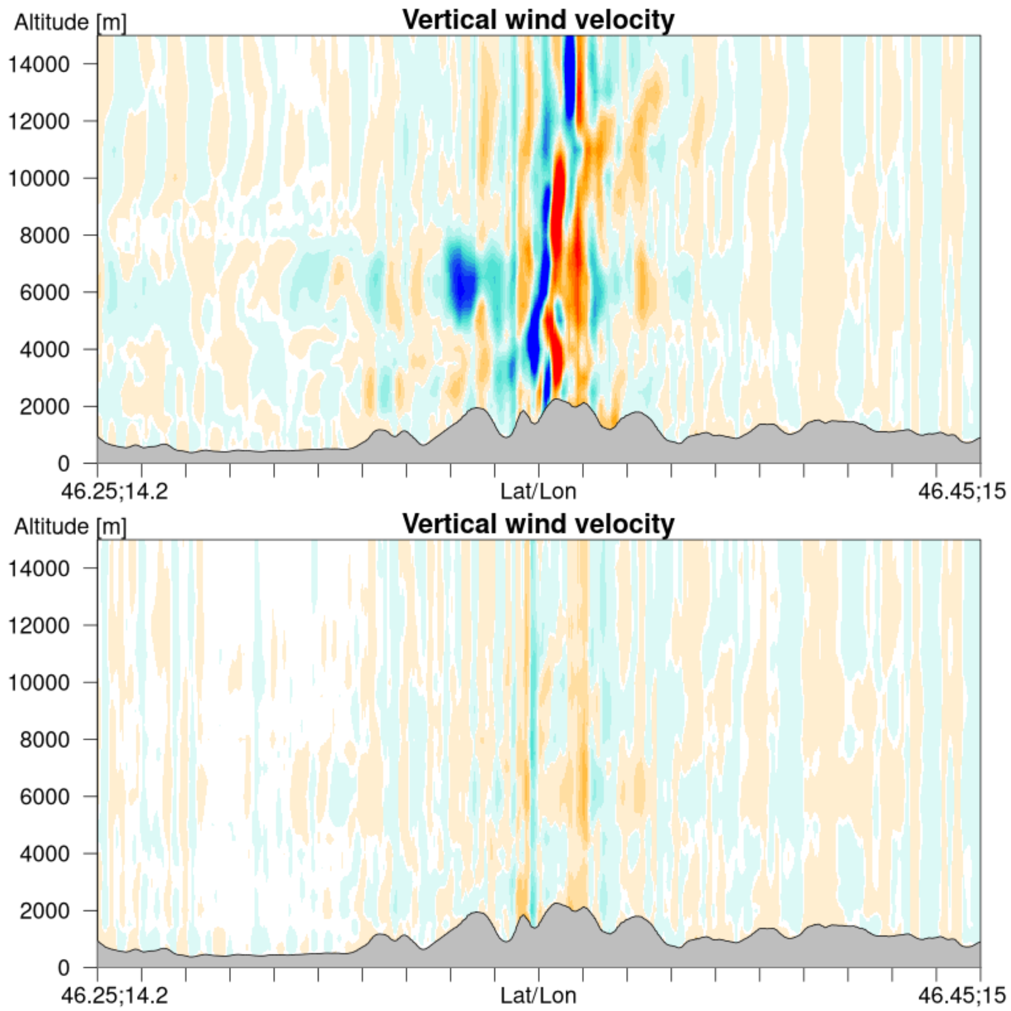
(courtesy of Nika Kastelec)

Comparison of reference and new dynamics

- in DEODE workflow @ 200m resolution
- Alpine domain, cubic truncation, 90 vertical levels
- case study for 3 September 2025 0 UTC + 24 hours
- AROME physics
- PC scheme, spectral diffusion and spectral nudging
- Vertical cross section over mountaneous ridge for vertical velocity field:

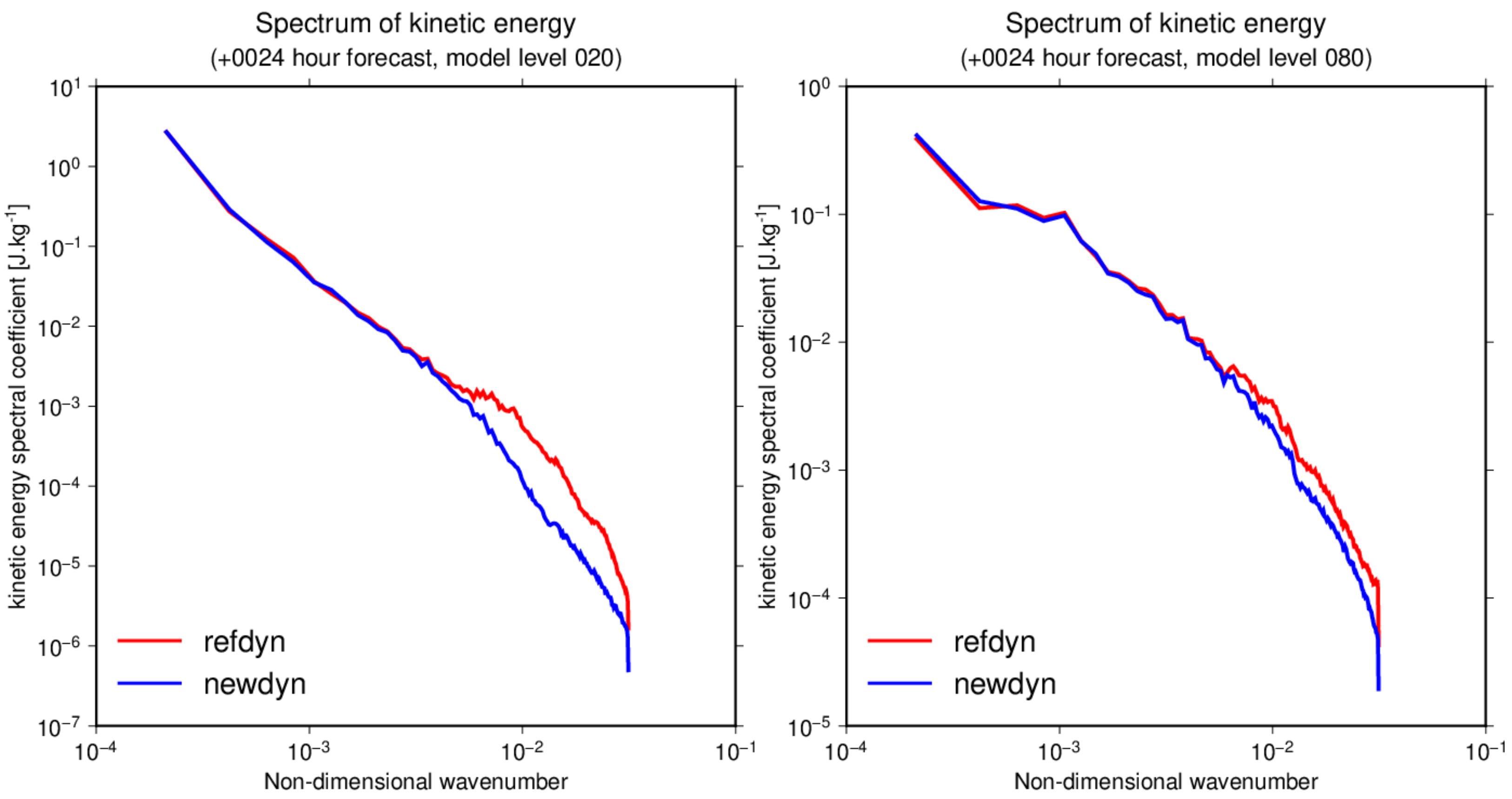
reference dynamics ↗

new dynamics ⟶



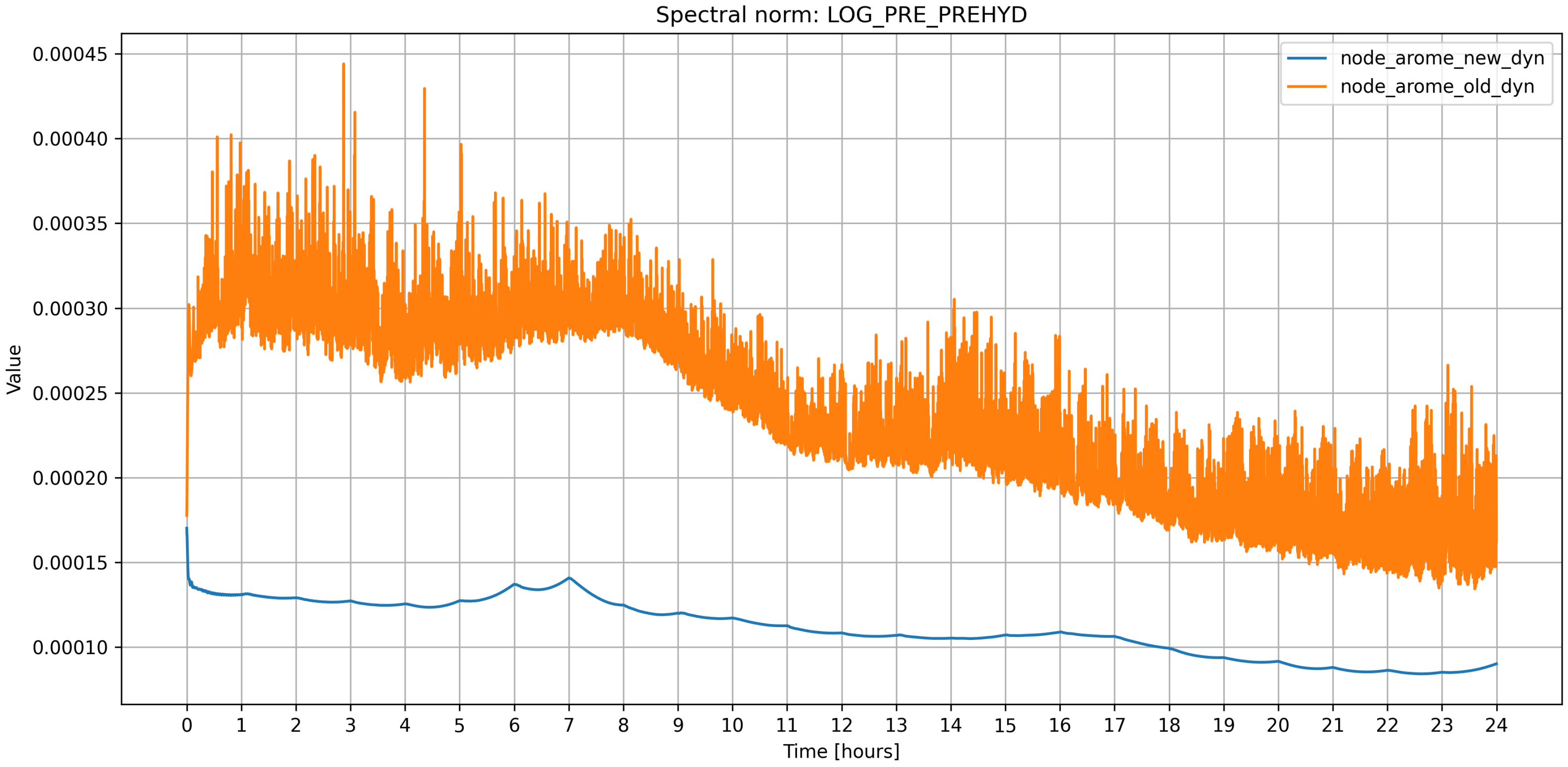
Comparison of reference and new dynamics

- Kinetic energy spectra for 20th and 80th model level: reference dynamics, new dynamics



Comparison of reference and new dynamics

- Time evolution of the domain averaged spectral norm for pressure departure: reference dynamics, new dynamics



(courtesy of Nika Kastelec)

Blended approach for fields initialization

We use one control parameter δ, $0 \leq \delta \leq 1$.

$$\frac{dT}{dt} = (1-\delta)\kappa T \frac{\dot{\pi}}{\pi} - \delta \frac{\kappa T}{(1-\kappa)}(D+d)$$

$$\frac{d\vec{v}}{dt} = -RT\frac{\vec{\nabla}\pi}{\pi} - \vec{\nabla}\phi - RT\vec{\nabla}\hat{q} - \frac{1}{m}\frac{\partial(p-\pi)}{\partial\eta}\vec{\nabla}\phi$$

$$\frac{dw}{dt} = \frac{g}{m}\frac{\partial(p-\pi)}{\partial\eta}$$

$$\frac{d\hat{q}}{dt} = -\frac{\delta}{(1-\kappa)}(D+d) - \delta\frac{\dot{\pi}}{\pi}$$

$$\frac{\partial q_s}{\partial t} = -\frac{1}{\pi_s}\int_0^1 D \; d\eta$$

where

$$\frac{\dot{\pi}}{\pi} = \frac{1}{1-\kappa}\mathbf{X}^d - \mathbf{S}D$$

$$\phi = \phi_s + \int_\eta^1 \frac{mRT}{p}\; d\eta'$$

HPE system $\sim \delta = 0$

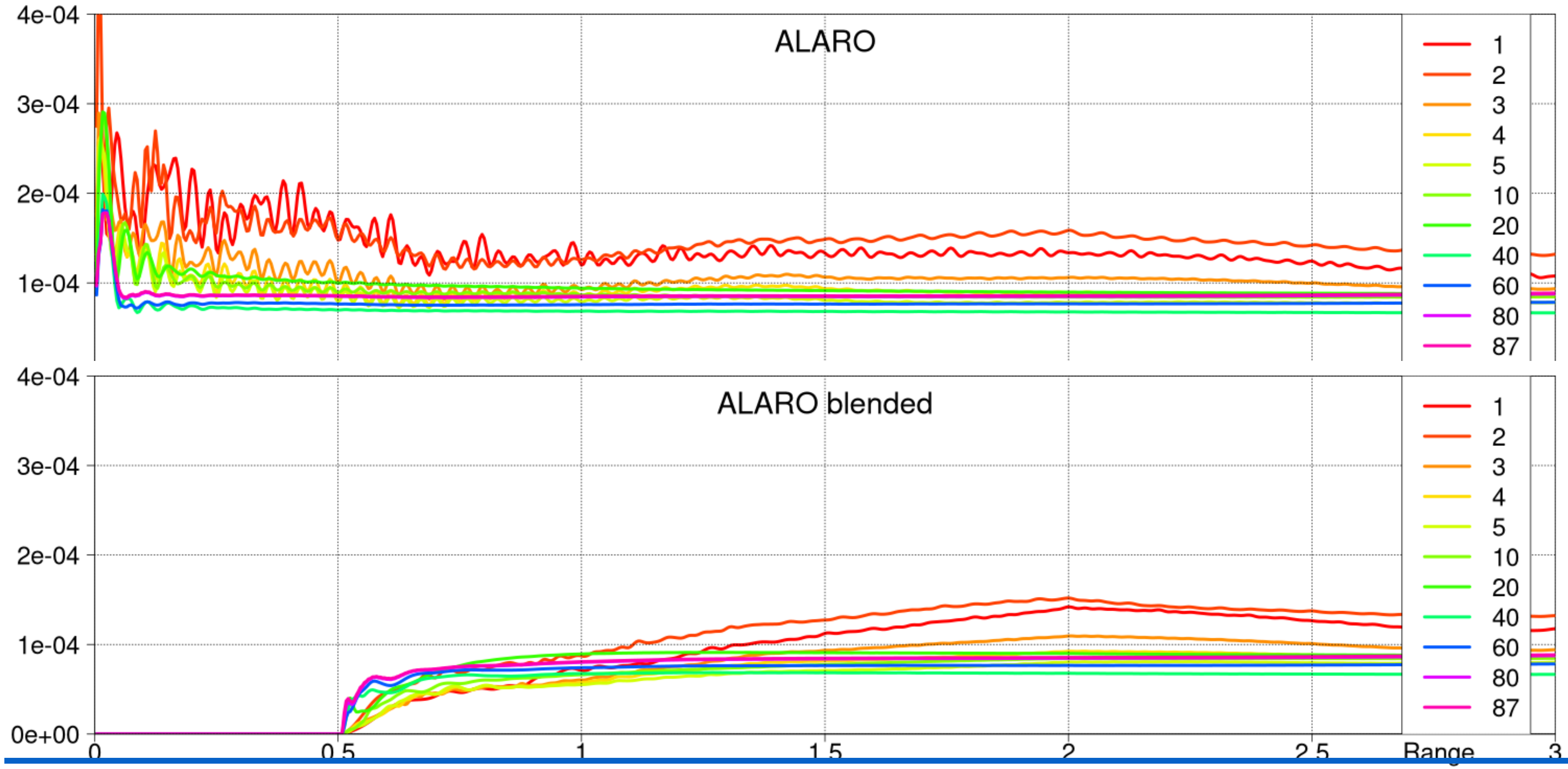
EE system $\sim \delta = 1$

a blended system is available with $0 < \delta < 1$

- filtering fast moving waves without meteorological relevance
- garantees higher numerical stability

Blended approach for fields initialization

- ALARO in DEODE workflow, 500m horizontal resolution, 87 vertical levels
- reference dynamics
- Time evolution of horizontal domain averaged spectral norms of pressure departure at several vertical levels:

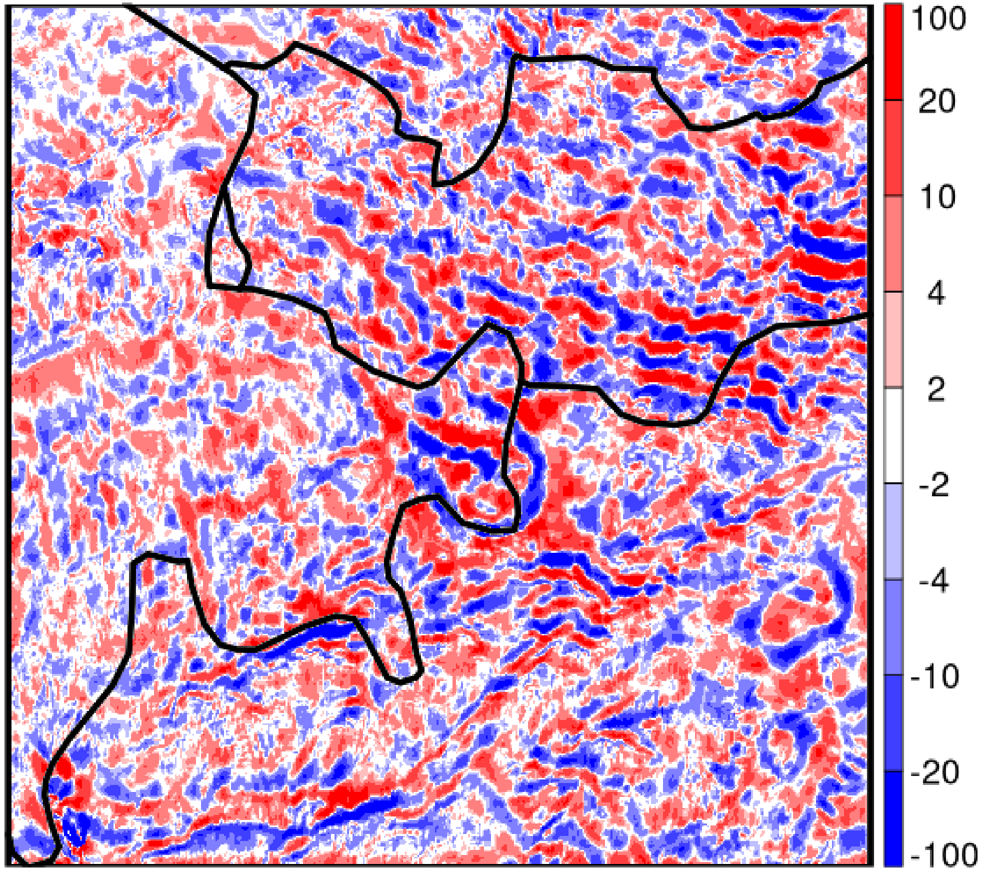


ALARO

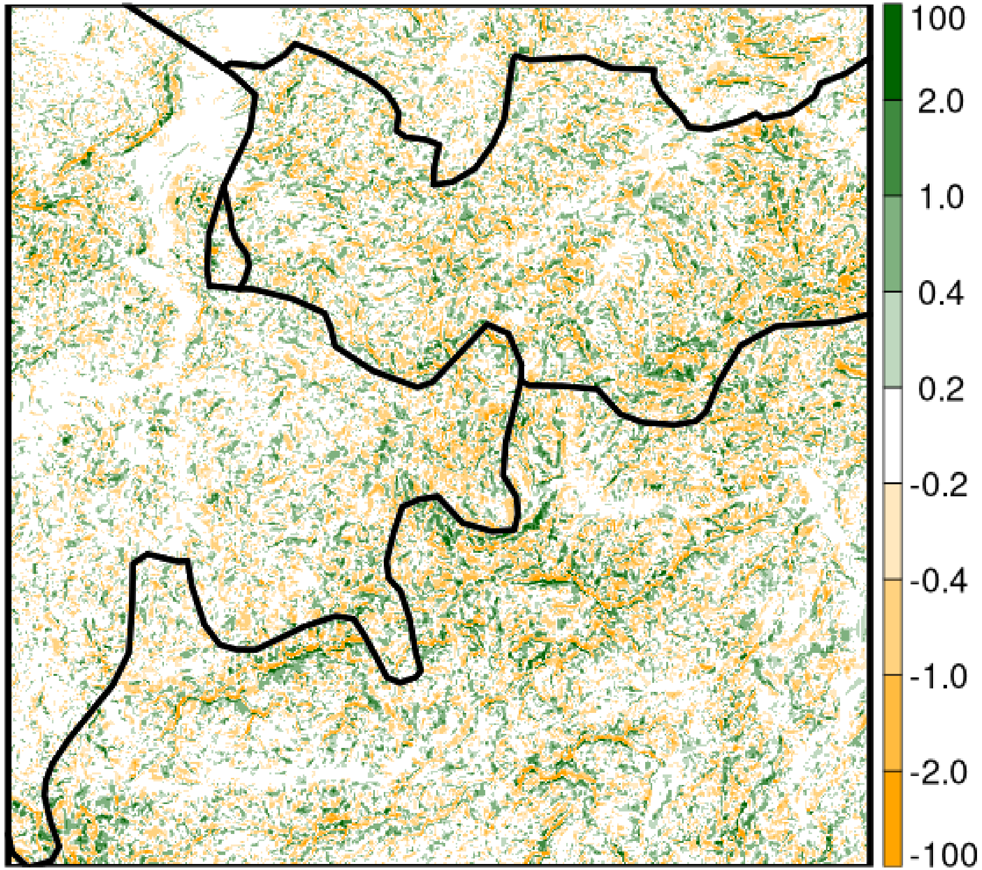
ALARO blended approach

Blended approach for fields initialization

Difference between NH and blended approach



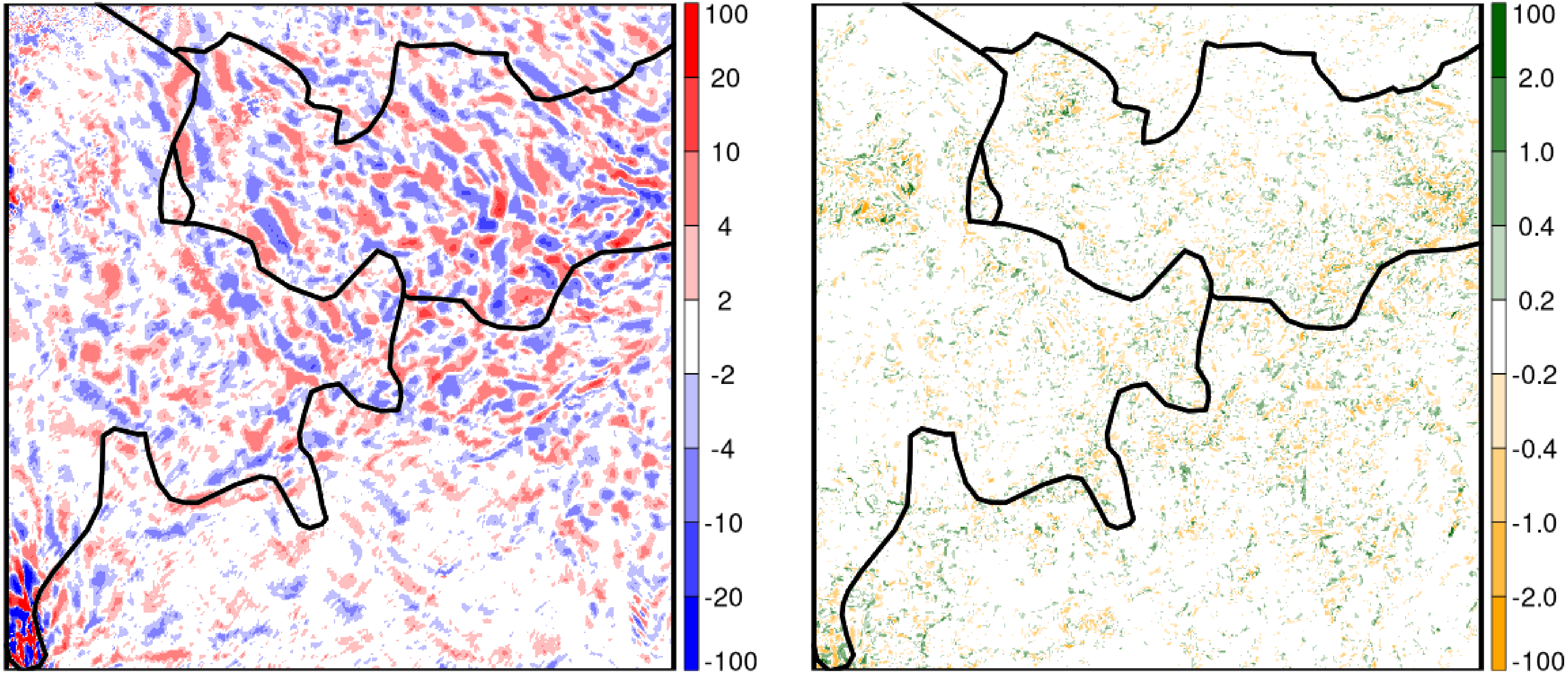
+ 30 minuts: pressure departure



vertical divergence

Blended approach for fields initialization

Difference between NH and blended approach

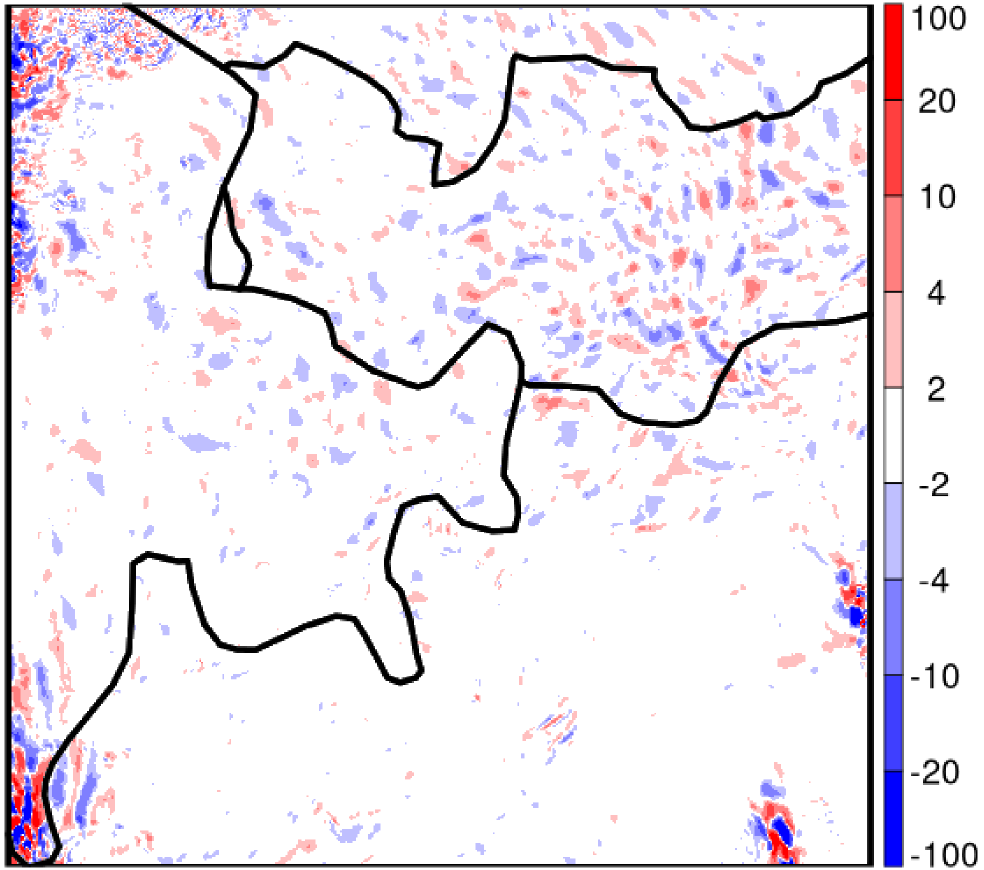


+ 1 hours: pressure departure

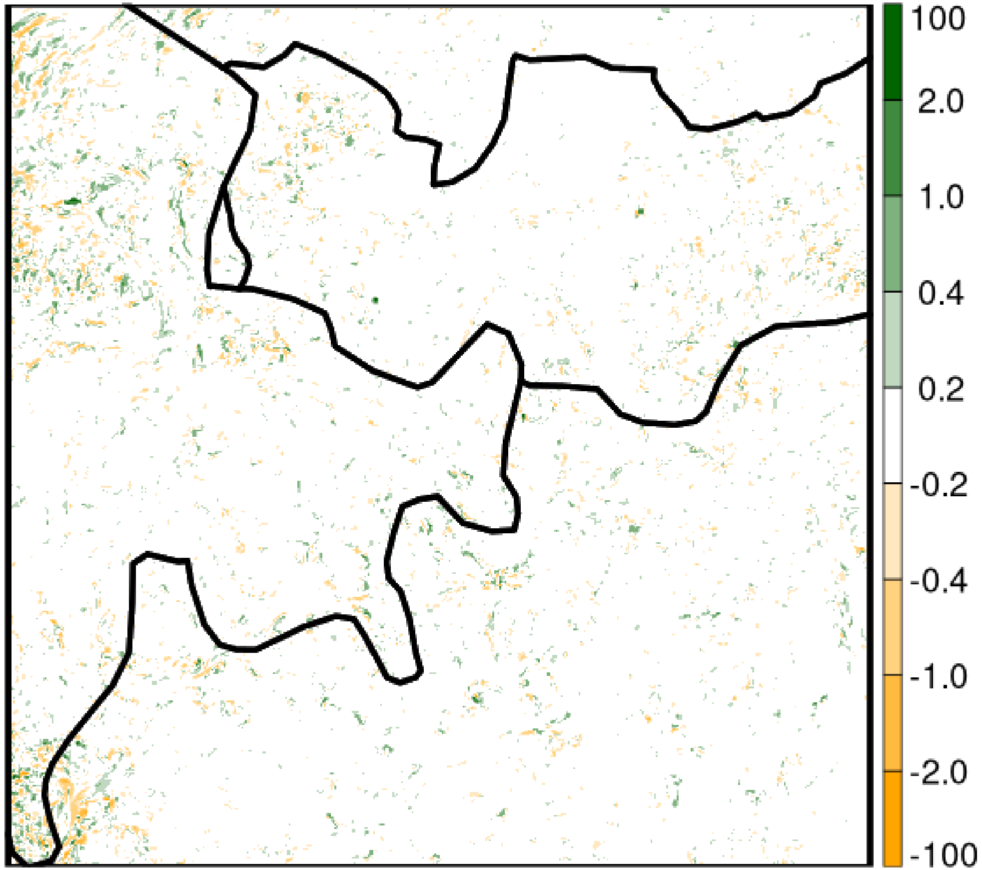
vertical divergence

Blended approach for fields initialization

Difference between NH and blended approach



+ 2 hours: pressure departure

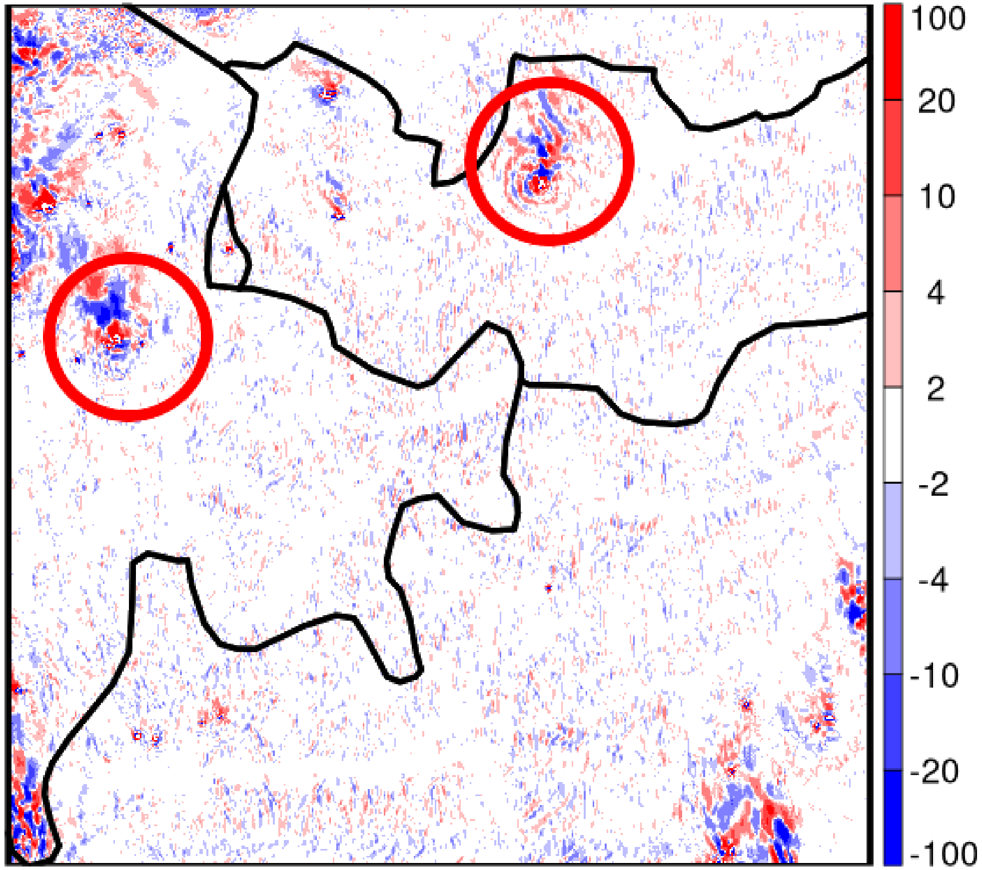


vertical divergence

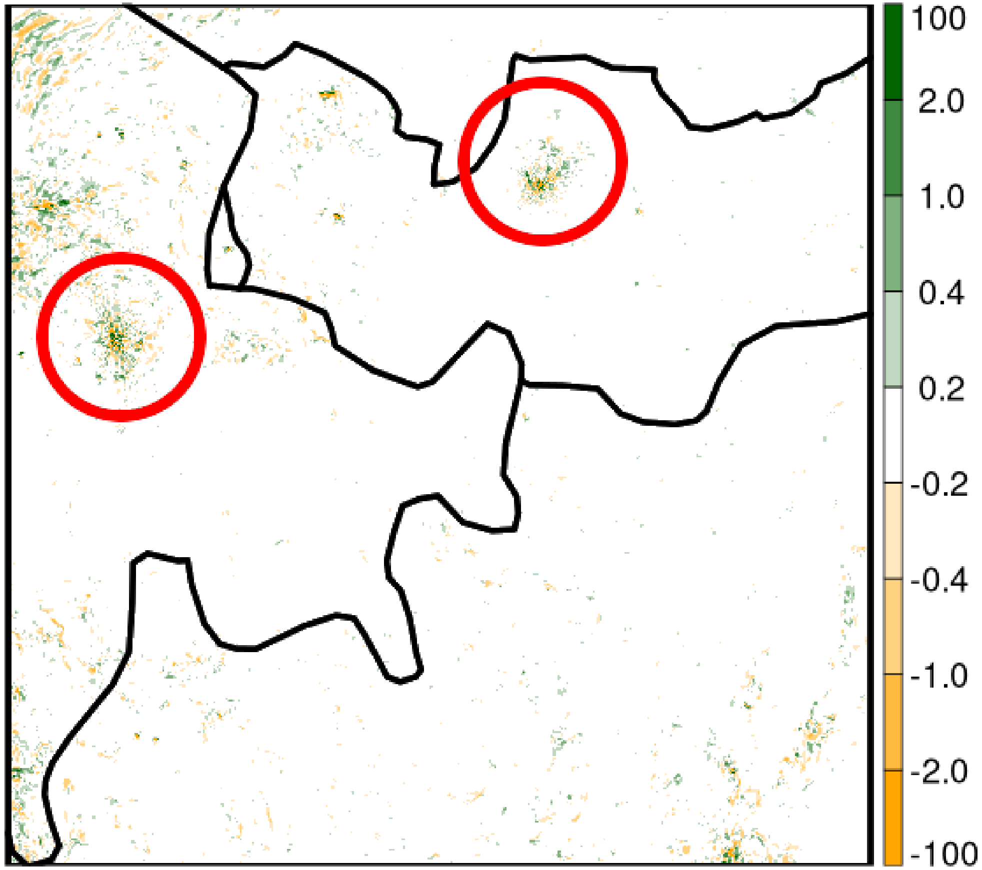
Blended approach for fields initialization

LACE

Difference between NH with reference dynamics and NH with new dynamics - persistent in time



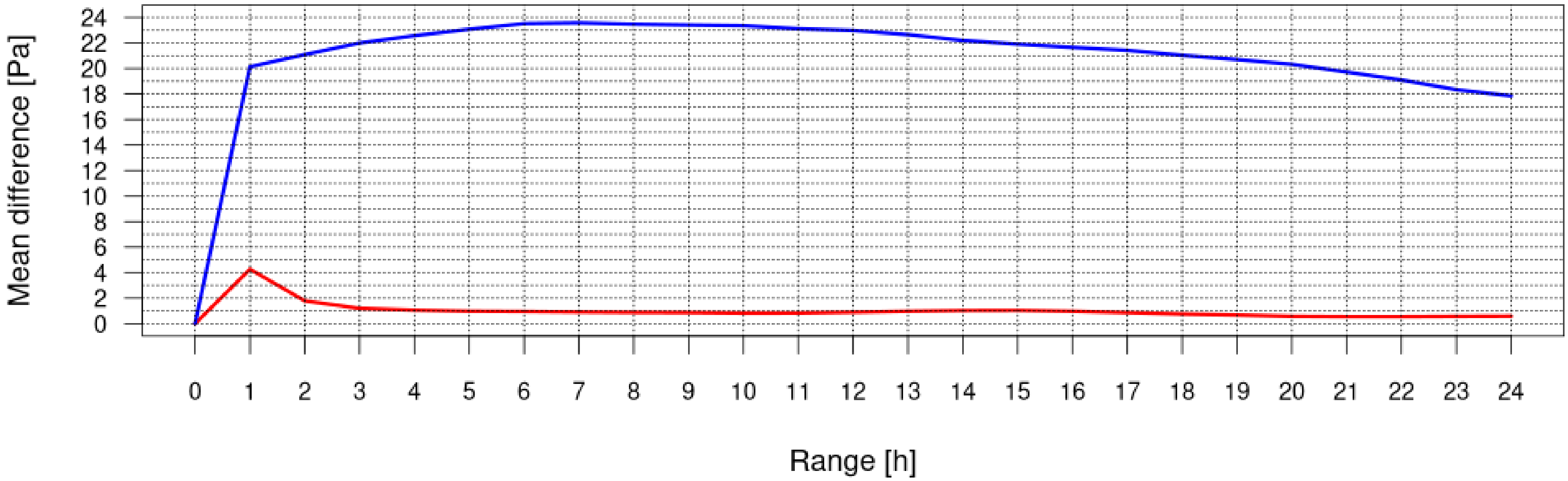
+ 2 hours: pressure departure



vertical divergence

Blended approach for fields initialization

The time evolution of the mean absolute difference between experiments with the blended approach and with NH dynamics for ALARO compared to the same metrics for the difference between NH run with ALARO and AROME.

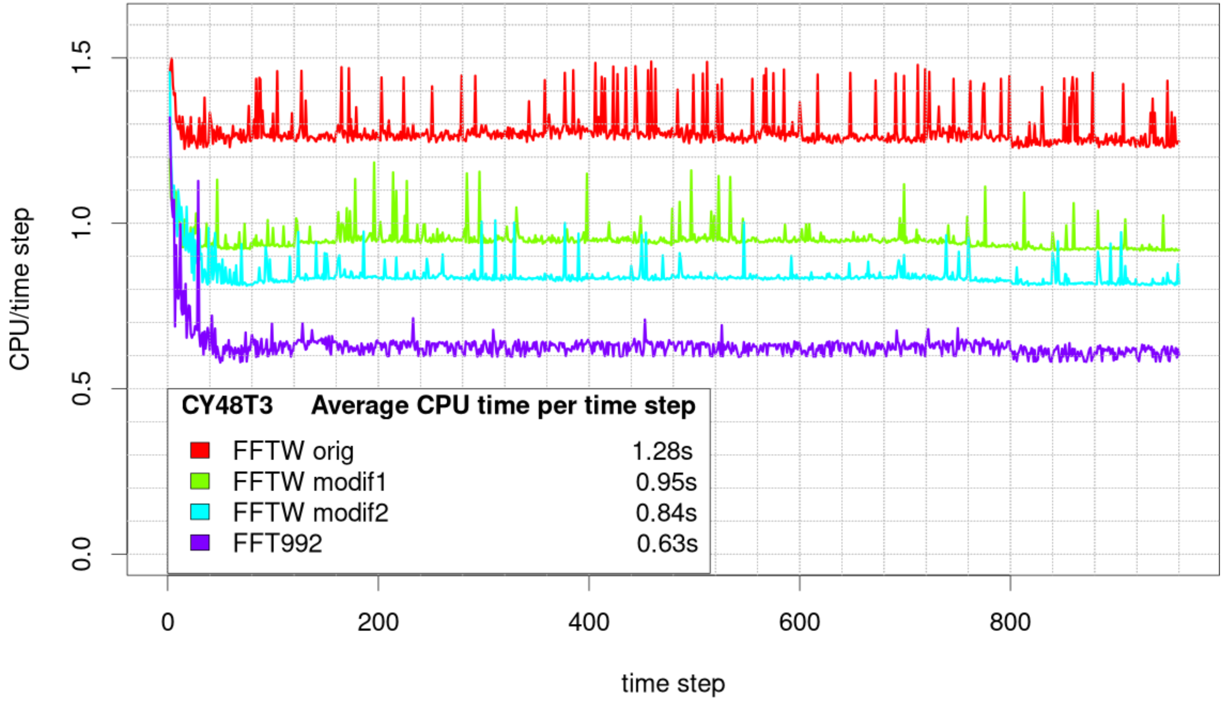


Fast spectral transforms ?

LACE nwp central europe

Comparison of FFT992 classical transformation with FFTW (licensed by MIT to ECMWF and sub-licenced to the ACCORD consortium, until 2031)

- Hungarian operations on Intel based scalar machine:
  - 0-4% advantage of FFTW
  - bigger advantage with "expensive choises" in domain and model setup
- Czech operations on NEC vector engine based machine:
  - no gain with FFTW so far
  - two improvements thanks to Ryad ElKhatib



GeoSphere Austria DHMZ Czech Hydrometeorological Institute HungaroMet METEO ROMANIA SHMU ARSO METEO Slovenia

Tack för din uppmärksamhet!