Further developments in the Discontinuous Galerkin based dynamical core for ICON

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Michael Baldauf (DWD)

BRIDGE (**B**asic **R**esearch for **I**CON with **D**iscontinuous **G**alerkin **E**xtension)

- an informal project at DWD (runs since ~ 2020)
- ‚Classical' discontinuous Galerkin (DG) solver for Euler equations (with thermodyn. variable either $\rho\Theta$ or total energy) or shallow water, advection, … eqns.
- Equations are formulated in covariant form using Ricci tensor calculus (on spherical or ellipsoidal or flat manifolds)
  → clear distinction between: equations – metrics – DG discretization
- Collocated nodal finite elements; Riemann solver: Lax-Friedrichs flux
- Treatment of diffusion by Bassi, Rebay (1997) ‚RB1' scheme
- Grid: horizontally (icosahedral) triangular, prismatic grid cells
- Time integration: explicit RK or HEVI with IMEX-RK
- Stabilization via exponential filtering

Baldauf (2020) JCP: Discontinuous Galerkin solver for the shallow-water equations in covariant form on the sphere and the ellipsoid

Baldauf (2021) JCP: A horizontally explicit, vertically implicit (HEVI) discontinuous Galerkin scheme for the 2-dimensional Euler and Navier-Stokes equations using terrain-following cordinates

Very first steps towards moisture in BRIDGE

Thermodynamics of the moist atmosphere

Gibbs-function for an atmosphere with dry air, water vapor and liquid water (cloud particles):

$$E_{int} = f(S, V, N_d, N_v, N_c)$$

$\Rightarrow$ equation of states:

1. ideal gas equation

$$p = R_m \rho T, \quad \text{with } R_m = R_d \frac{\rho_d}{\rho} + R_v \frac{\rho_v}{\rho}$$

   (total density $\rho = \rho_d + \rho_v + \rho_c$)

2. density of internal energy

$$E_{int} = C_v T + L_v \rho_v, \quad \text{with heat capacity density } C_v = c_{vd}\,\rho_d + c_{vv}\,\rho_v + c_{wl}\,\rho_c$$

3. phase transition regime (vapor $\leftrightarrow$ liquid)

$$\mu_v \overset{!}{=} \mu_l \;\Rightarrow\; \text{Clausius-Clapeyron-eq.} \;\Rightarrow\; p_{sat}(T) \text{ or } \rho_{sat}(T) = \frac{p_{sat}(T)}{R_v T}$$

   (e.g. use Tetens formula for $p_{sat}(T)$)

(Thanks to Sebastian Borchert for discussions about this topic)

Dynamics of the moist atmosphere

Moisture transport equations

$$\frac{\partial \rho_v}{\partial t} + \nabla_k v^k \rho_v = \left.\frac{d\rho_v}{dt}\right|_{cond/evap}$$

$$\frac{\partial \rho_c}{\partial t} + \nabla_k v^k \rho_c = -\left.\frac{d\rho_v}{dt}\right|_{cond/evap}$$

Relaxation approach for **condensation/evaporation** (instead of saturation adjustment)

$$\left.\frac{d\rho_v}{dt}\right|_{cond/evap} = \begin{cases} -\frac{\rho_v - \rho_{sat}(T)}{\tau}, & \rho_v > \rho_{sat}(T), \\ \frac{\min(\rho_c, \rho_{sat}(T) - \rho_v)}{\tau}, & \rho_v < \rho_{sat}(T), \end{cases}$$

relaxation time constant $\tau = 4\Delta t$ (quite arbitrary at the moment).

2 variants of the **energy equation**:

1. $E = \frac{1}{2}\rho v_j v^j + \rho\Phi(\mathbf{r}) + C_v T + L_v \rho_v \quad \Rightarrow \quad \frac{\partial E}{\partial t} + \nabla_k v^k (E + p) = 0$
2. $E = \frac{1}{2}\rho v_j v^j + \rho\Phi(\mathbf{r}) + C_v T \quad \Rightarrow \quad \frac{\partial E}{\partial t} + \nabla_k v^k (E + p) = -L_v \left.\frac{d\rho_v}{dt}\right|_{cond/evap}$

BRIDGE uses the 2nd version at the moment.

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Bryan, Fritsch (2002) MWR test case

Set a warm bubble (+2 K) in an environment atmosphere with
- *dry case*: pot. temp. Θ = 300 K, without moisture
- *moist case*: equiv. pot. temp. Θ_e = 300 K, saturated atm.+clouds everywhere
- dx=dz=100m

In both cases the w field should look quite similar (but not exactly identical)

Remarks:
- the moist initialization procedure is not exactly clear … at least, for me …
  (iteration is necessary for the environment, but also for the bubble setting?)
- the buoyancy term must be adapted (not so nice…)
- „No physical or computational diffusion is applied“.
  Is this really the case …?

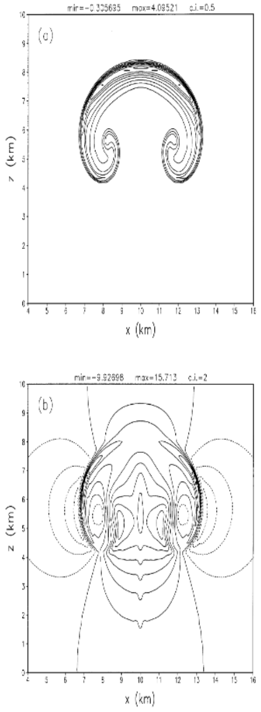
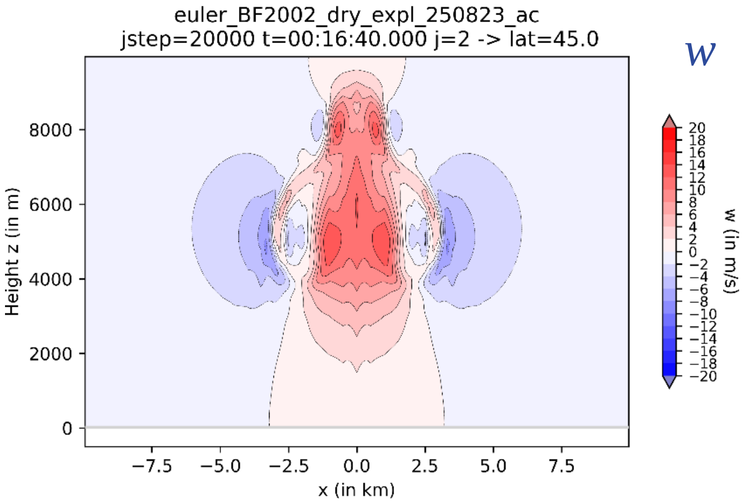


FIG. 3. Results of the moist thermal simulation for θ_e = 320 K and r_t = 0.020. (a) Perturbation wet equivalent potential temperature (θ_e') is contoured every 0.5 K. The zero contour is omitted. (b) Vertical velocity, contoured every 2 m s^{-1}.

Bryan , Fritsch (2002) test, dry case: no diffusion, weak filtering



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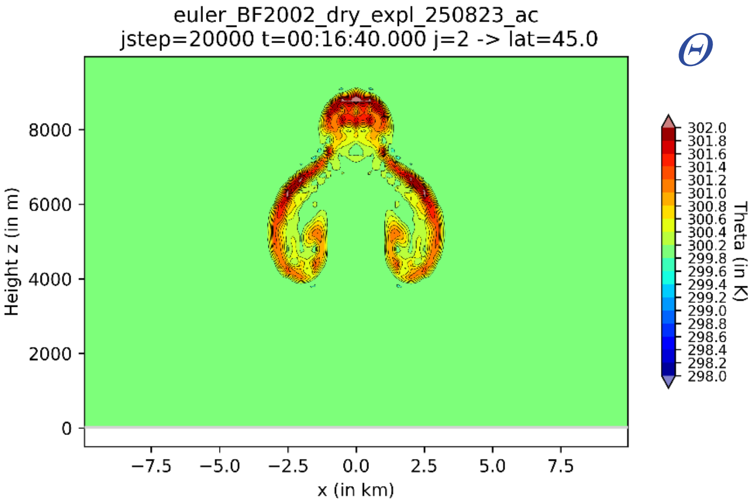

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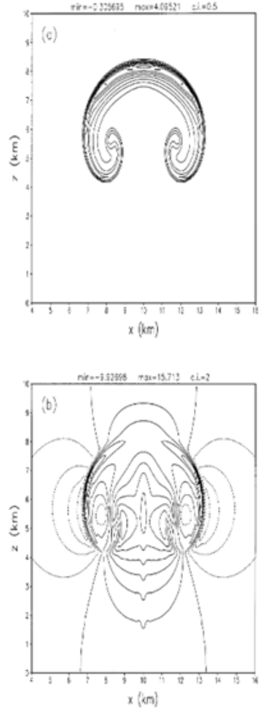
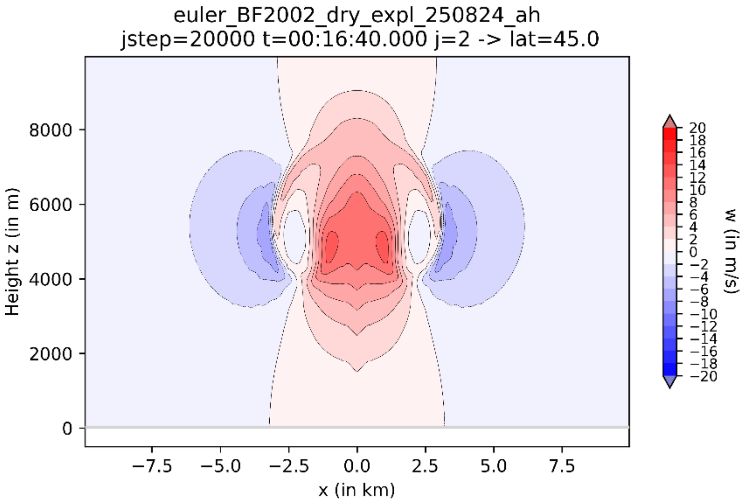


FIG. 3. Results of the moist thermal simulation for $\theta_s = 320$ K and $r_t = 0.020$. (a) Perturbation wet equivalent potential temperature (θ_e') is contoured every 0.5 K. The zero contour is omitted. (b) Vertical velocity, contoured every 2 m s^{-1}.

Bryan , Fritsch (2002) test, dry case: with diffusion K=20m²/s, no filtering



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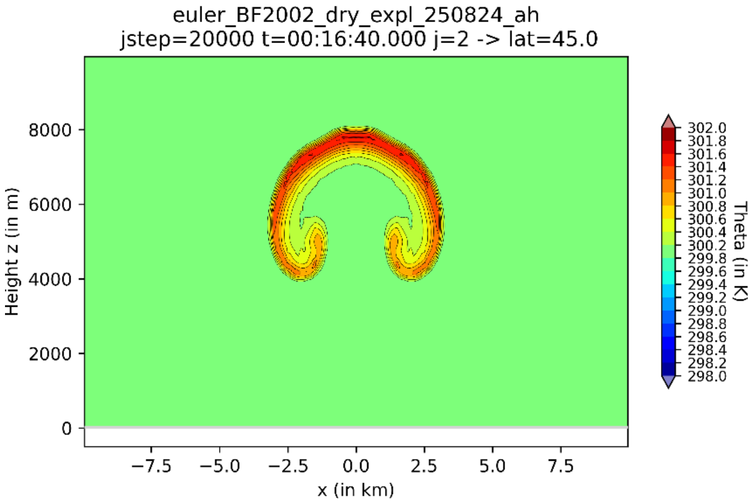

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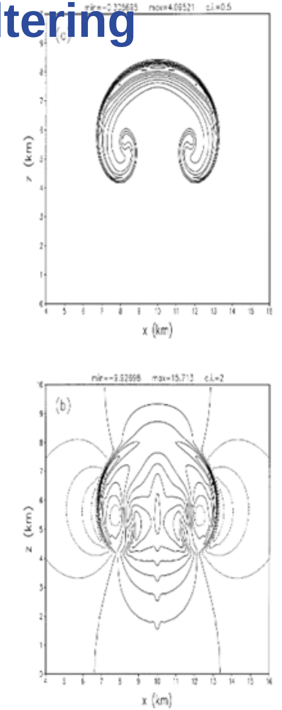
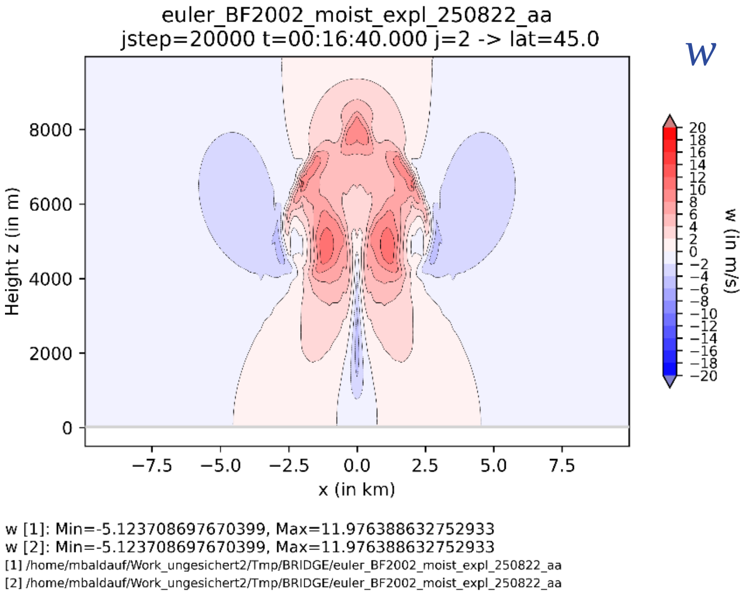
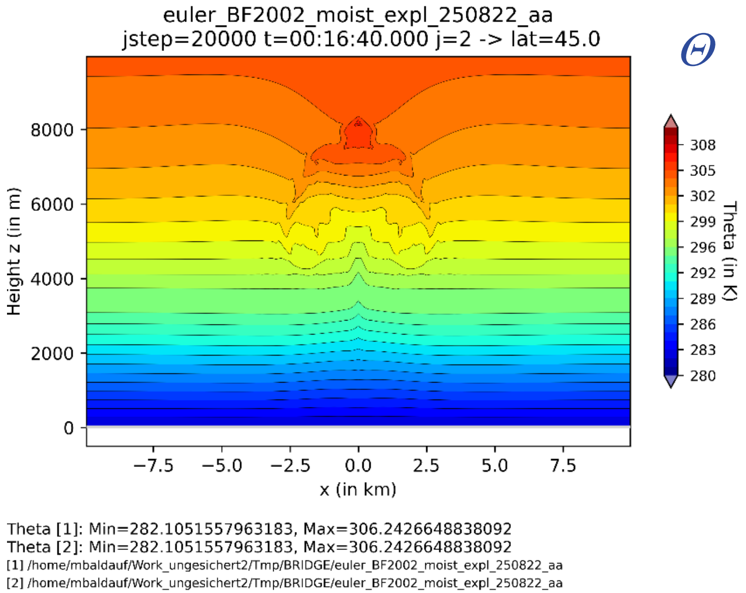


FIG. 3. Results of the moist thermal simulation for θ_e = 320 K and r_t = 0.020. (a) Perturbation wet equivalent potential temperature (θ_e') is contoured every 0.5 K. The zero contour is omitted. (b) Vertical velocity, contoured every 2 m s^{-1}.

Bryan , Fritsch (2002) test, moist case: with diffusion K=10m²/s, no filtering

Max. oversaturation: 0.8%, relative change in water mass: 1.0e-15





Weisman, Klemp (1982) MWR setup

Prescibe Θ and rel. humidity of an environment atmosphere; set in a warm bubble.
Use Kessler microphysics and a TKE turbulence scheme.

Remark: the purpose of the paper is the investigation
of supercell storms;
it is to a less extent a test case setup.

BRIDGE setup:

- 4th order DG, dx=8 km, dz=1300 …3600m
- RK4, dt=0.05s
- With diffusion, Km=Kh=10 m²/s
- With filtering

508 MONTHLY WE

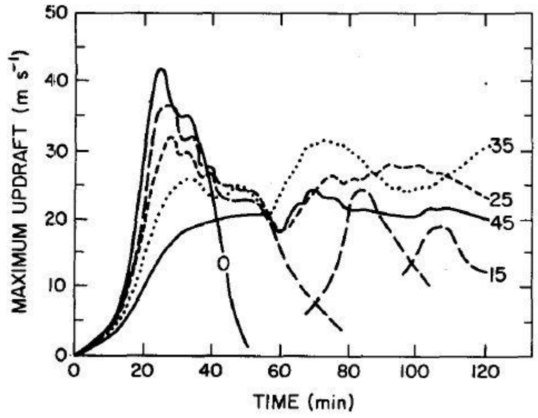
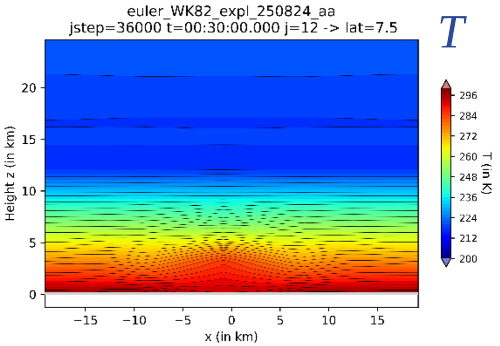


FIG. 3. Time series of maximum vertical velocities for the U_S = 0, 15, 25, 35 and 45 m s^{-1} wind shear experiments. q_{v0} = 14 g kg^{-1}.

Very first steps towards moisture in BRIDGE



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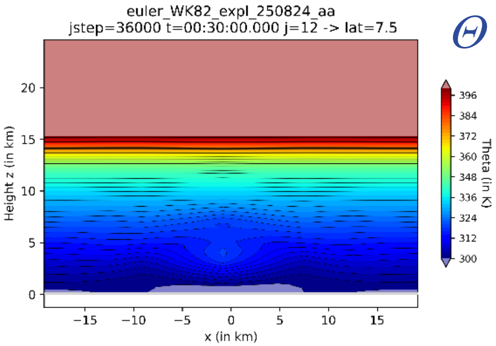

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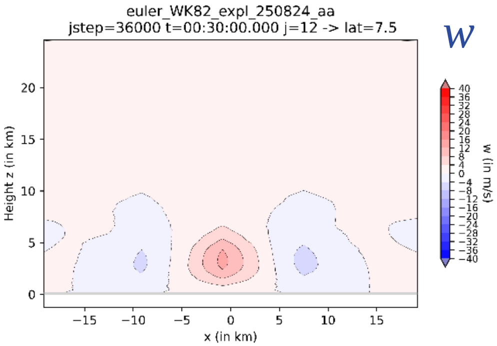

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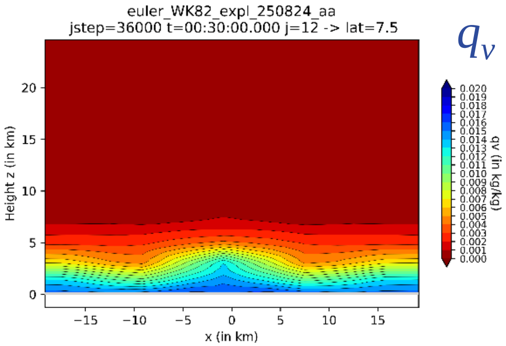

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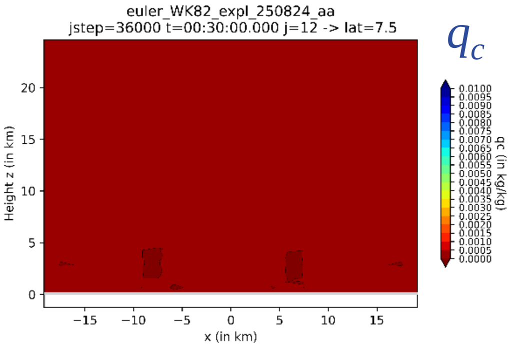

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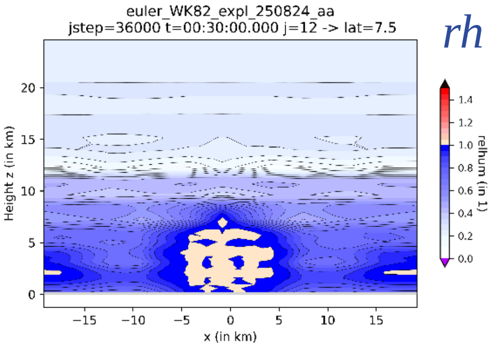

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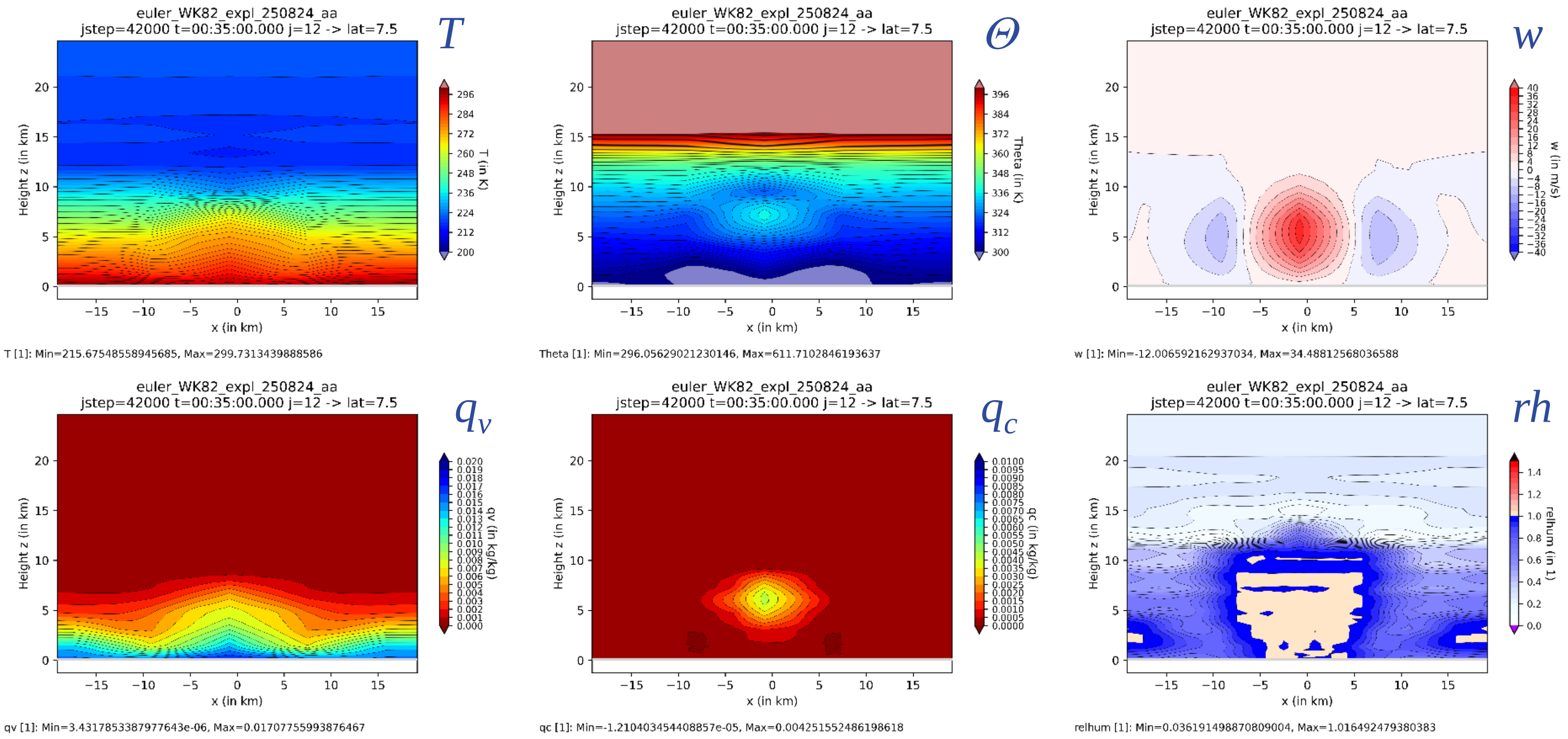
Very first steps towards moisture in BRIDGE

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



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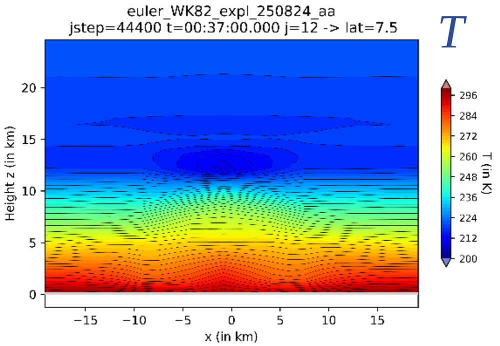

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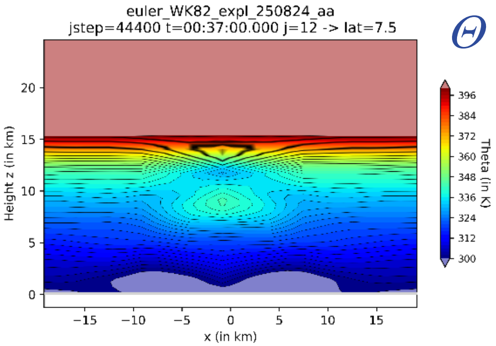

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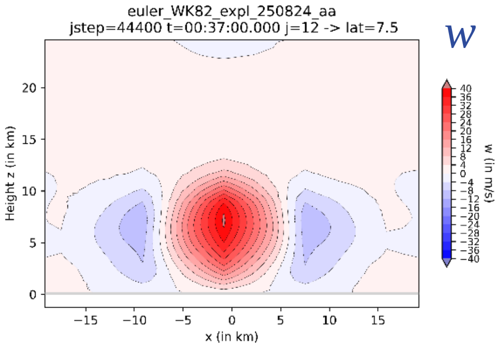

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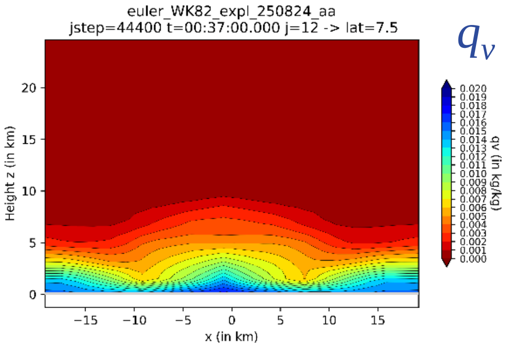

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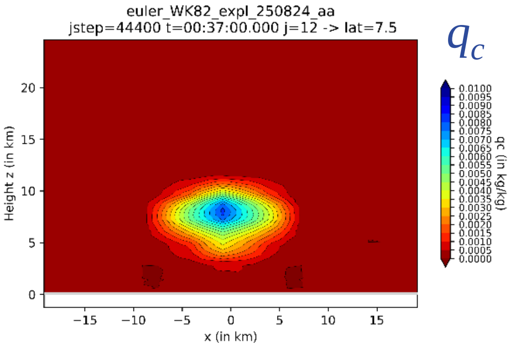

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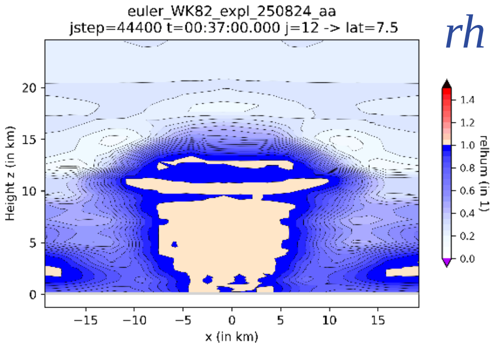

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Very first steps towards moisture in BRIDGE

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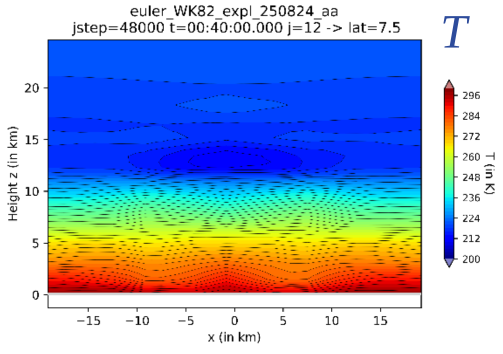
Wetter und Klima aus einer Hand



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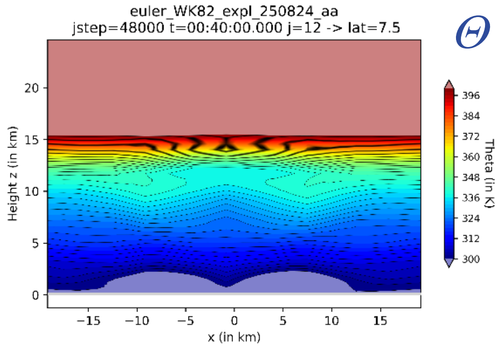

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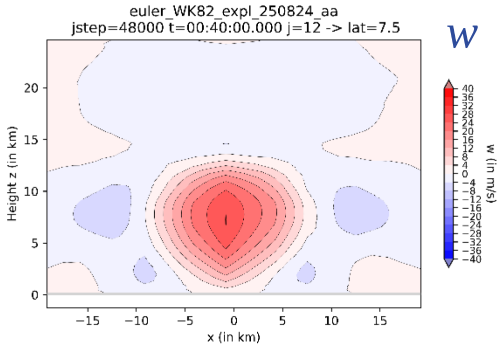

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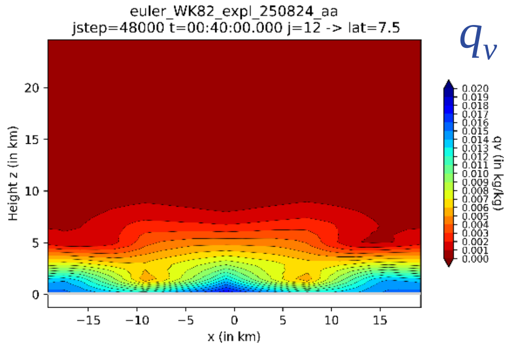

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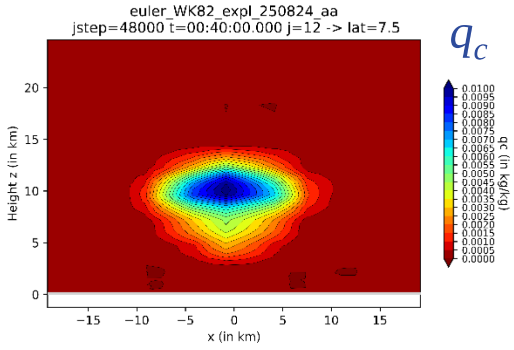

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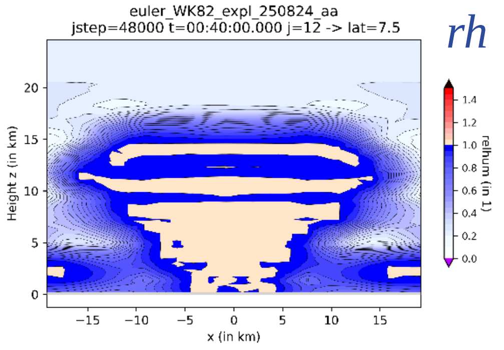

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Very first steps towards moisture in BRIDGE

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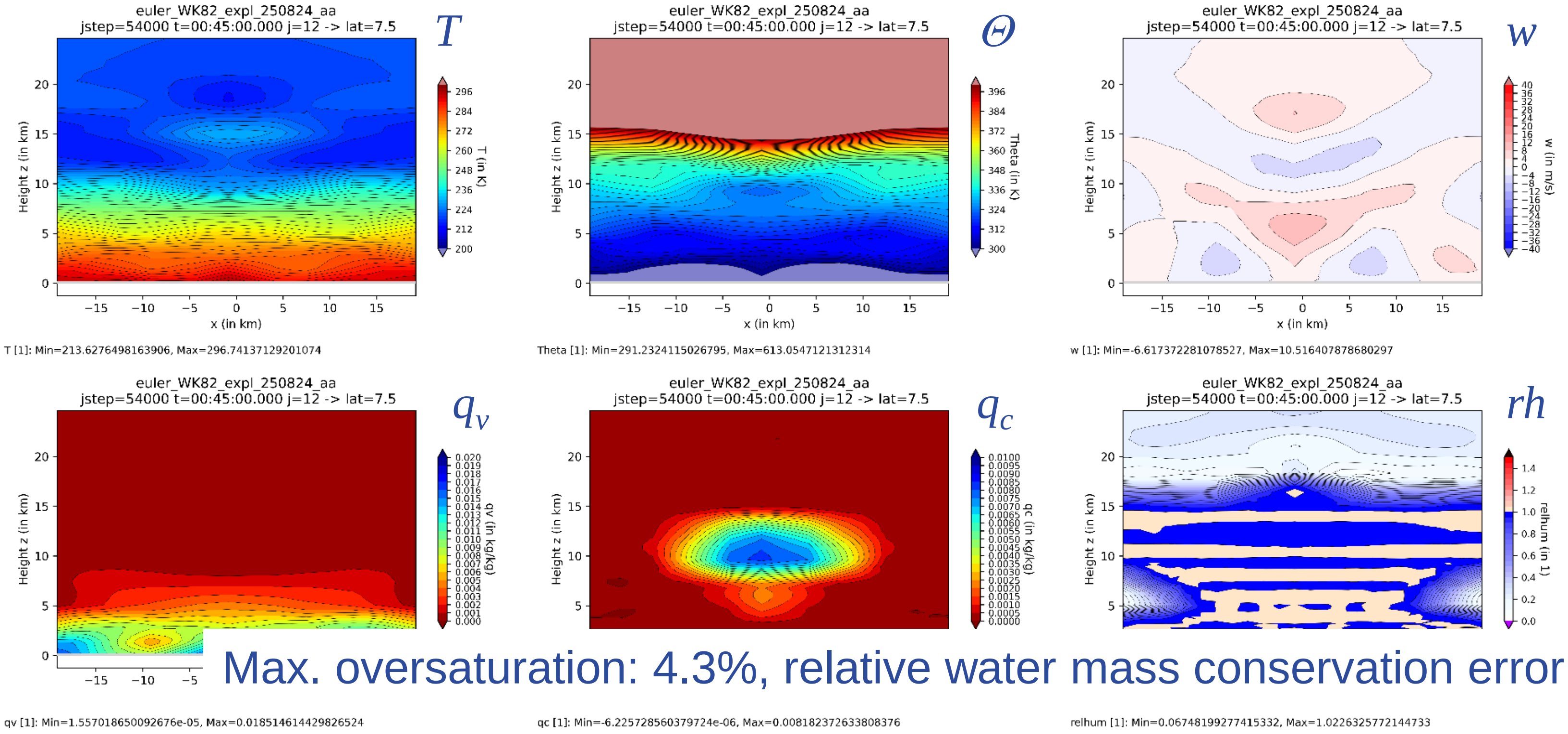
Wetter und Klima aus einer Hand



Max. oversaturation: 4.3%, relative water mass conservation error ~ 3%

Summary

- Some very first steps towards using microphysics in BRIDGE have been made:
  - condensation/evaporation implemented;
  - reduced conservation violation (=2nd step of Zhang, Shu, 2010)
    in a positivity pres. transport scheme
  - with explicit time integration
    works satisfyingly in idealized test cases.

- A lot of heavy todo‘s remain:
  - True conserving, posit. pres. (and mass consistent?) advection scheme
  - Couple with IMEX-RK (→ much larger dt), and splitting schemes
  - Full Kessler with (implicit) sedimentation (and saturation adjustment?)

About the spherical and ellipsoidal geopotential approximation
- Tests with a discontinuous Galerkin dynamical core

Ellipsoidal geopotential approximation (EGA) by *Staniforth, White (2015) QJRMS*

Ellipsoidal approximation:
coordinate surfaces with R=const.

$$\frac{x^2+y^2}{A^2}+\frac{z^2}{C^2}=1, \qquad (13)$$

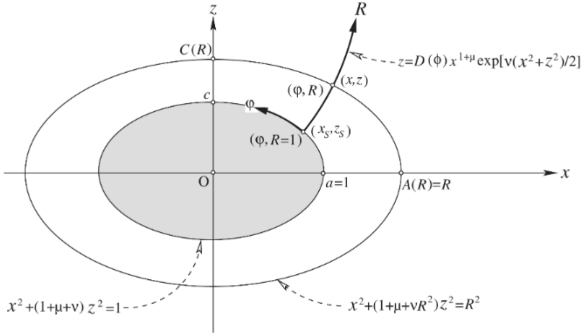
with $A(R)=R$, $C(R)=\ldots$
and the geopotential

$$\Phi(R)=\frac{\gamma M_E}{R}\left\{1+\left(\frac{8\widetilde{\varepsilon}-7m}{12}\right)+\left(\frac{11m-4\widetilde{\varepsilon}}{12}\right)\frac{R^2}{a^2}\right\}. \qquad (33)$$

Monopole   Quadrupole   Centrifugal

→ Φ=const. on surfaces with R=const.!

This approach in particular retains Clairaut's fraction g_{pole}/g_{eq} !



Key features of the coordinate system in a meridional section; see text for details.

To get an *orthogonal* coordinate system one needs to solve *iteratively*:

$$z^2=D^2(\varphi)\left\{R^2-\left(1+\mu+\nu R^2\right)z^2\right\}^{1+\mu} \times\exp\left[\nu\left\{R^2-\left(\mu+\nu R^2\right)z^2\right\}\right]. \qquad (71)$$

In the following $D(\varphi)$ is chosen so that φ is the *geographic* latitude.

What is a fair comparison between sphere and ellipsoid?

Idea: use a *geostrophic+hydrostatic balanced, stationary* initial state

$$\nabla_i p = -\rho \nabla_i \Phi - 2 E_{ijl} \Omega^j \rho v^l - \nabla_k (\rho v^k v_i)$$

Prescribe a purely zonal velocity field of the form $\mathbf{v} = u(\varphi, z)\, \mathbf{e}_\lambda$

p exists, if the *integrability condition* $\nabla\times$ rhs = 0 is fulfilled
→ (generalized) thermal wind relationship = PDE for $T(\varphi, z)$

What is the correct boundary condition?
Method of characteristics → prescription of a $T(z)$-profile is possible (e.g. at the equator)

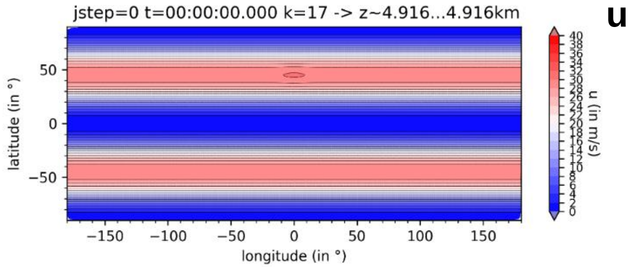
Solve the PDE for $T(\varphi, z)$ e.g. via a spectral solver.
Determine p via line integration.

Finally, add some perturbation (e.g. on $\mathbf{v}$)

Baroclinic instability for u_{max}=35 m/s

Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines)

u



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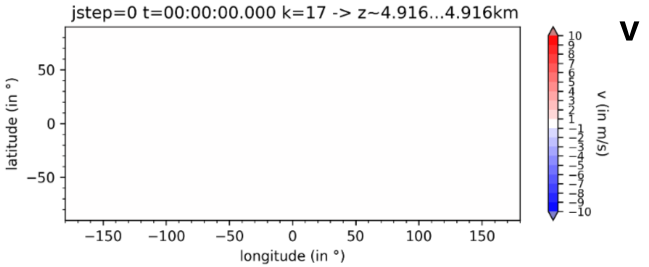
v



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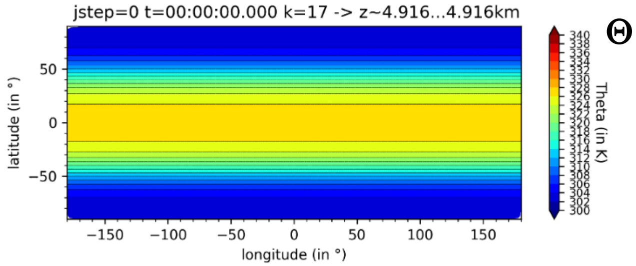

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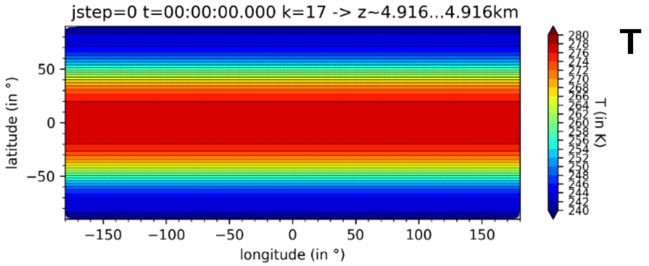

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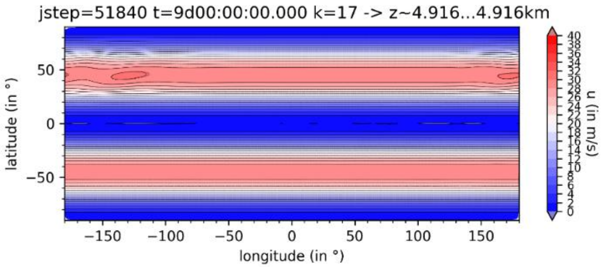
Baroclinic instability for u_{max}=35 m/s

Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines)

u



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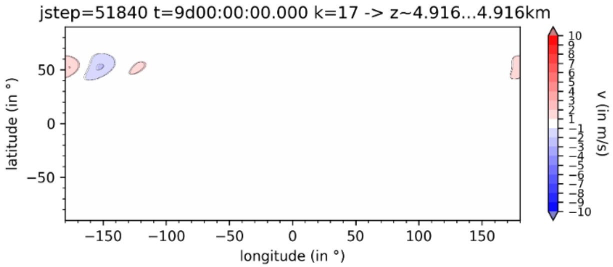
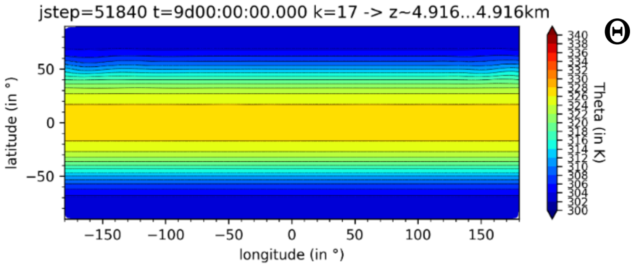
v



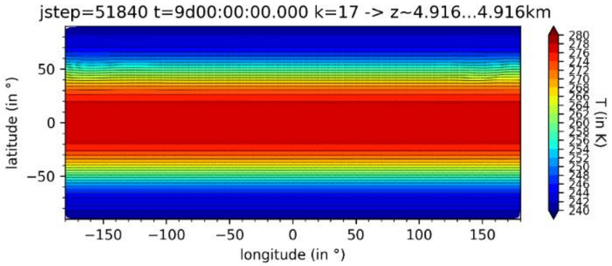
v [1]: Min=-2.0486195186807237, Mean=0.0011252580415168454, Max=2.0132038360081816
v [2]: Min=-2.081614414041501, Mean=0.0012041504001246474, Max=1.9956302274911828

Θ



Theta [1]: Min=302.39600096749126, Mean=319.2265917018228, Max=326.5524426607136
Theta [2]: Min=302.1071606275113, Mean=319.11203063283455, Max=326.48146546031023

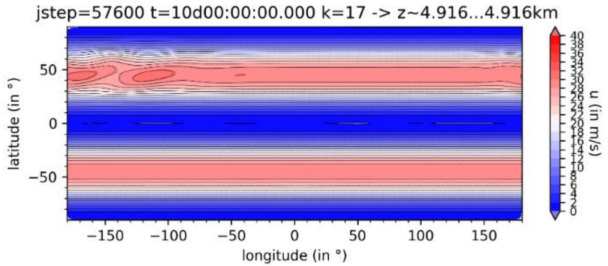
T



T [1]: Min=241.80244524316842, Mean=266.97707202654715, Max=277.1130068402645
T [2]: Min=241.36576822066175, Mean=266.64515161972014, Max=276.7963734872096

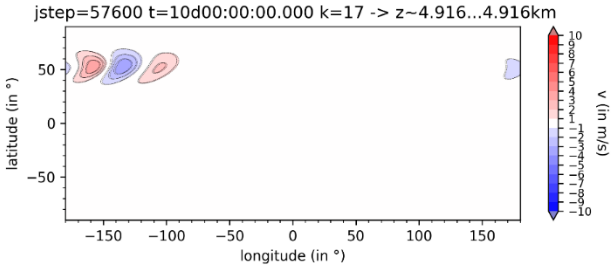
Baroclinic instability for u_{max}=35 m/s

Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines)



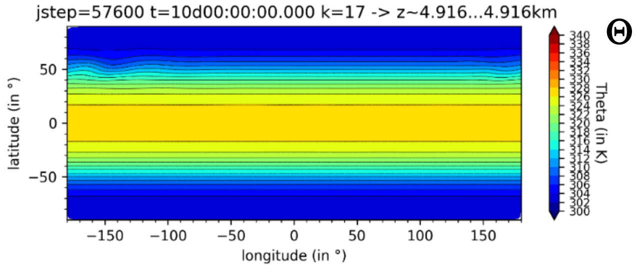
u

u [1]: Min=-0.02744685299260993, Mean=15.897202650368966, Max=31.262131466097287
u [2]: Min=-0.026708850879797578, Mean=15.859582904788567, Max=31.20413949176768



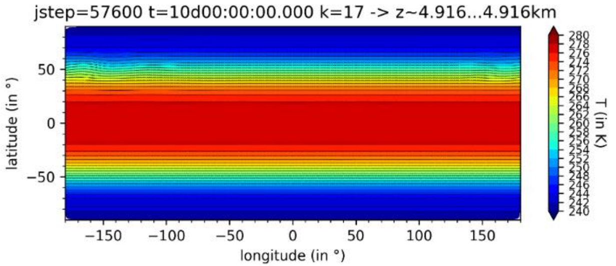
v

v [1]: Min=-3.8533966943565465, Mean=0.0003623418636674577, Max=3.872466370852339
v [2]: Min=-3.9005151719328195, Mean=0.00012601756289855985, Max=3.8653585677757774



Θ

Theta [1]: Min=302.40328535291843, Mean=319.2251539461623, Max=326.5551490909007
Theta [2]: Min=302.1558545336776, Mean=319.11071928584124, Max=326.48157495962386



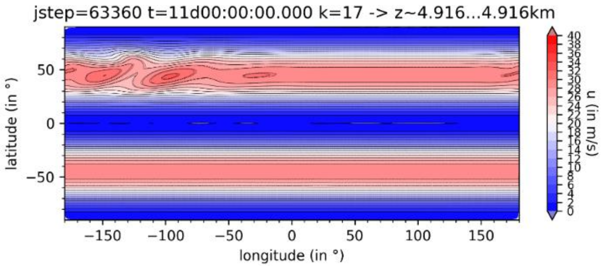
T

T [1]: Min=241.80819032934264, Mean=266.97598404932853, Max=277.1164755908529
T [2]: Min=241.39797553663567, Mean=266.64409756960185, Max=276.80217075717945

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

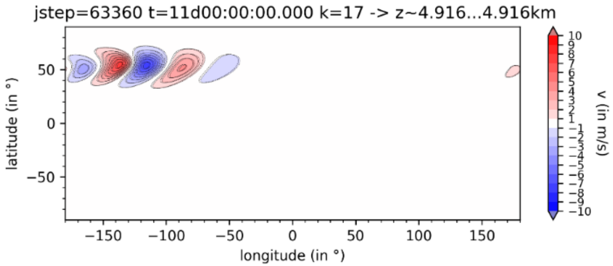
Baroclinic instability for u_{max}=35 m/s
Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines)

u



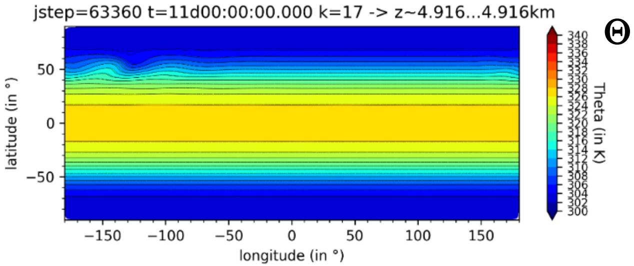
u [1]: Min=-0.04755494145870814, Mean=15.892896911587734, Max=32.431628334936896
u [2]: Min=-0.04610836152847958, Mean=15.852287066945996, Max=32.38488242537634

v



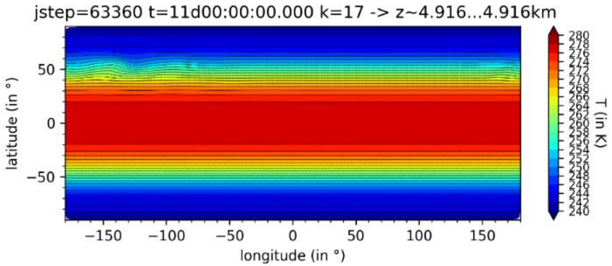
v [1]: Min=-7.443500852280272, Mean=-0.0009432031235488105, Max=7.413802936498804
v [2]: Min=-7.512956498080756, Mean=-0.0009686794754680668, Max=7.448932416873414

Θ



Theta [1]: Min=302.3973060854562, Mean=319.2240763721599, Max=326.55608432808464
Theta [2]: Min=302.14007191348907, Mean=319.1098899705293, Max=326.48268781514696

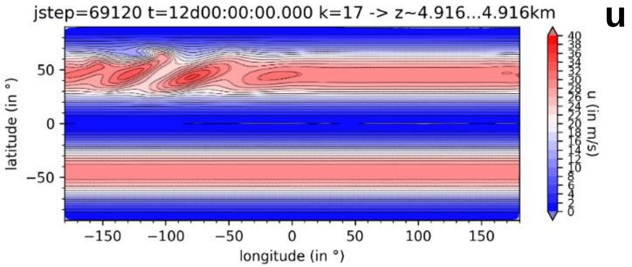
T



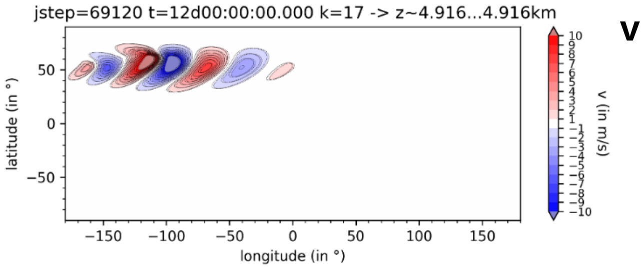
T [1]: Min=241.80890770415434, Mean=266.9753616627286, Max=277.11791500071746
T [2]: Min=241.39094891361643, Mean=266.64365042791445, Max=276.8043564362708

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

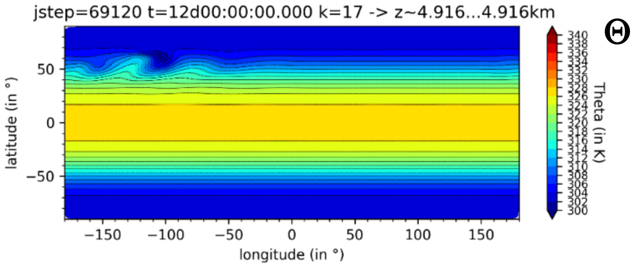
Baroclinic instability for u_{max}=35 m/s
Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines)



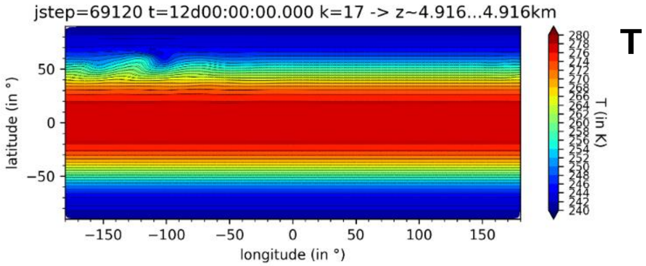
u [1]: Min=-0.14260027741605596, Mean=15.890459158363731, Max=34.483888378460314
u [2]: Min=-0.14175643985426373, Mean=15.858687905782103, Max=34.461864732379595



v [1]: Min=-14.861315275865994, Mean=-0.00020828994502579884, Max=12.583079402577228
v [2]: Min=-14.983452167986442, Mean=7.526415048724225e-05, Max=12.630566574789635



Theta [1]: Min=300.1082805795721, Mean=319.2226862043839, Max=326.5525453024551
Theta [2]: Min=299.84362467377576, Mean=319.1085078503491, Max=326.4824606853182

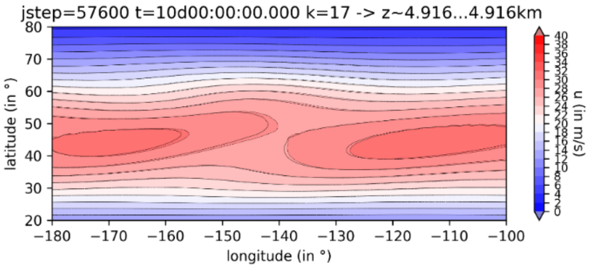


T [1]: Min=241.79524830493844, Mean=266.9748270653365, Max=277.1094915704254
T [2]: Min=241.38292754482174, Mean=266.6432040201498, Max=276.791914661595

Baroclinic instability for u_{max}=35 m/s

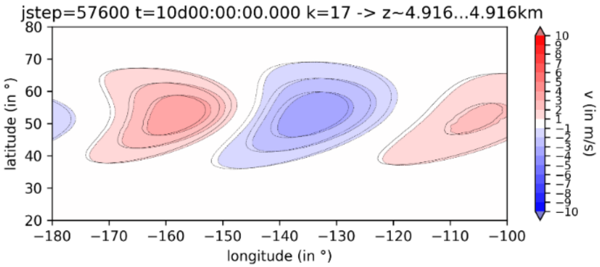
Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines) (zoom in)

u



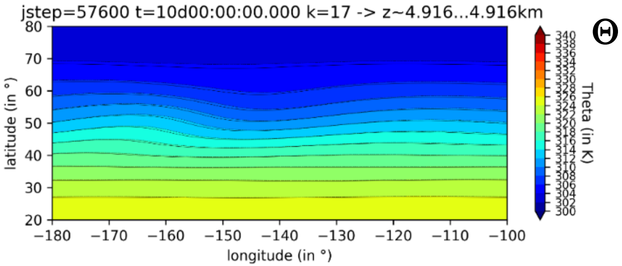
u [1]: Min=-0.02744685299260993, Mean=15.897202650368966, Max=31.262131466097287
u [2]: Min=-0.026708850879797578, Mean=15.859582904788567, Max=31.20413949176768

v



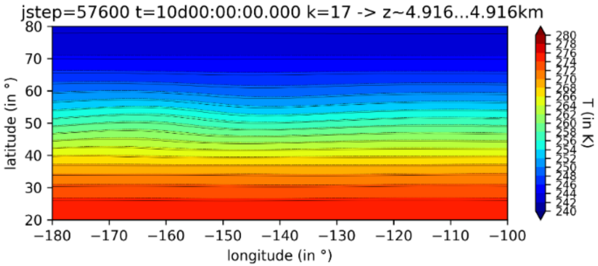
v [1]: Min=-3.8533966943565465, Mean=0.0003623418636674577, Max=3.872466370852339
v [2]: Min=-3.9005151719328195, Mean=0.00012601756289855985, Max=3.865358567757774

Θ



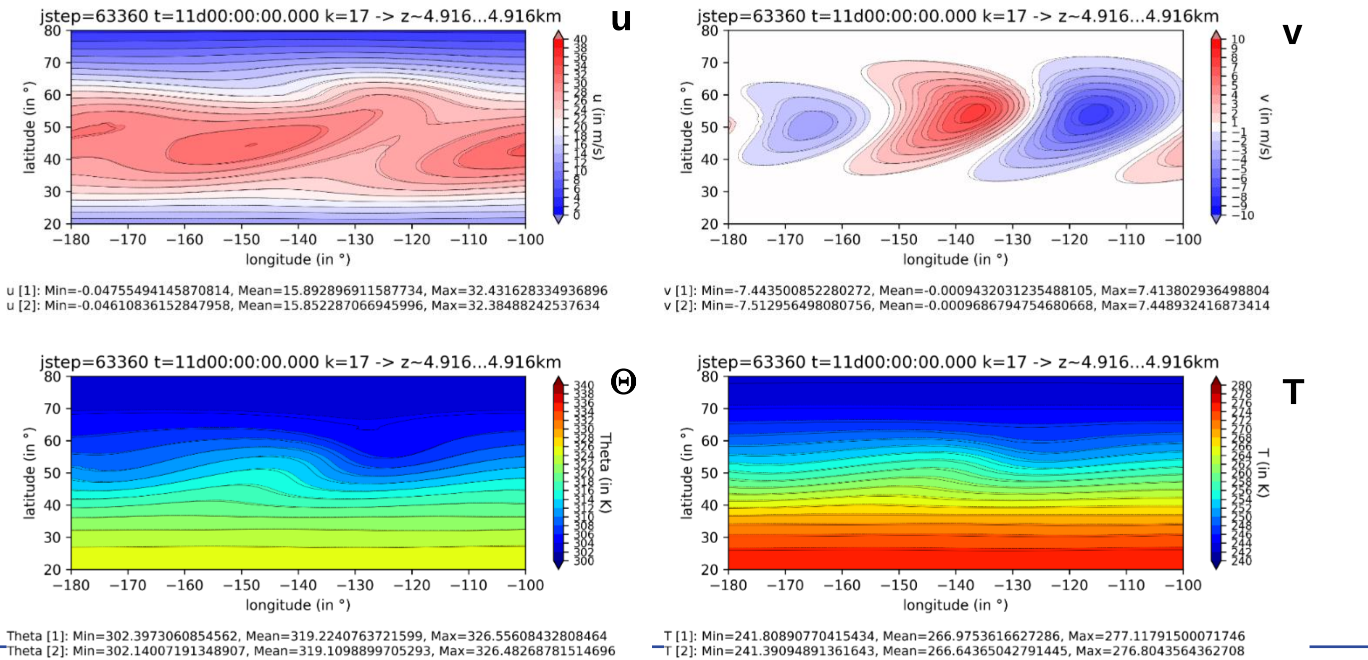
Theta [1]: Min=302.40328535291843, Mean=319.2251539461623, Max=326.5551490909007
Theta [2]: Min=302.1558545336776, Mean=319.11071928584124, Max=326.48157495962386

T

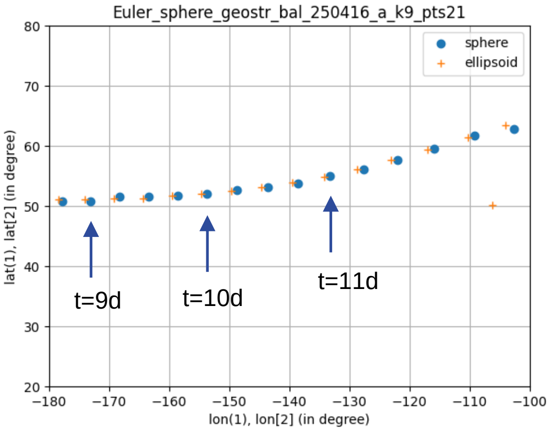


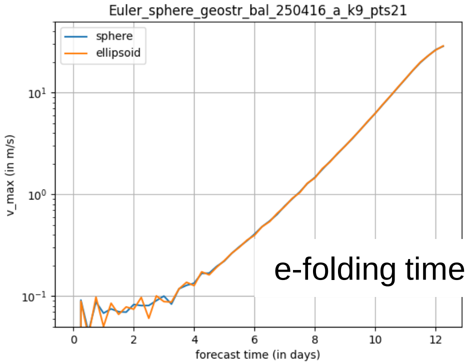
T [1]: Min=241.80819032934264, Mean=266.97598404932853, Max=277.1164755908529
T [2]: Min=241.39797553663567, Mean=266.64409756960185, Max=276.80217075717945

Baroclinic instability for u_{max}=35 m/s
Fields in z ~ 5 km, sphere (colors, dashed lines), ellipsoid (solid lines) (zoom in)

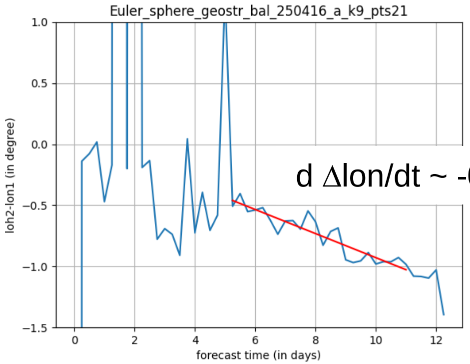


Determine Position of max v:





e-folding time ~ 1,46 d



d Δlon/dt ~ -0.1°/d

Fair comparison:
consider the range of exponential perturbation growth

Summary

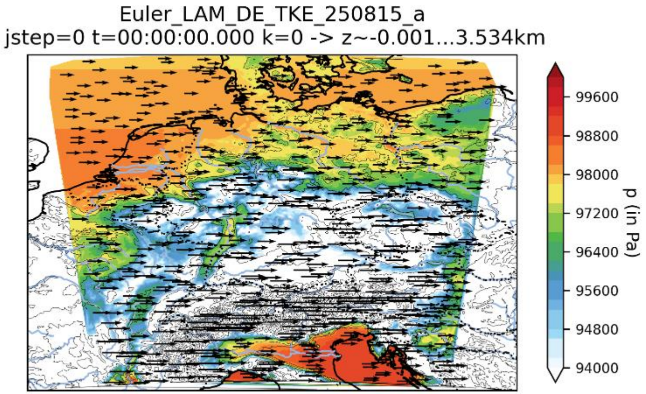
- Effects of the *ellipsoidal geopotential approximation (EGA)* have been investigated by use of a higher order Discontinuous Galerkin solver
- Test scenario: development of a baroclinic instability;
compare positional shifts during the exponential growing phase
- Effects are small and in the order of the ellipticity ε~0.003:
frontal shifts up to ~100 km / 7days for stronger jets
wave velocity slightly smaller for the ellipsoid
- Nevertheless EGA might be relevant in the near future for NWP models
- In this test scenario, EGA vs. SGA effects are significantly larger than deep vs. shallow atm. effects

Outlook

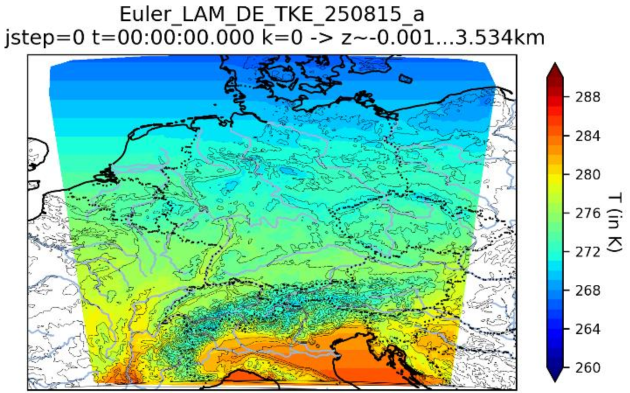
- Investigate influence of large scale mountains
- Use higher resolution

A slightly more realistic simulation with BRIDGE …

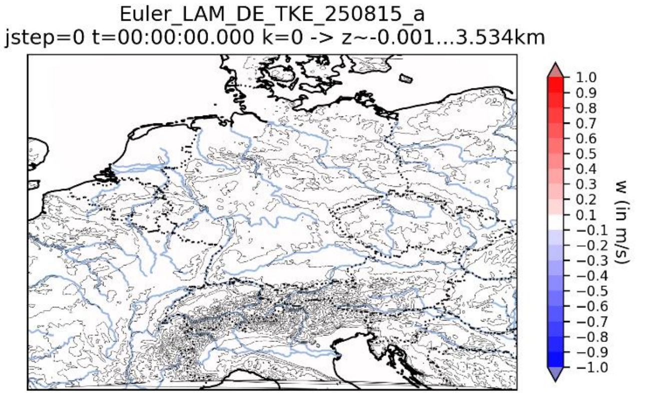
- ‚Limited area' simulation with initial/boundary values taken from the geostropic/hydrostatic balancing scheme (see ellip./sphere topic above)
- Nearly ICON-D2 area, R2B8 grid (dx~10km)
- Orography on R2B10 (~2.5km), generated from R2B13 (~300m) ASTER data
- DG 4th order SSP3(3,3,2) IMEX-RK-HEVI time integration
- With TKE turbulence scheme



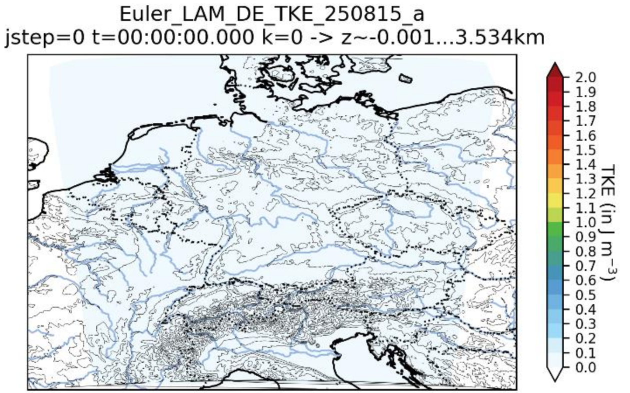
p [1]: Min=63825.847542353986, Mean=94436.47807947196, Max=99053.18068709542



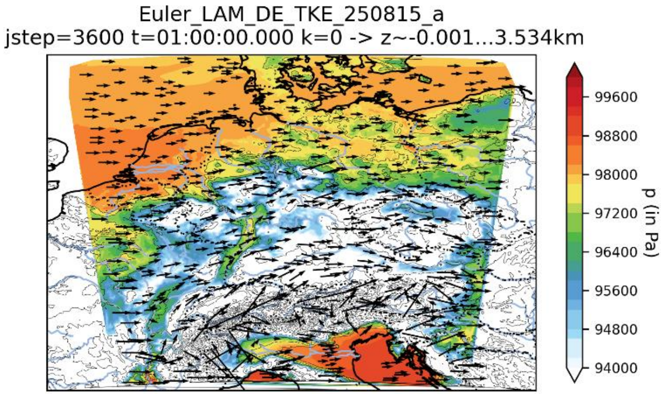
T [1]: Min=265.75150775006364, Mean=273.88979383829997, Max=284.94134029646756

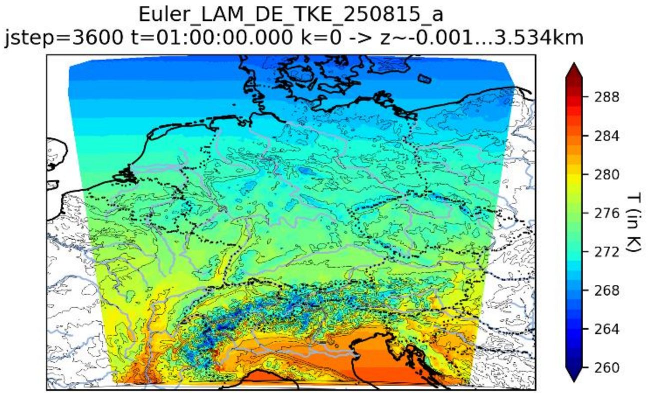


w [1]: Min=0.0, Mean=0.0, Max=0.0



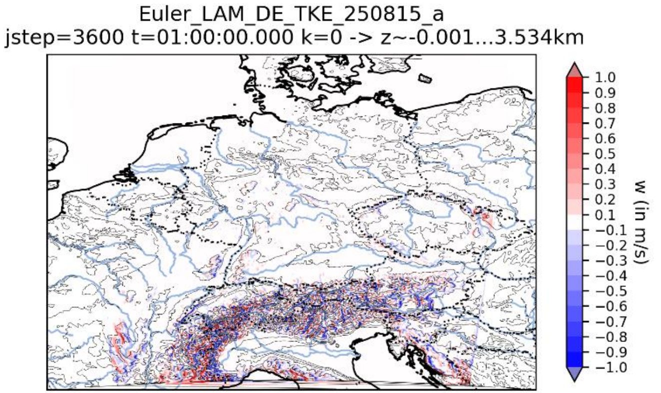
TKE [1]: Min=0.0009999999999999998, Mean=0.0010000000000000009, Max=0.00100000000000000

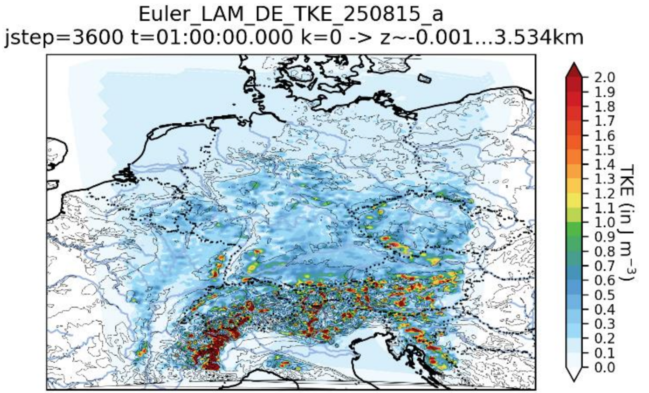




p [1]: Min=64029.97873917685, Mean=94431.08778743642, Max=99054.62016560104

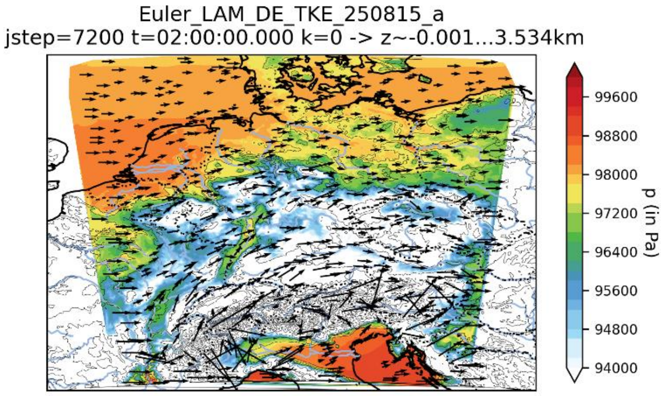
T [1]: Min=261.40731679653027, Mean=274.02739216994803, Max=288.84587205180935



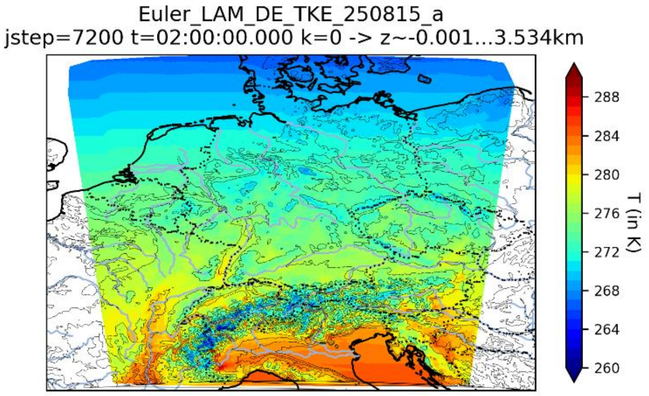


w [1]: Min=-7.817653678763975, Mean=-0.024482507471938083, Max=1.9581107449702417

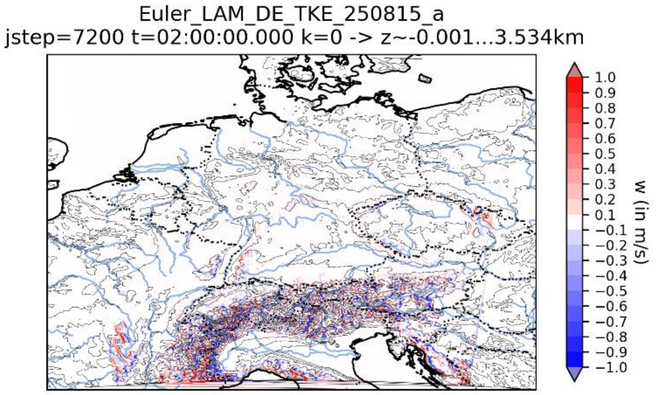
TKE [1]: Min=0.0, Mean=0.25543136591157006, Max=16.44887574307054



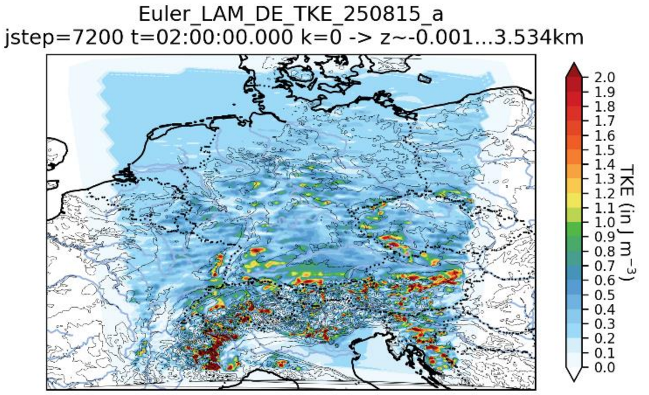
p [1]: Min=64049.16434112612, Mean=94441.71500753362, Max=99054.65052455927



T [1]: Min=262.3708749536576, Mean=274.434297760023, Max=288.9251002339021



w [1]: Min=-6.075530241694701, Mean=-0.020395862363194 76, Max=2.0103999531259364



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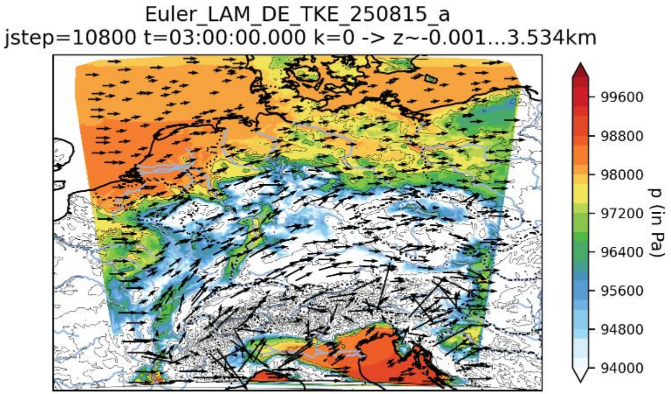
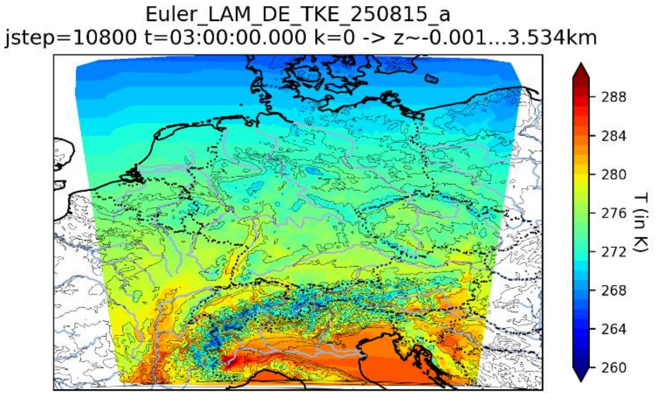
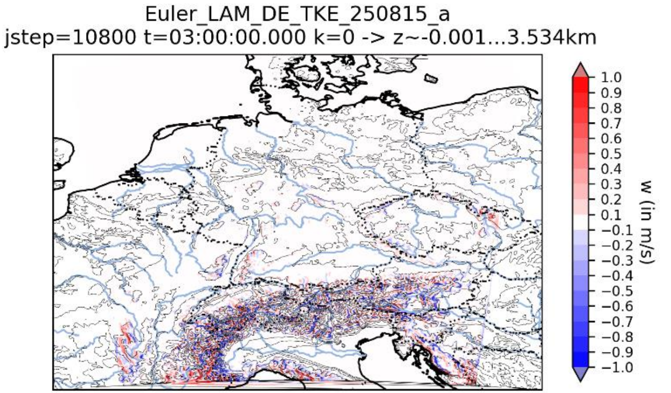


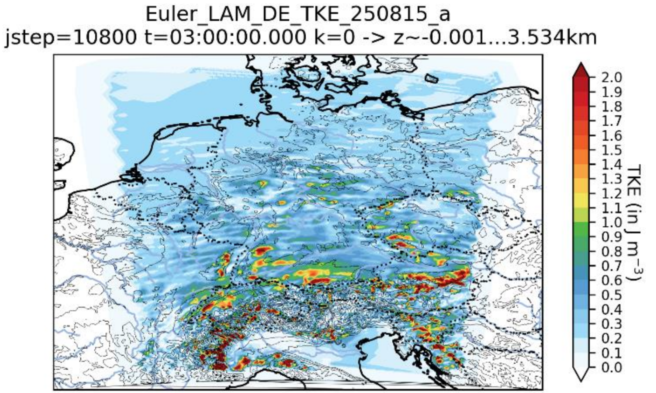
p [1]: Min=64092.69837587956, Mean=94445.11520493958, Max=99054.64899518441



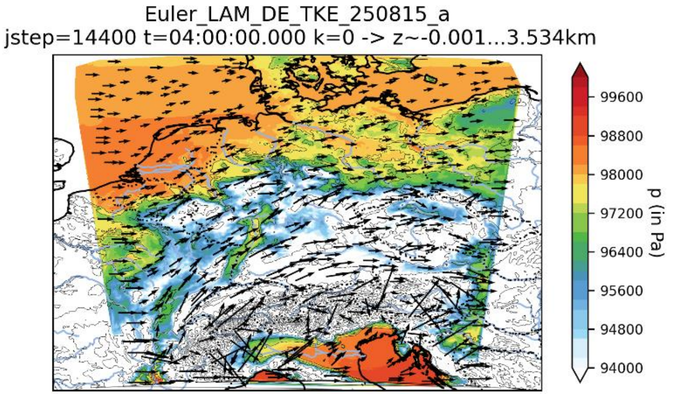
T [1]: Min=261.4386728877804, Mean=274.7417128700661, Max=289.8146760563638



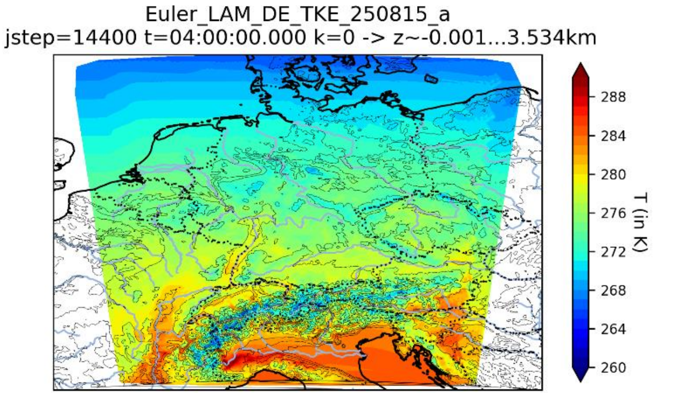
w [1]: Min=-5.538907673113814, Mean=-0.018837196919420526, Max=1.9837644395759697



TKE [1]: Min=0.0, Mean=0.313602459720235, Max=12.399520690875882



p [1]: Min=64091.999716856095, Mean=94448.96638282166, Max=99054.64791438483



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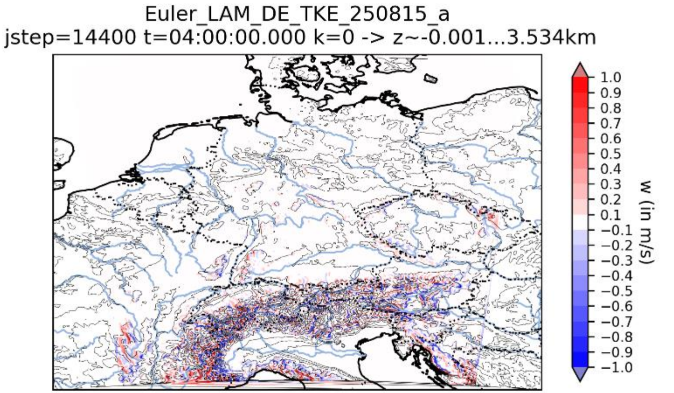
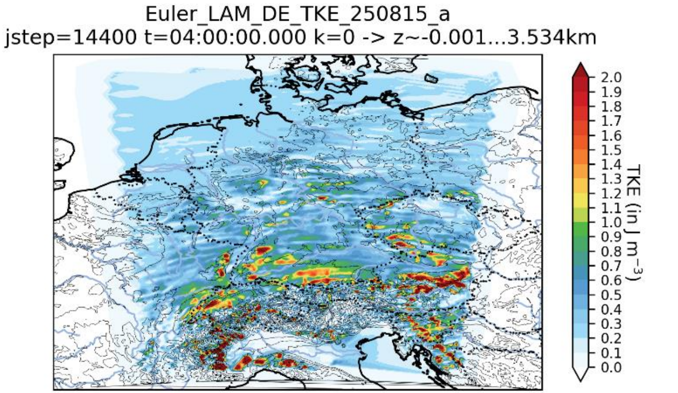


w [1]: Min=-5.230606119564826, Mean=-0.018301710654113526, Max=1.905437206873609



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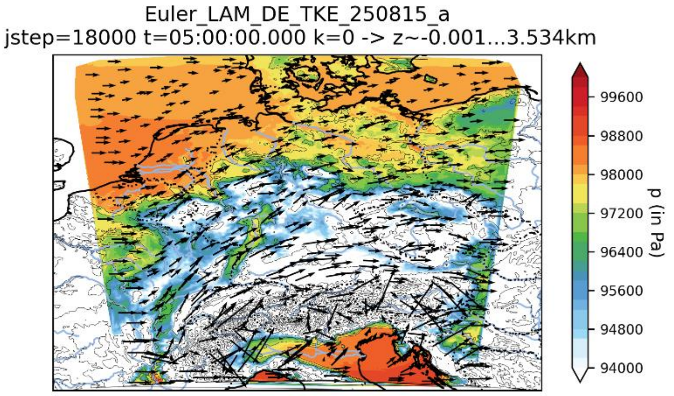
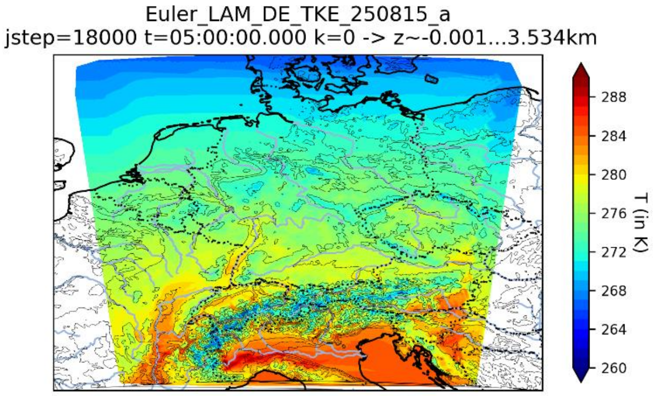
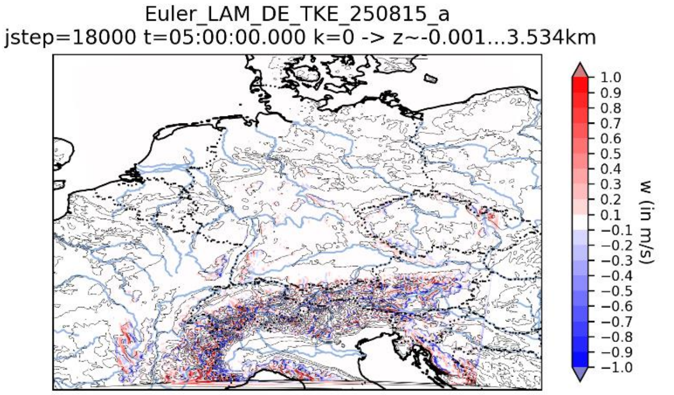


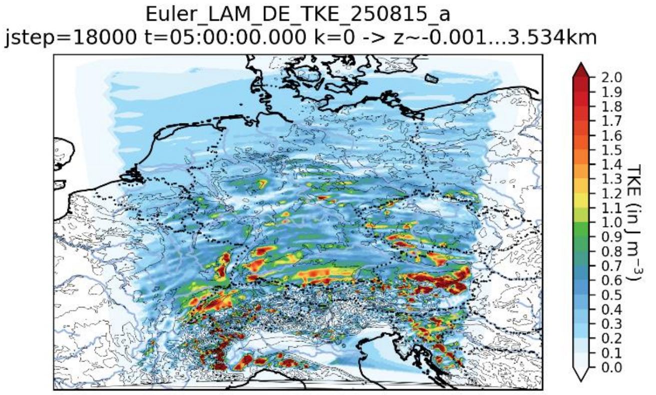
p [1]: Min=64066.16365682049, Mean=94449.24749298352, Max=99054.64555854397



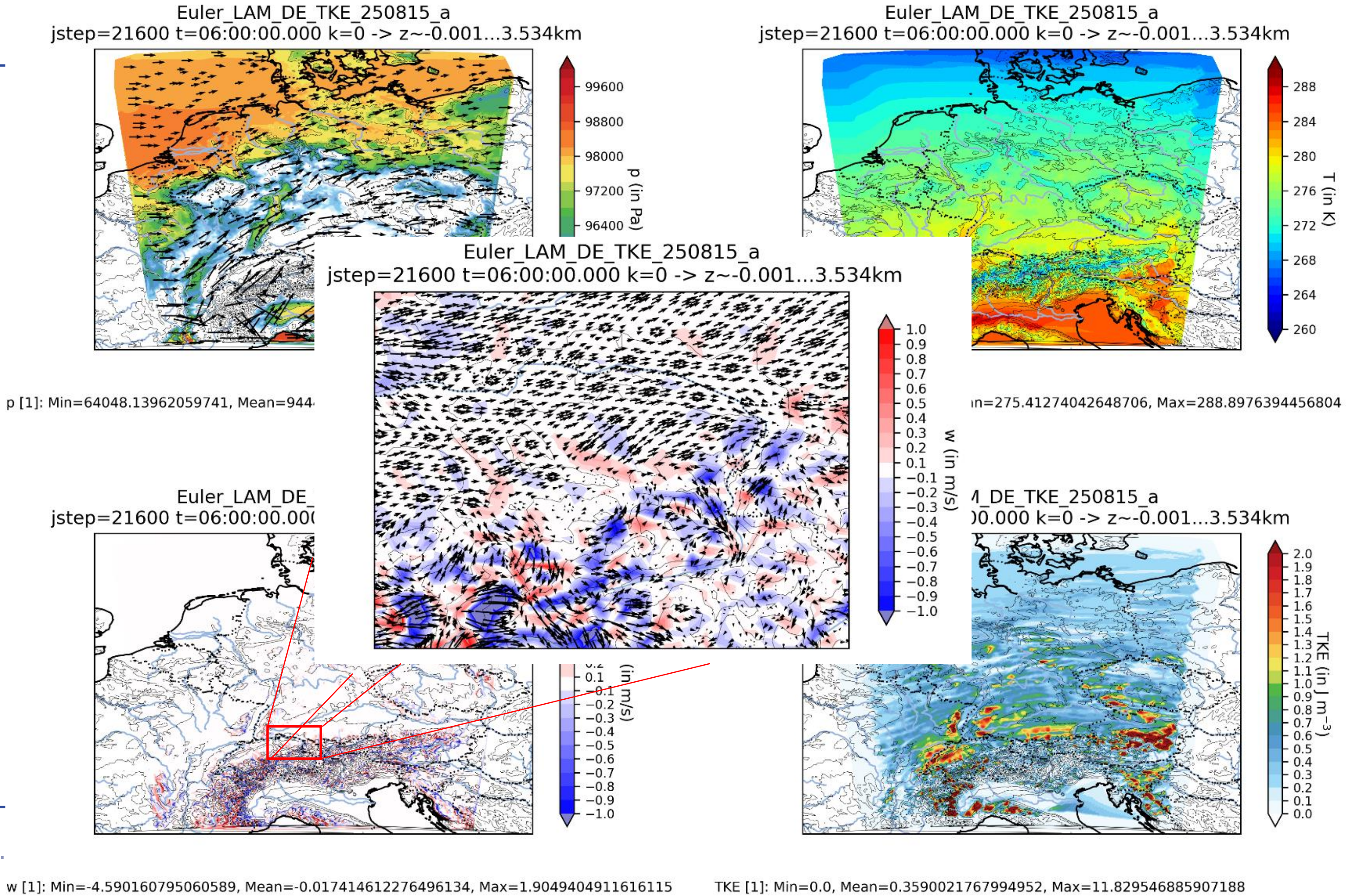
T [1]: Min=262.32031311043204, Mean=275.21670956453767, Max=290.14234285938136



w [1]: Min=-4.664740684903363, Mean=-0.017835676825043244, Max=1.9047854804155668



TKE [1]: Min=0.0, Mean=0.3473749296520968, Max=11.289697653671336


Euler_LAM_DE_TKE_250815_a
jstep=21600 t=06:00:00.000 k=0 -> z~-0.001...3.534km
p (in Pa)
99600
98800
98000
97200
96400
p [1]: Min=64048.13962059741, Mean=944
T (in K)
288
284
280
276
272
268
264
260
n=275.41274042648706, Max=288.8976394456804
w (in m/s)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1.0
w [1]: Min=-4.590160795060589, Mean=-0.017414612276496134, Max=1.9049404911616115
TKE (in J m−3)
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
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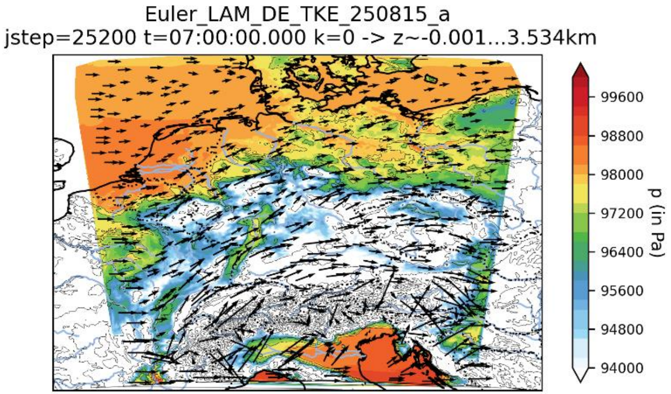
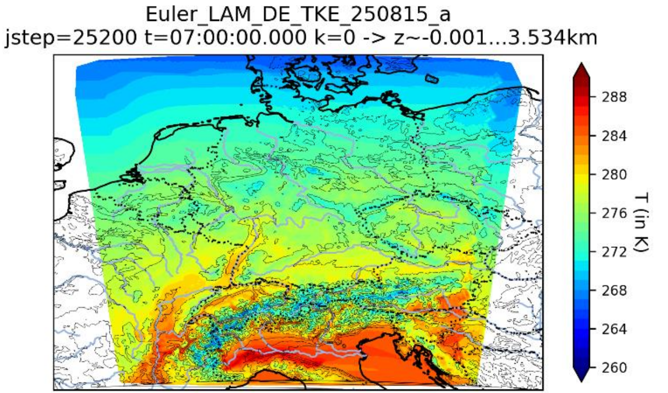


p [1]: Min=64046.93315474689, Mean=94444.71455761425, Max=99054.64293139536



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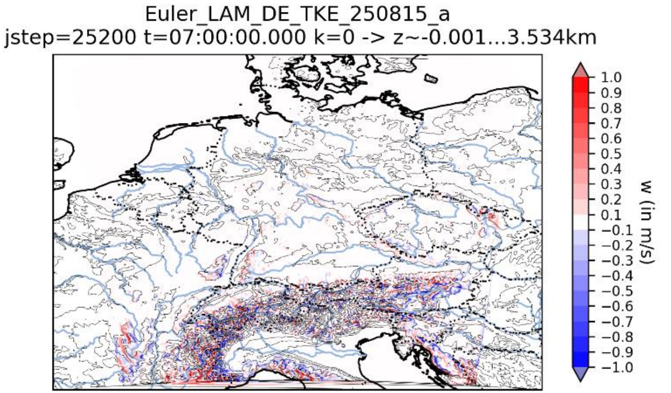

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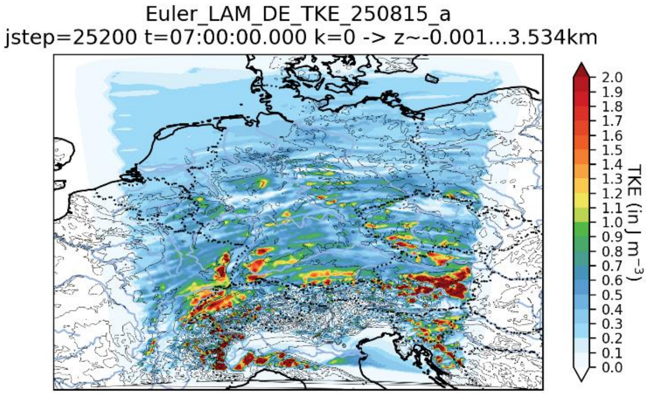

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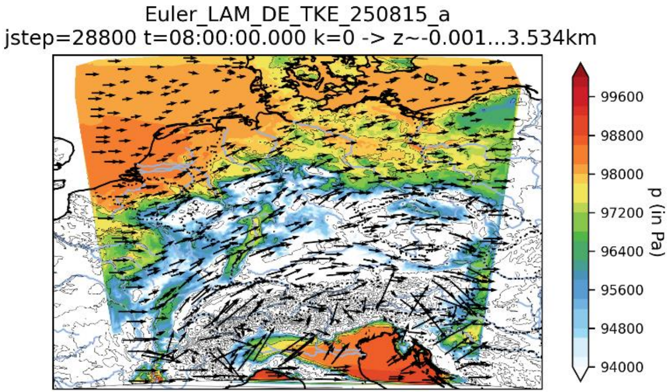

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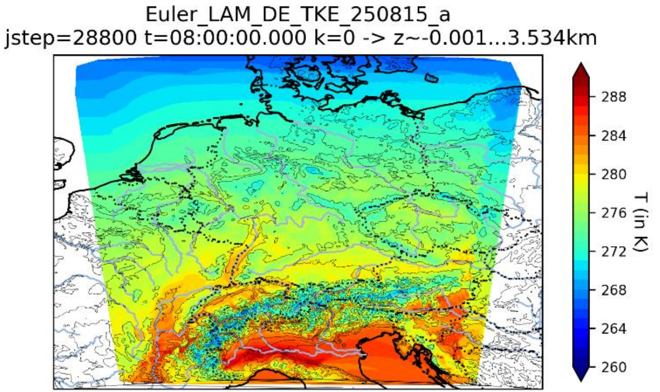

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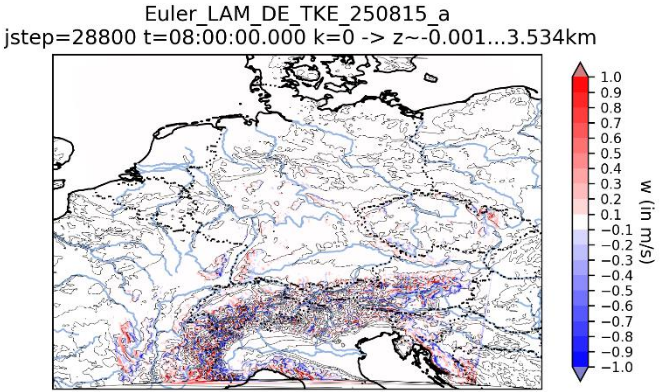

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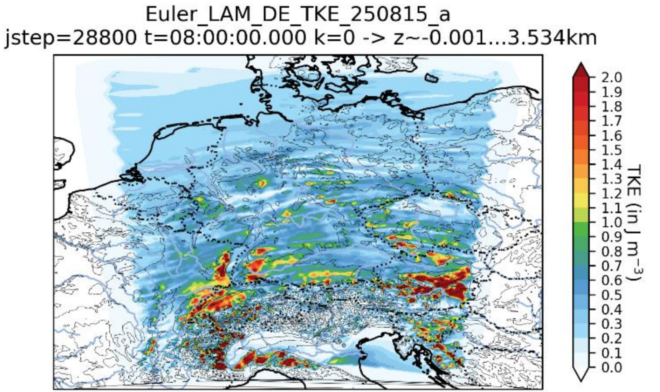

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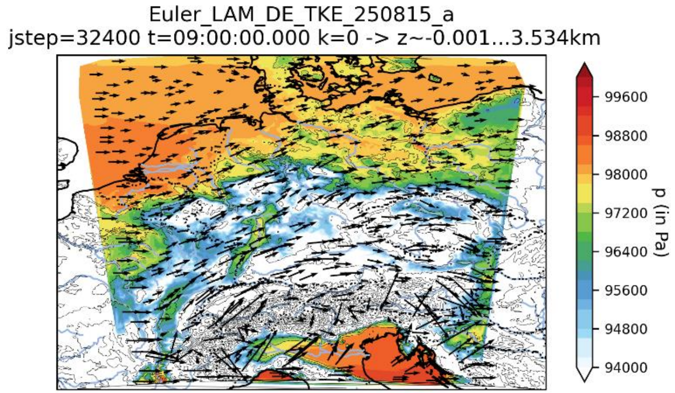

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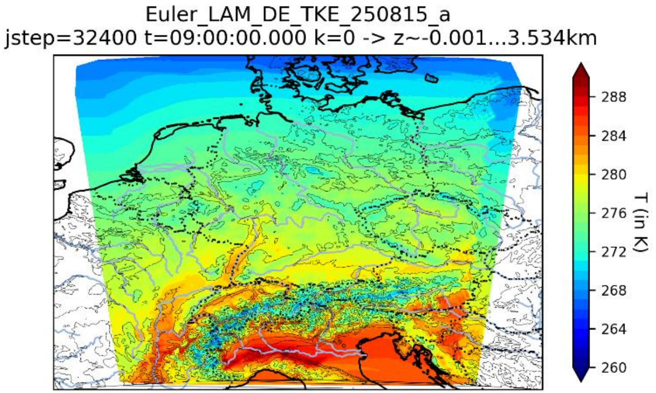

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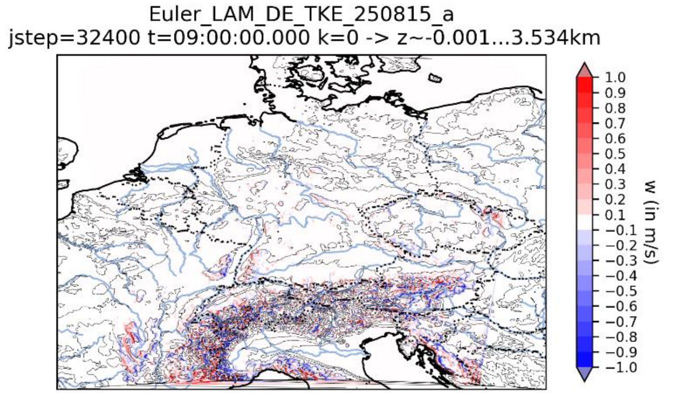

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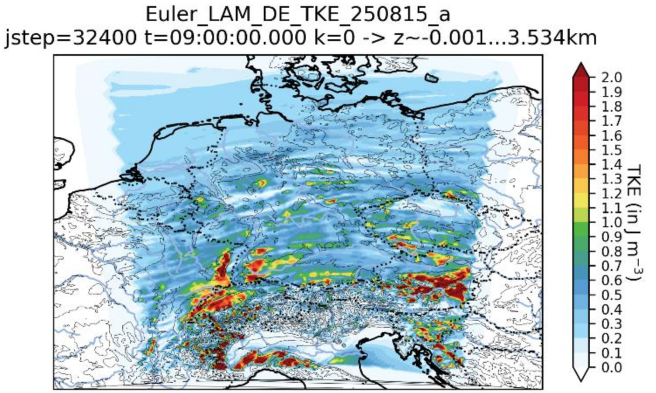

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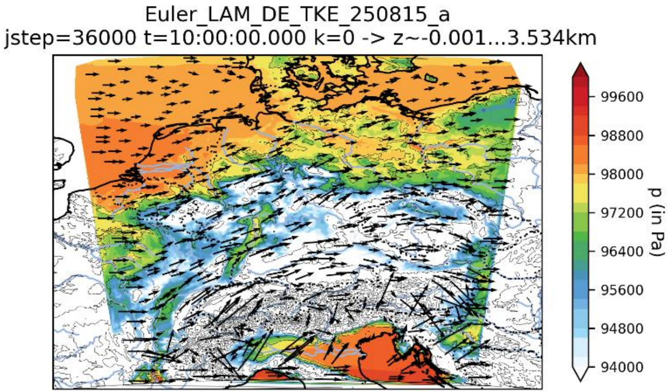

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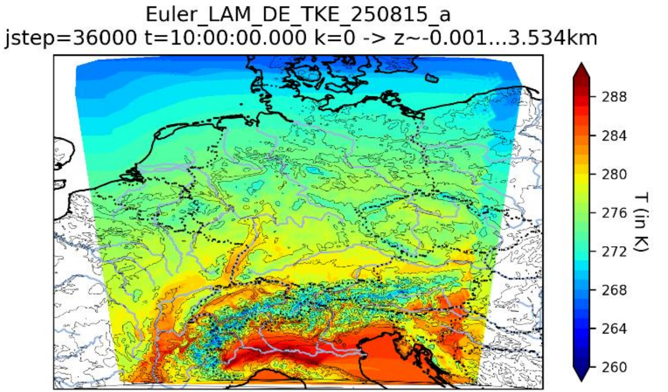

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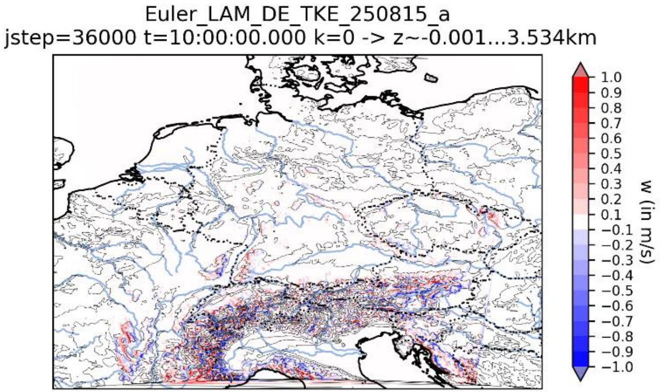

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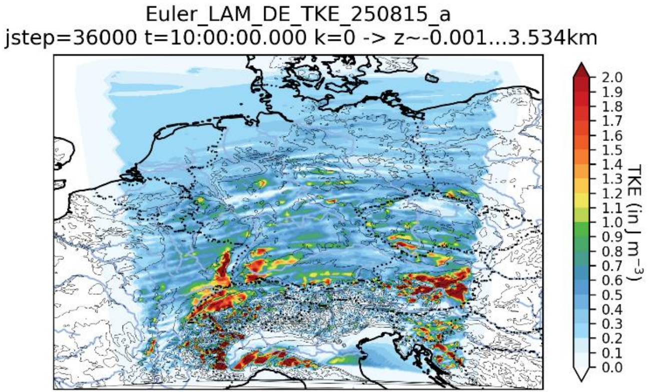

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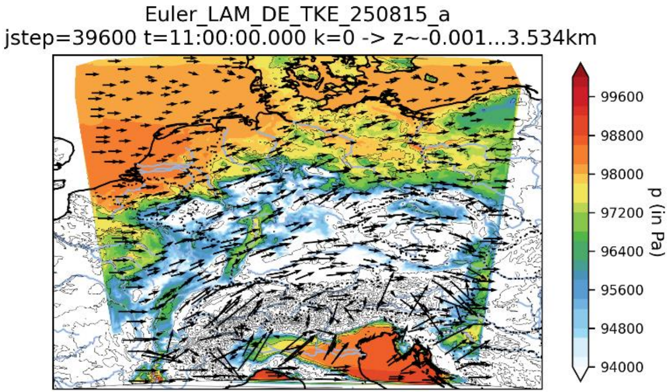

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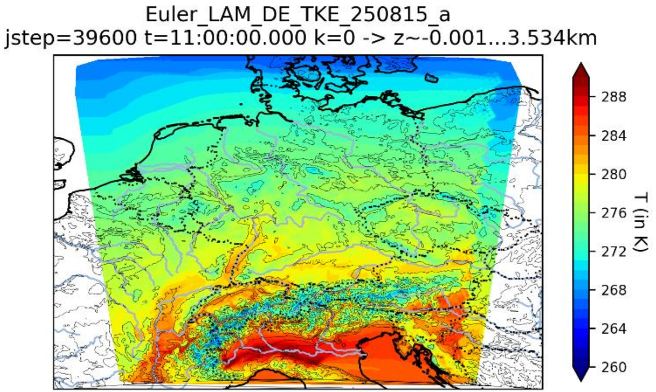

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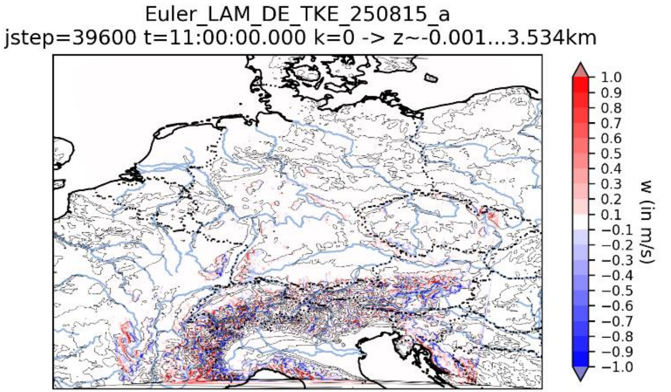

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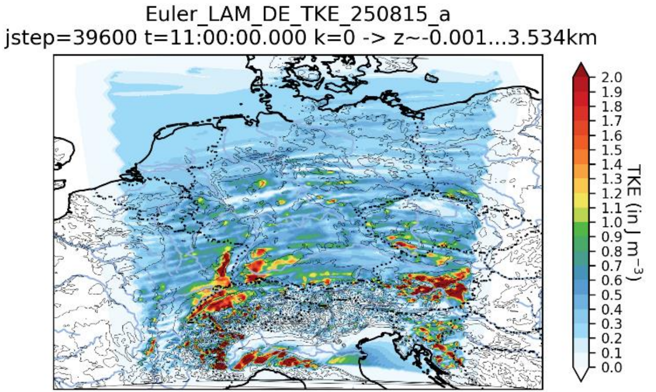

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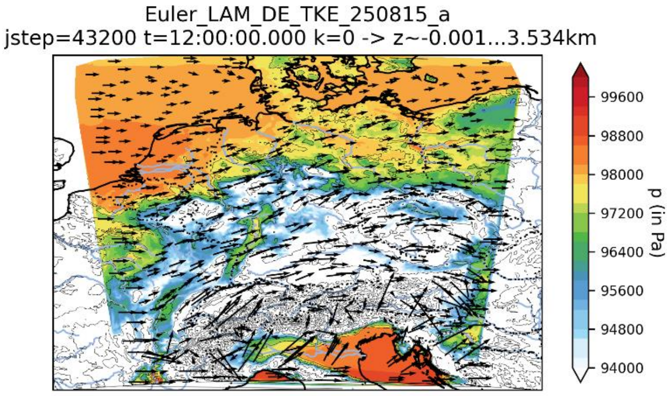

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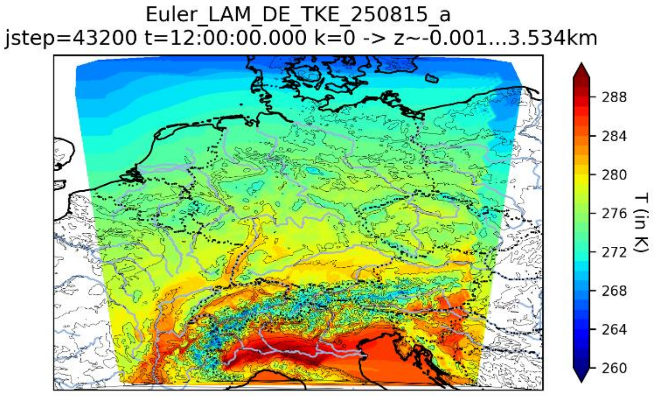

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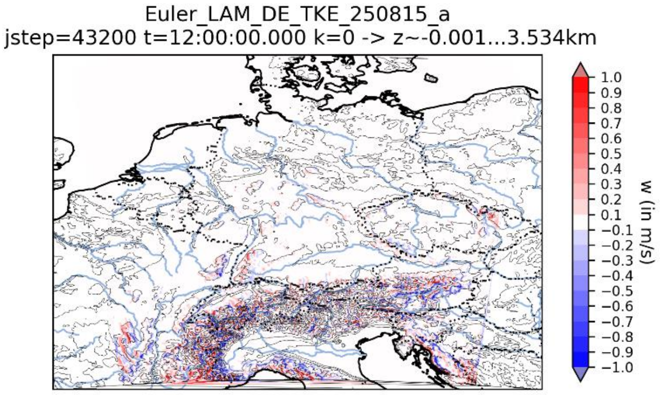

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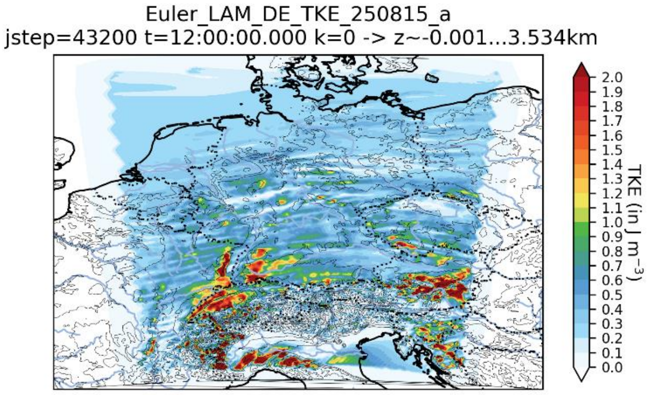

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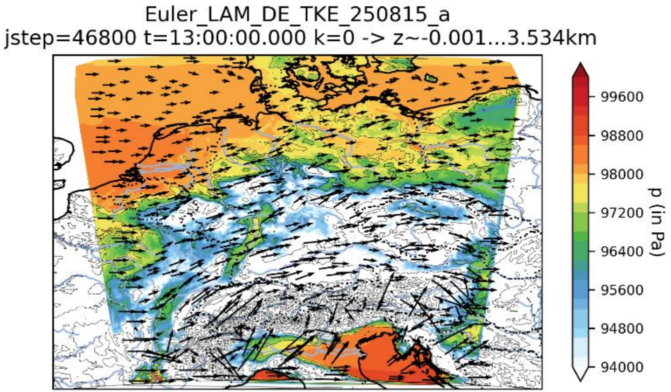

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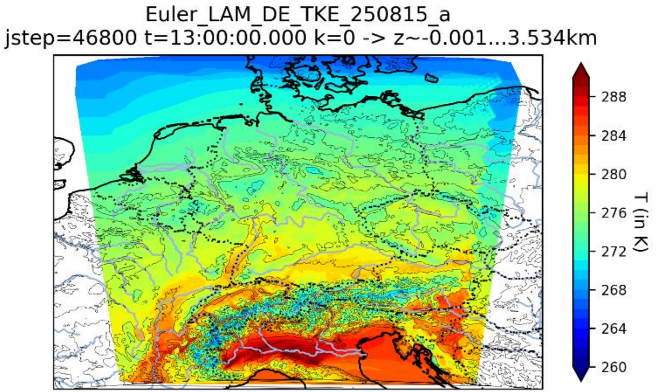

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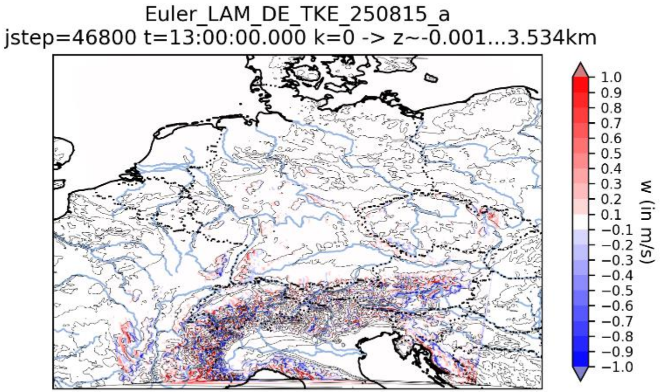

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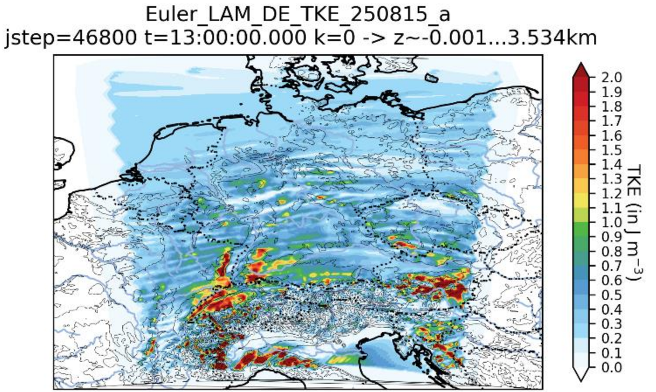

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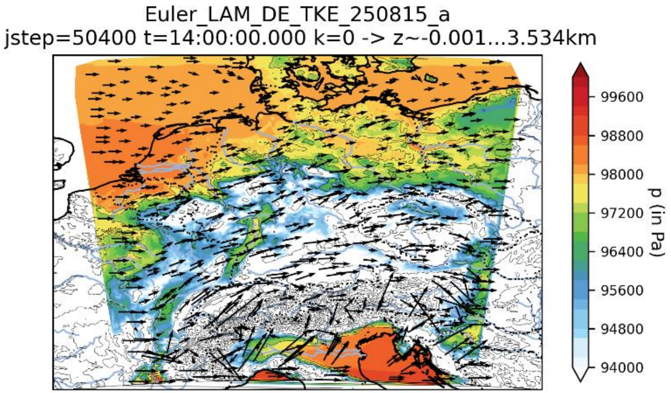

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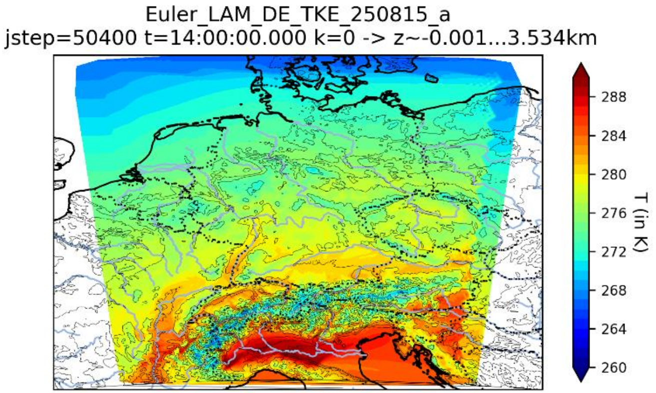

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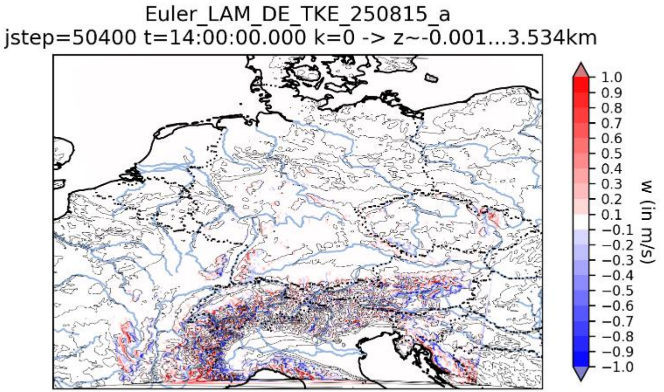

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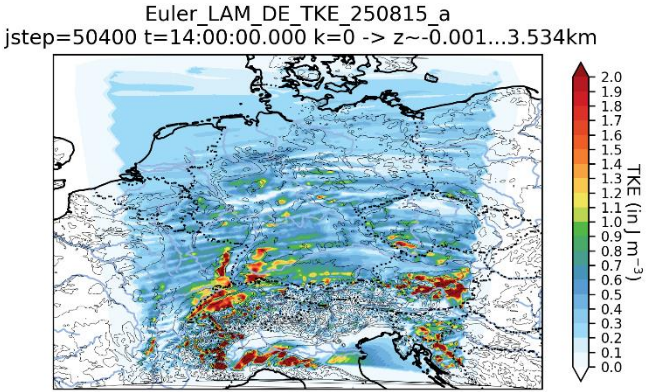


TKE [1]: Min=0.0, Mean=0.34244680341542716, Max=9.894663026030852

Milestones of the BRIDGE-project (status Sept. 2025)

0.	first version of a **FE/DG framework** available, **MPI** parallelized	Q2/2021	✓
1.	**shallow water** equations on the sphere, **explicit** time integration (RK)	Q3/2021	✓
2.	**3D Euler** equations on the sphere, **explicit** time integration (RK)	Q1/2022	✓
2.b	with 3D diffusion (+ a simple turbulence scheme) BR1/BR2	Q2/2022	✓/🛠
2.c	grid refinement works	Q2/2022	✓
3.	**Euler** equations, **HEVI** time integration (IMEX-RK)	Q3/2022	✓
3.b	optimization of the vertically implicit solver (Schur compl.,...)	Q4/2022	✓
3.c	with 3D diffusion (+ a simple turbulence scheme), HEVI	Q1/2023	✓
4.	cloud **microphysics** (Kessler) + **tracer advection** (positive definit) + explicit sedimentation scheme	Q4/2022	🛠
4.b	cloud microphysics (cloud ice, Graupel) + vertically implicit sedimentation scheme	Q1/2023	!
5.	HEVI diffusion on dt(fast phys.); **physics-dynamics coupl**./time integr.	Q4/2023	✓/🛠
6.	**limited area (**=use **lateral BCs)** version available	Q1/2023	✓/...
7.a	**vectorized** version available	Q2/2023	!
7.b	**GPU** version	Q1/2024	!
7.c	**general code optimization**	Q4/2024	🛠
8.	coupling of a one-equation **turbulence + BL** scheme	Q2/2023	✓

BRIDGE – collaboration with academia

Project: **ICON-DG: Gearing up the ICON Model with Smarter Discretizations**
in the call ‚**WarmWorld Module 4 Smarter**'
by German Ministry of Research and Educ. (BMBF)
has been accepted.
Duration: 01.07.2024 - 30.06.2027

Project partners:

- *DLR Köln, Inst. for software technology*, project leader: Johannes Markert
- *Univ. Köln, Division of Mathematics*, group Prof. Gregor Gassner
  postdoc: Tristan Montoya
- *DWD*, M. Baldauf, BRIDGE group

Goal: develop further stabilization of DG schemes by designing
entropy stable / entropy conserving schemes (=obeying 2nd law of
thermodynamics) especially for the prismatic grid cells used in
ICON / BRIDGE with HEVI and horiz. flux transformations.
→ **‚Next Generation DG methods'**

e.g. Gassner, Winters (2021) Front. Phys.

Discontinuous Galerkin (DG) methods in a nutshell

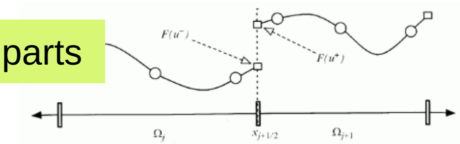
$$\frac{\partial q^{(k)}}{\partial t} + \nabla \cdot \mathbf{f}^{(k)}(q) = S^{(k)}(q), \qquad k = 1, ..., K$$

e.g. *Cockburn, Shu (1989) Math. Comput.*
Cockburn et al. (1989) JCP
Hesthaven, Warburton (2008)

1.) Finite-element representa

$$q^{(k)}(x,t) = \sum_{l=0}^{p} q_{j,l}^{(k)}(t)\; p_l(x - x_j)$$

via Integration by parts



From *Nair et al. (2011)* in ‚*Numerical techniques for global atm. models'*

2.) Galerkin approach → weak formulation

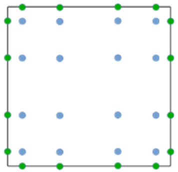
$$\frac{d}{dt}\int_{\Omega_j} q^{(k)}\, v\, dV + \int_{\partial\Omega_j} f^{(k)num,\perp}\, v\, da - \int_{\Omega_j} \mathbf{f}^{(k)} \cdot \nabla v\, dV = \int_{\Omega_j} S^{(k)}\, v\, dV$$

3.) Finite-volume ingred

$$\mathbf{f}(q) \to f^{num,\perp}(q^+, q^- \quad \ldots \quad - q^-)$$

Discrete analogon:
summation by parts
→ also allows derivation of
stability statements.

4.) Gaussian quadratur ... ntegrals



→ ODE-system for $q^{(k)}{}_{jl}(t)$

5.) Use a time-integration scheme (Runge-Kutta, …)

Work done until now in the Warmworld project:

Task: adapt ‚summation by parts‘ (SBP) operators to the *covariant formalism* of *Baldauf (2020, 2021).*
This has been successfully achieved for the shallow-water equations on a cubed sphere grid (*T. Montoya*).
Related paper (almost ready for publication):
T. Montoya, A. M. Rueda-Ramirez, G. J. Gassner: Entropy-stable discontinuous spectral-element methods for the spherical shallow water equations in covariant form

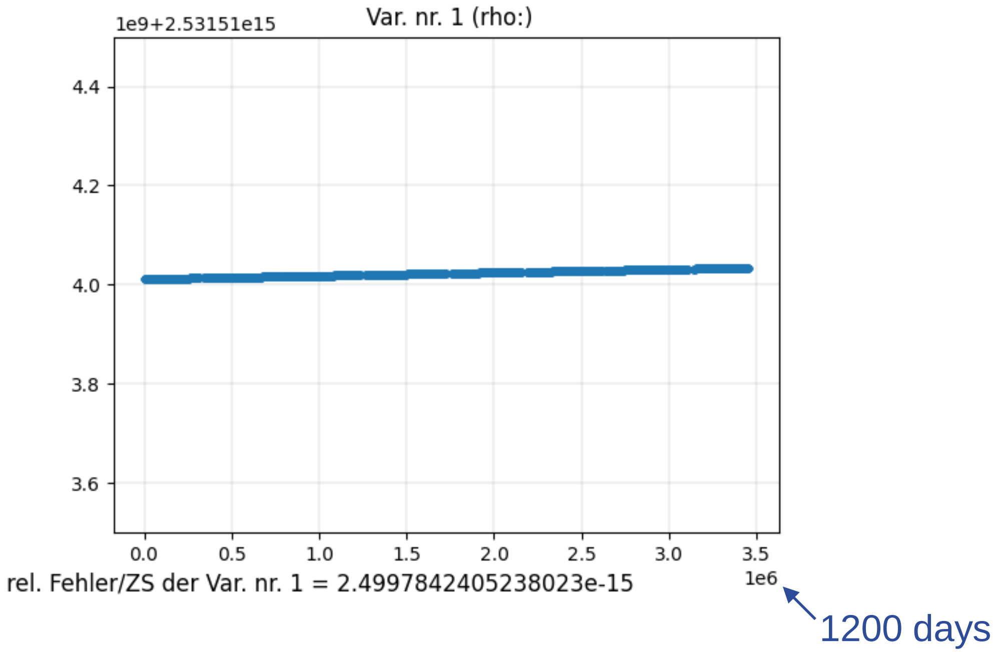
Next steps:

- SBP operators for the full 3D problem with the covariant Euler equations.
- Achieve SBP property on triangles (heavy problem!)

Meetings

- First kick-off meeting 19./20. Sept. 2024 at DLR Köln
- Project pres. at the Warmworld General Assembly, 14-16 Oct 2024, Bremerhaven
- Project meeting at DWD am 27./28.03.
- Bi-weekly online meetings between project partners and communication via Slack

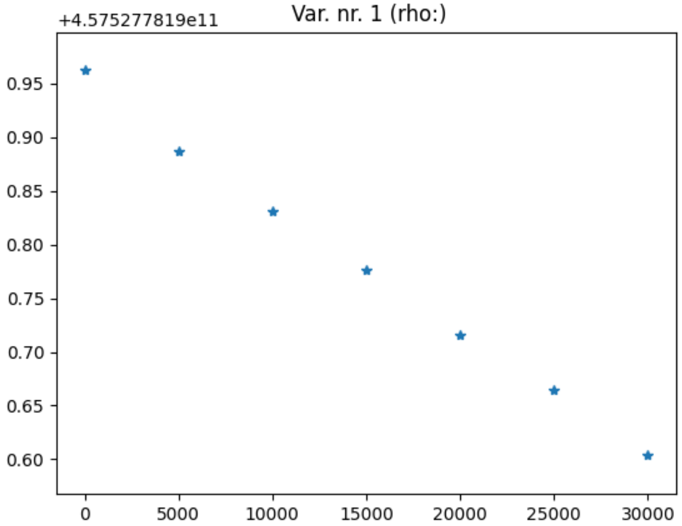
Held, Suarez (1994) test: global mass conservation



Relative global mass increase of only 3 $\cdot 10^{-9}$ / yr !

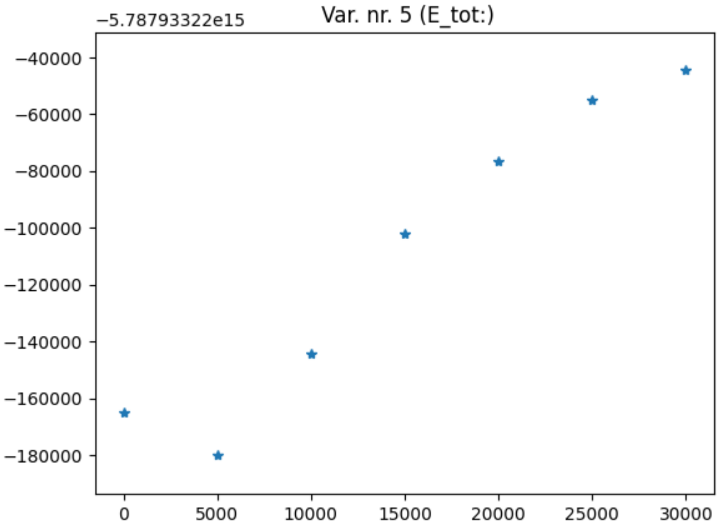
Falling cold bubble test (Straka et al (1993)): global conservation

Density



rel. Fehler/ZS der Var. nr. 1 = -2.614680703169673e-17

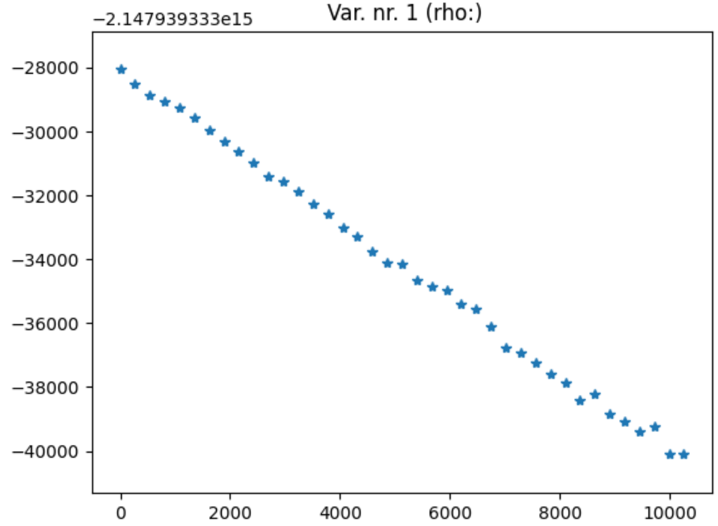
Energy



rel. Fehler/ZS der Var. nr. 5 = 6.943871868987909e-16

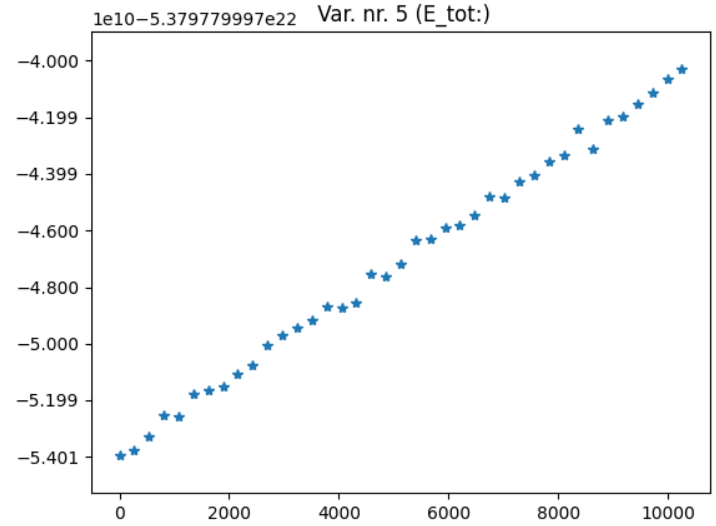
Baroclinic instability test (Jablonowski, Williamson (2006): global conservation

Density



rel. Fehler/ZS der Var. nr. 1 = -5.469224933099075e-16

Energy



rel. Fehler/ZS der Var. nr. 5 = 2.4665875978841478e-17