

# **Examination of the impact of mixing-length formulation on mesoscale simulation results**

A Lokal-Modell case study

*Jürgen Helmert*

Leibniz-Institute for Tropospheric Research, Leipzig

Fifth International SRNWP-Workshop on Non-Hydrostatic Modelling  
Bad Orb, 27-29 October 2003

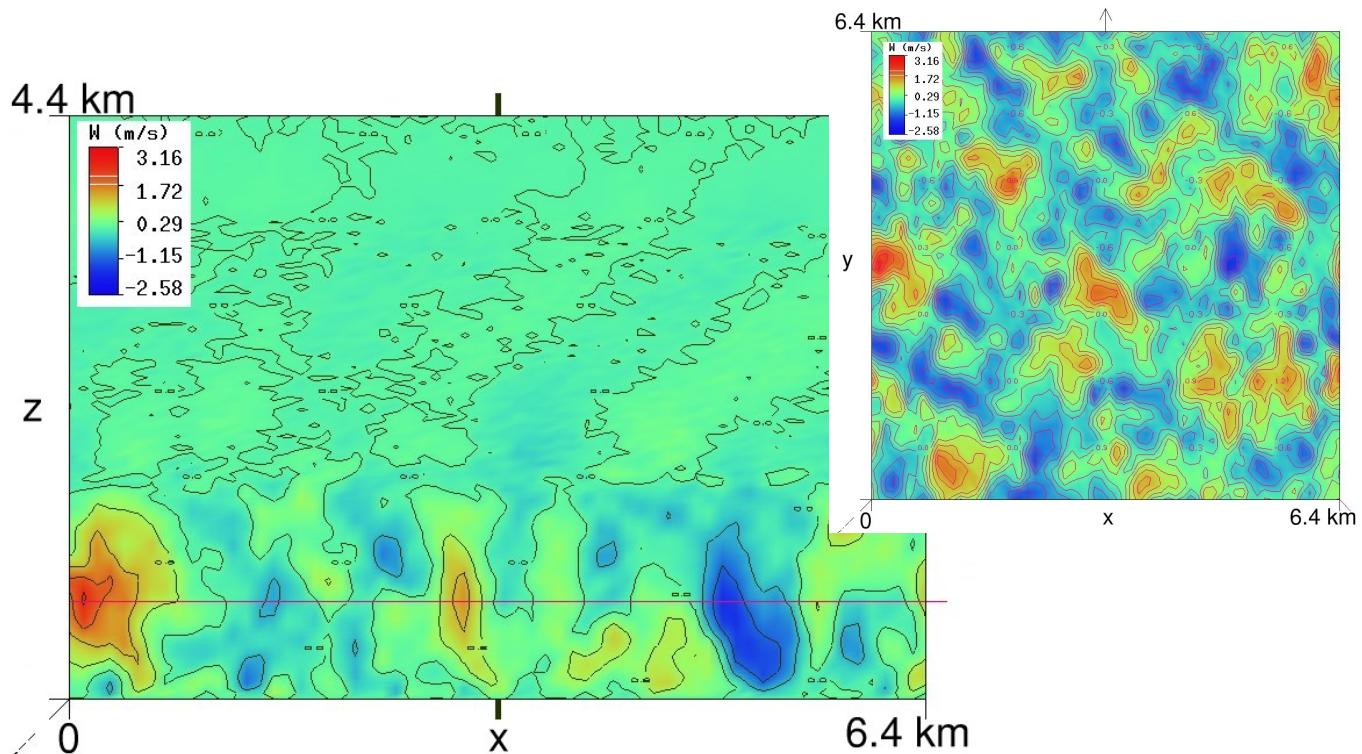
# Overview

- Introduction
- Turbulence length-scales derived from large-eddy simulation
- Mesoscale response to mixing-length formulation
- Conclusions and outlook

# Introduction

## Boundary layer turbulence

### Coherent structures in convective boundary layers

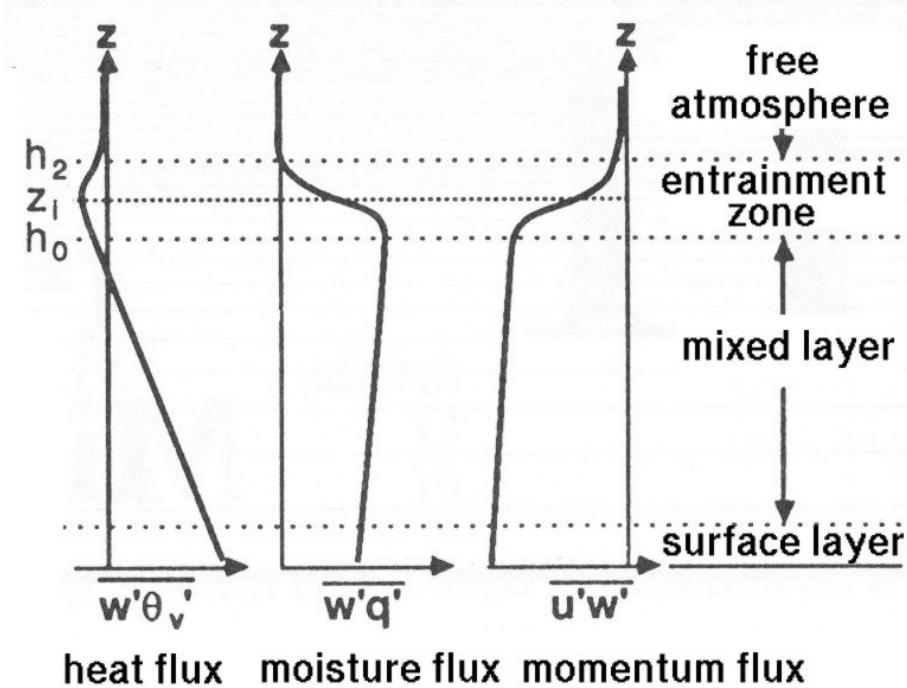


# Introduction

## Turbulence closure

Parameterisation of turbulent fluxes

local – eddy diffusivity approach:  $\overline{w'\phi'} = -K_\phi(z, t) \frac{\partial \bar{\phi}}{\partial z}$



# Introduction

## Turbulence closure – Mellor-Yamada approach

- Aim: K-Parameterisation in dependence on atmospheric stability state<sup>1</sup>
- Requirements:
  - Specification of a characteristic velocity scale  $e^{1/2}$
  - Specification of a characteristic mixing length scale  $l$

$$K \sim le^{1/2}$$

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<sup>1</sup>Mellor and Yamada, 1974; Mellor and Yamada, 1982

# Introduction

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- Specification of the characteristic velocity scale by a prognostic TKE-equation:

$$\frac{\partial \bar{e}}{\partial t} = \dots$$

- Specification of the characteristic mixing length scale by a prognostic equation is difficult!

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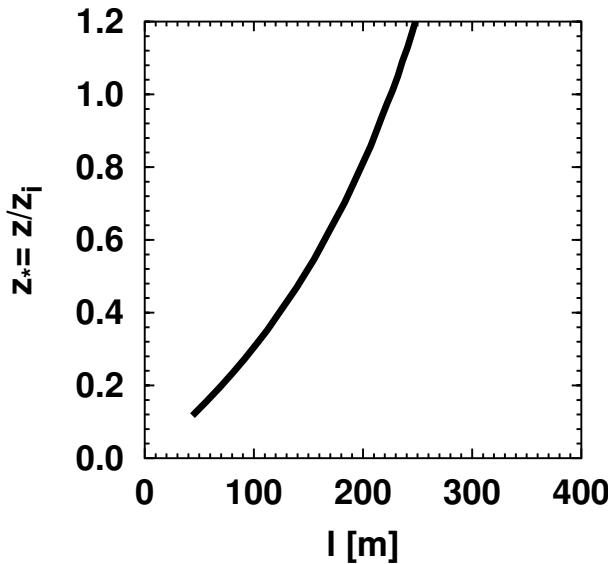
<sup>1</sup>Mellor and Yamada, 1974; Mellor and Yamada, 1982

# Introduction

## Turbulence closure – Specification of the mixing length

- Master length scale approach<sup>1</sup>: neutral boundary layer<sup>2</sup>

$$l = \left( \frac{1}{\kappa z} + \frac{1}{l_0} \right)^{-1}$$

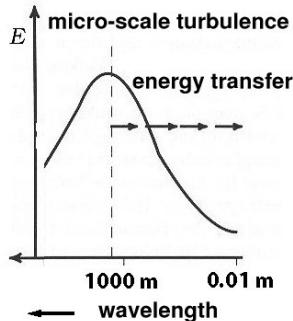
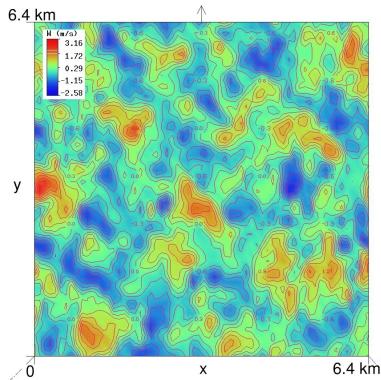


<sup>1</sup> Mellor and Yamada, 1974, 1982

<sup>2</sup> Blackadar, 1962

# Introduction

## Characteristic turbulence length scales

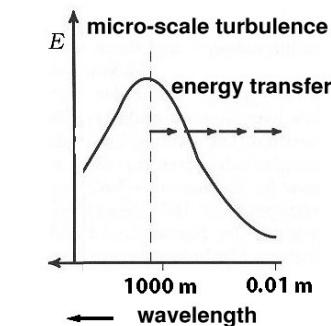
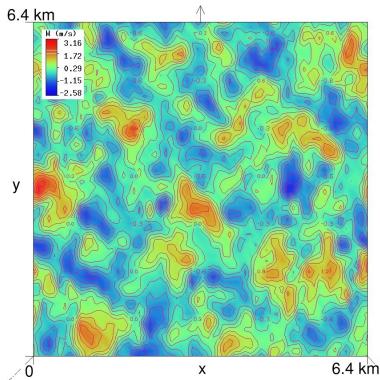


spectral peak wavelength

$$\Phi_w(\lambda) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} e^{-i\lambda \cdot \mathbf{r}} w(\mathbf{r}) d\mathbf{r}$$
$$E_w(\lambda) = \int_{-\infty}^{\infty} \frac{1}{2} \|\Phi_w(\lambda)\|^2 d\lambda$$
$$(\lambda_m)_w = \max \{E_w(\lambda)\}$$

# Introduction

## Characteristic turbulence length scales

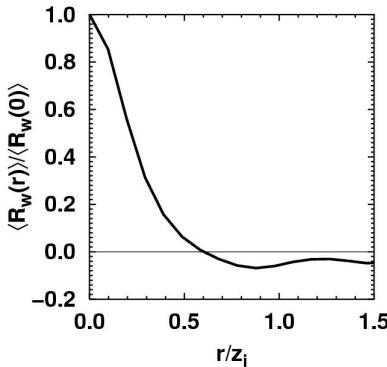
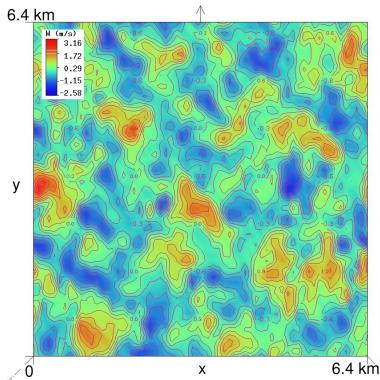


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integral length scale

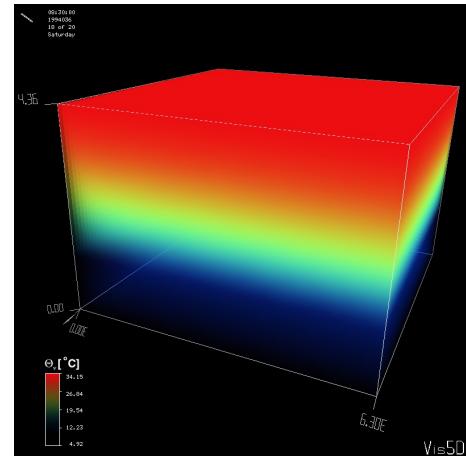
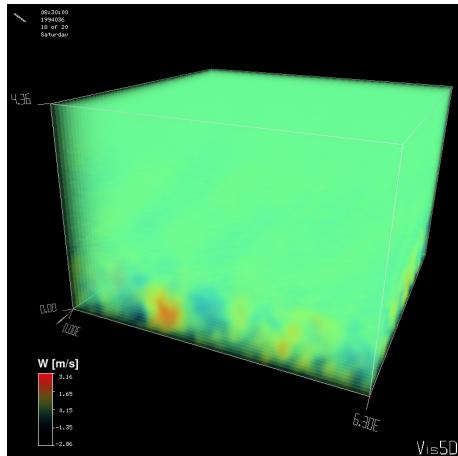
$$R_w(\mathbf{r}) = \int_{-\infty}^{\infty} w(\mathbf{x}) w(\mathbf{x} + \mathbf{r}) d\mathbf{x}$$

$$\Lambda_w = \frac{1}{R_w(0)} \int_0^{\infty} R_w(r) dr$$

# Introduction

## Determination of characteristic turbulence length scales

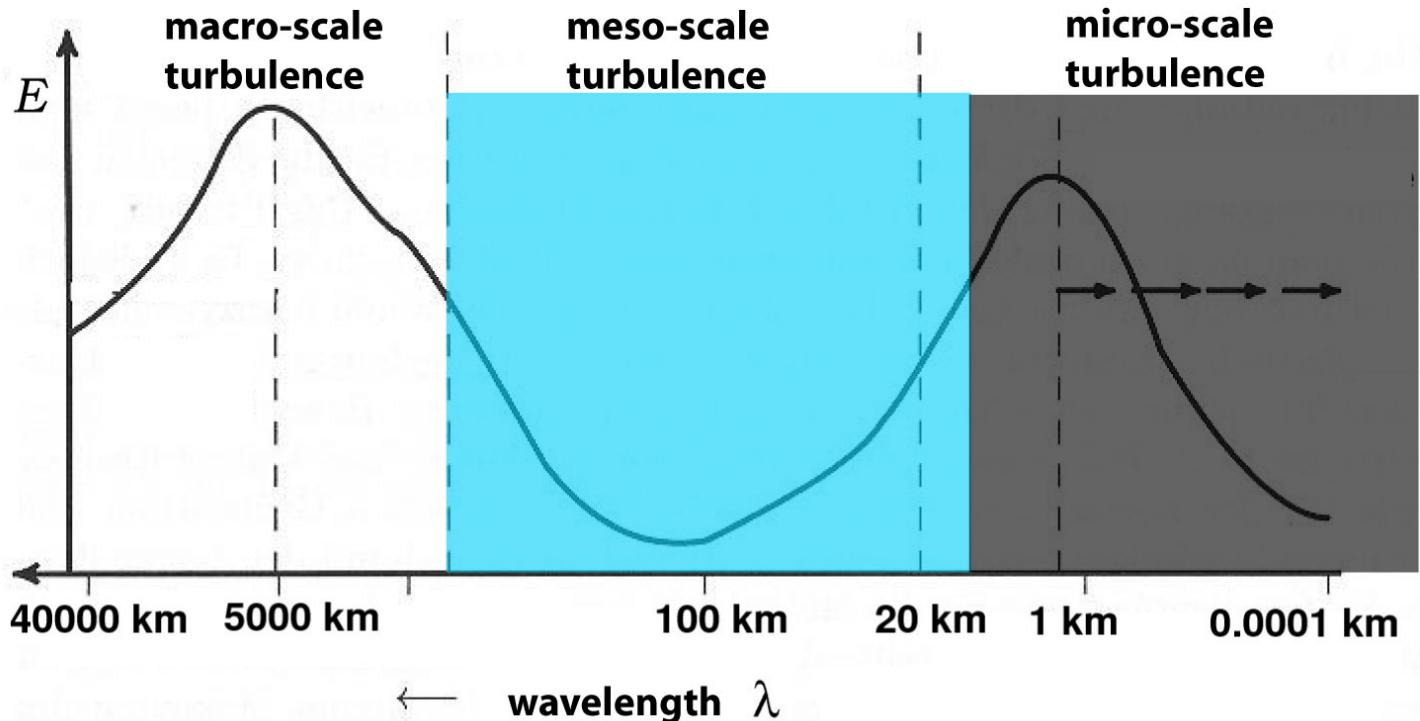
- Large-eddy simulation<sup>1</sup>
  - High-resolution numerical simulation of convective boundary layers
  - Grid space  $\Delta x \sim 0.1$  km —→ coherent structures are resolved
  - Modification of boundary layer parameters (wind, temperature, heat flux)
  - Supplement to laboratory experiments and field measurements



<sup>1</sup>e.g. Graf and Schumann, 1991; Khanna and Brasseur, 1998)

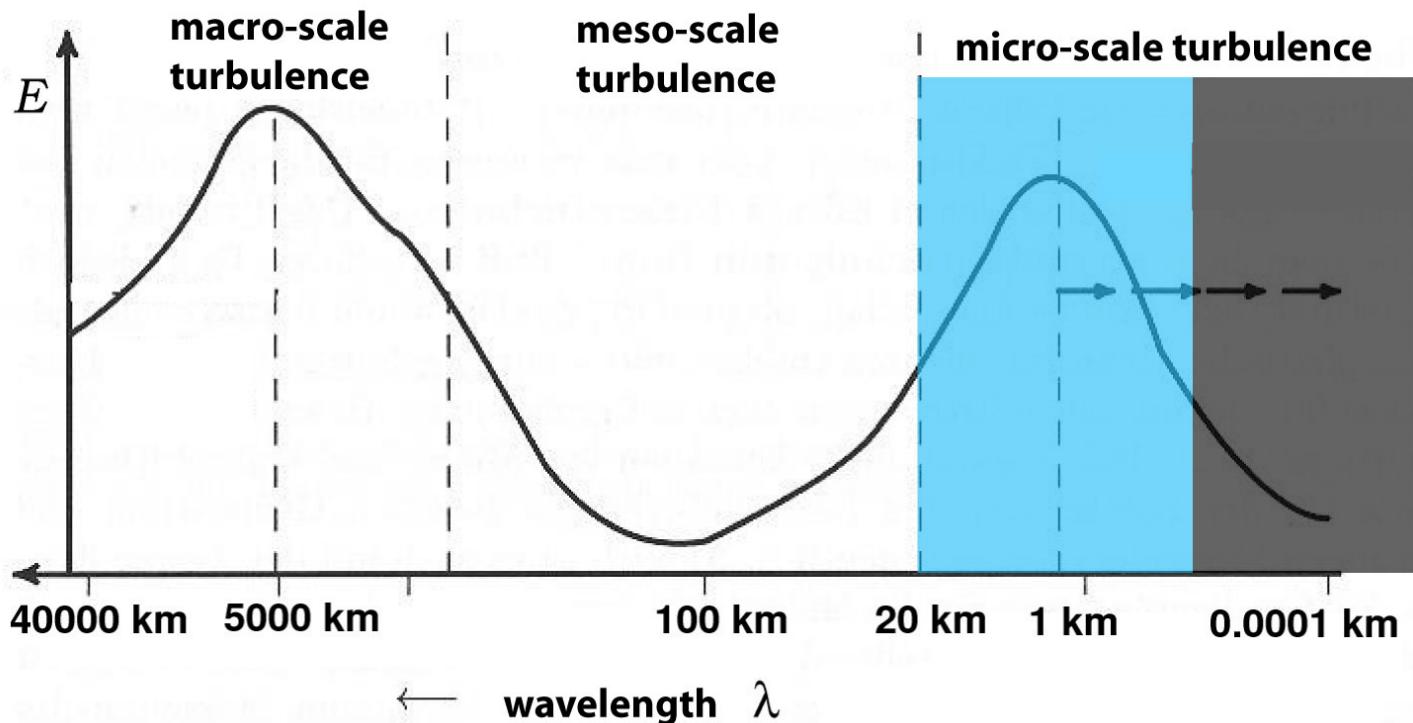
# Introduction

## Energy spectrum of atmospheric flows



# Introduction

## Energy spectrum of atmospheric flows



# Large-Eddy Simulation

## LES cases

Lauf	$-z_i/L$	$u_*$ [m s <sup>-1</sup> ]	$w_*$ [m s <sup>-1</sup> ]	$z_i$ [m]	$-L$ [m]	$\bar{u}_g$ [m s <sup>-1</sup> ]	$\bar{q}_t$ [g kg <sup>-1</sup> ]	$\overline{(w'\theta'_v)_s}$ [K m s <sup>-1</sup> ]
A	2.12	0.82	1.43	1593.30	751.00	20	24	0.058
B	5.91	0.50	1.23	1026.00	173.60	10	24	0.058
C	7.09	0.86	2.73	1600.00	225.80	20	12	0.231
D	9.16	0.73	2.06	1400.00	152.80	15	0	0.200
E	10.94	0.75	2.25	1586.70	145.00	15	12	0.231
F	11.41	0.74	2.27	1620.00	142.00	15	15	0.231
G	11.78	0.53	1.65	1246.70	105.80	10	24	0.115
H	17.29	0.56	1.98	1426.70	82.50	10	24	0.173
I	18.69	0.57	2.07	1400.00	74.90	10	0	0.200
J	23.22	0.57	2.21	1506.70	64.90	10	6	0.231
K	23.65	0.58	2.25	1586.70	67.10	10	24	0.231
L	24.89	0.57	2.24	1573.30	63.20	10	12	0.231
M	25.29	0.58	2.30	1686.70	66.70	10	18	0.231
N	48.25	0.46	2.26	1606.70	33.30	5	24	0.231

# Large-Eddy Simulation

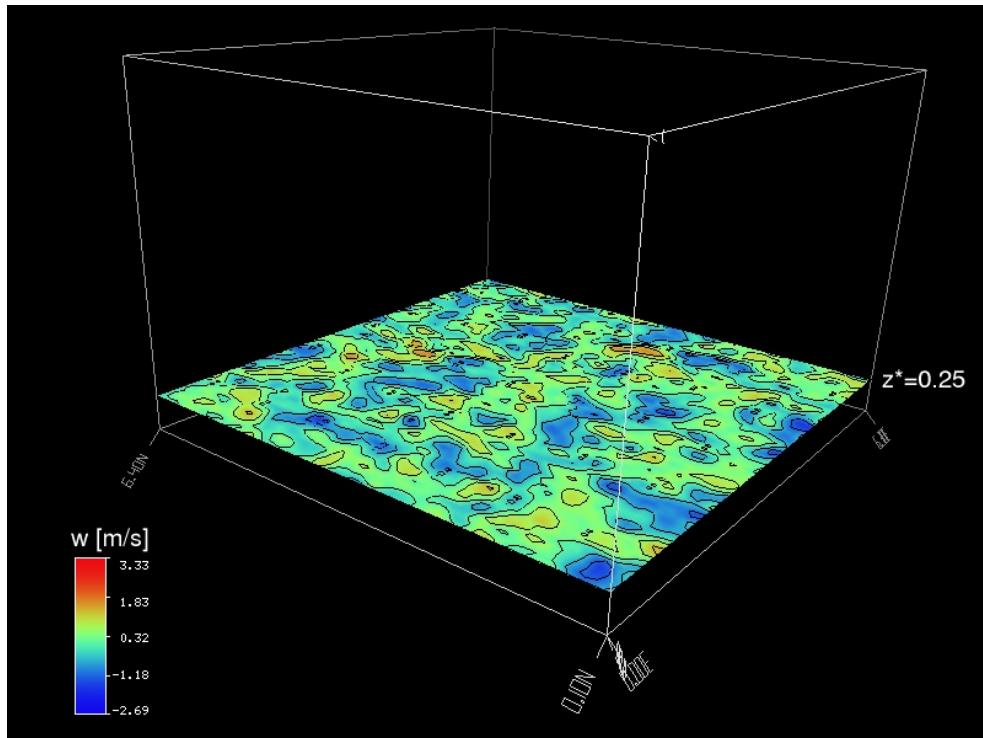
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# Large-Eddy Simulation

## Determination of characteristic turbulence length scales

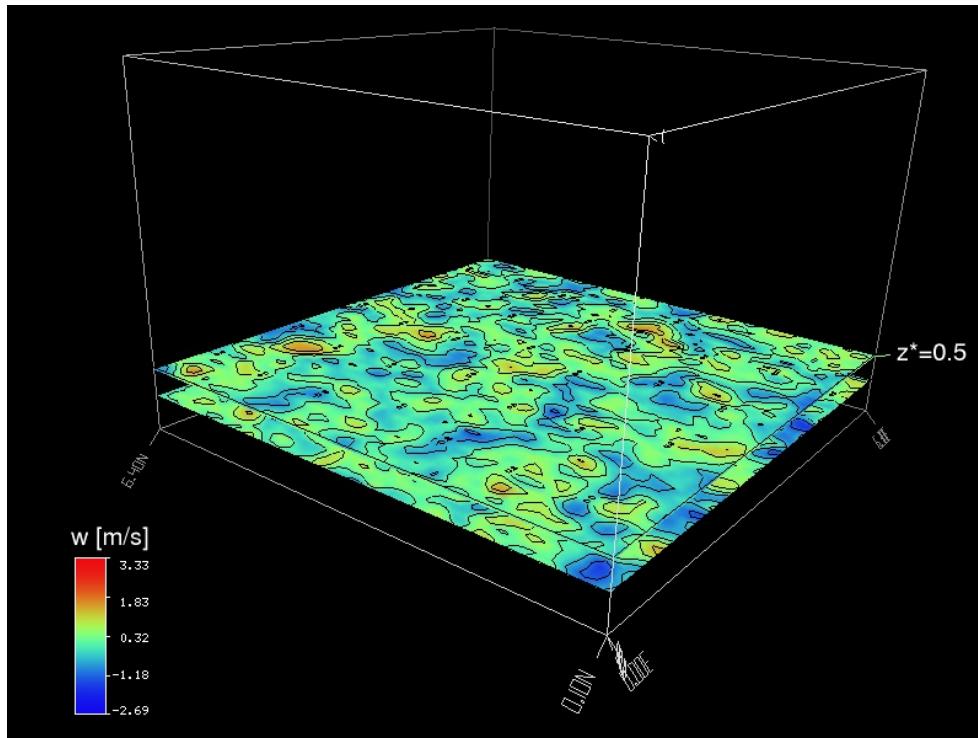
- 1D sampling of turbulent fluctuations at horizontal LES model levels
- Computation of 1D- spectra and auto-covariances for x- and y-pathways



# Large-Eddy Simulation

## Determination of characteristic turbulence length scales

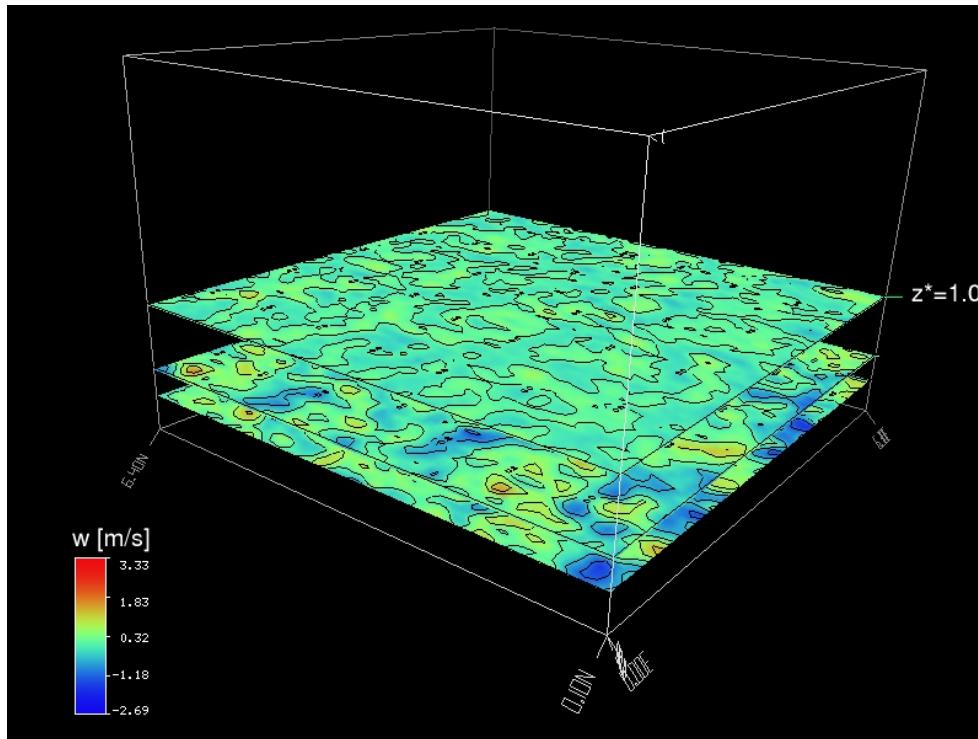
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# Large-Eddy Simulation

## Determination of characteristic turbulence length scales

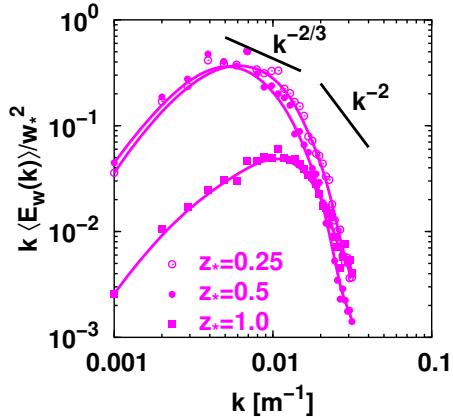
- 1D sampling of turbulent fluctuations at horizontal LES model levels
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# Large-Eddy Simulation

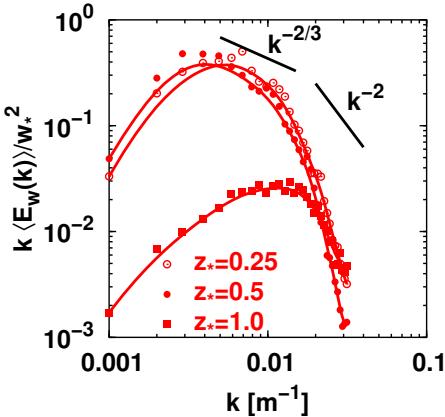
## Case B

Stability:  $-z_i/L = 6$



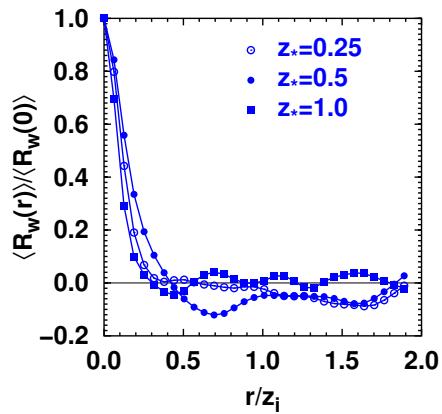
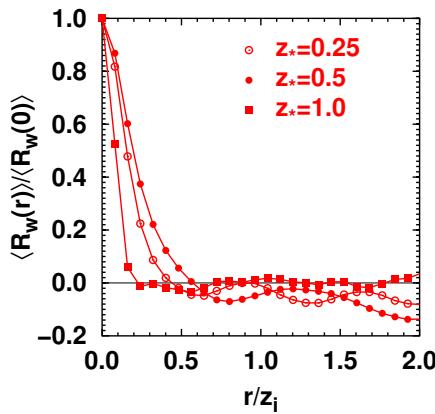
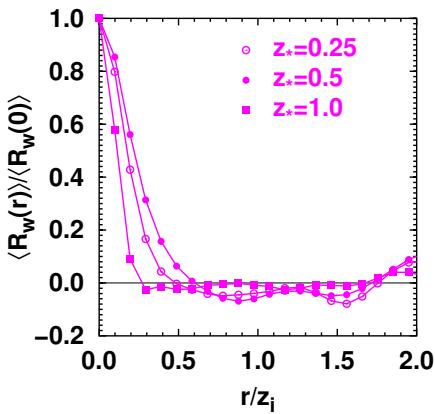
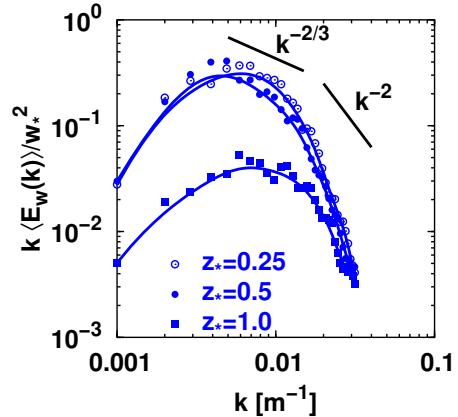
## Case G

$$-z_i/L = 12$$



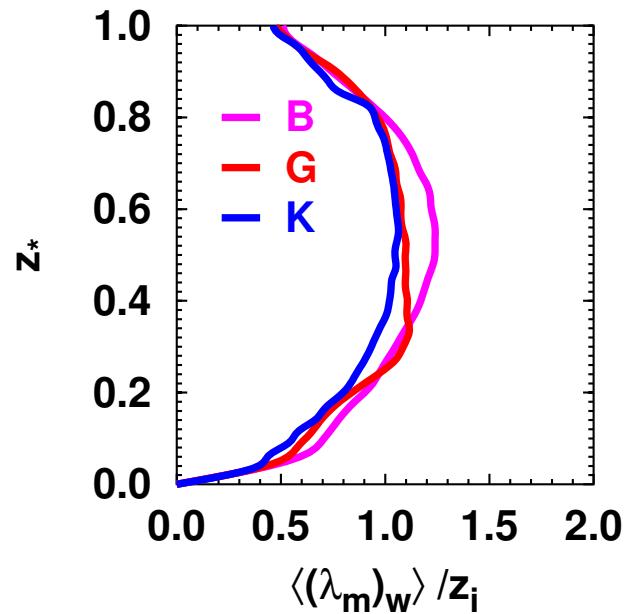
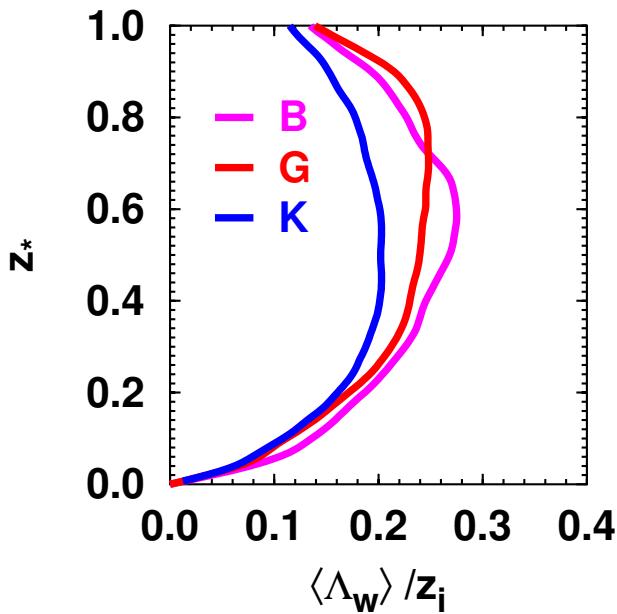
## Case K

$$-z_i/L = 24$$



# Large-Eddy Simulation

- Stability-dependent turbulence length scales
- Agreement with laboratory experiments<sup>1</sup>, atmospheric measurements<sup>2</sup>, and further numerical simulations<sup>3</sup>



<sup>1</sup>Wind tunnel exp.: *Kaiser and Fedorovich, 1998*

<sup>2</sup> SEMAFORE: *Durand et al., 2000*

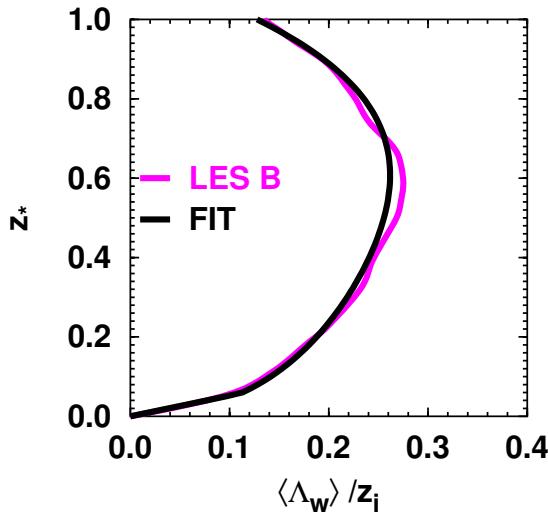
<sup>3</sup> LES: *Graf and Schumann, 1991, Khanna and Brasseur, 1998*

# Large-Eddy Simulation

## Parameterisation of turbulence length scales

### Integral length scale

$$\frac{\langle \Lambda_w \rangle}{z_i} = \begin{cases} a_0 z_*, & z_* < z_{*0} \\ a_1(z_*)^{1/2}(1 - a_2 z_*)(1 + a_3 z_*)^2, & z_* \geq z_{*0} \geq 1 \end{cases}$$



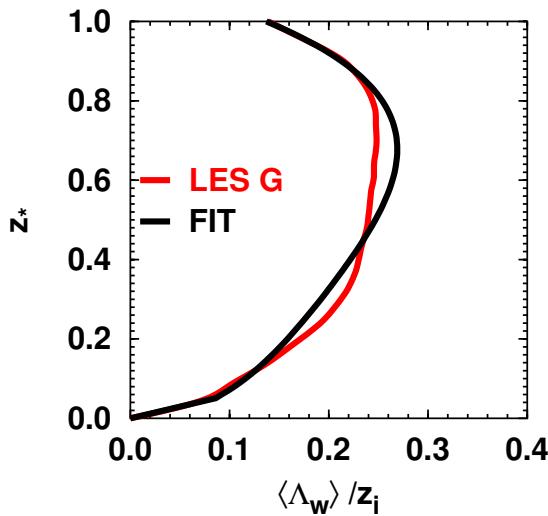
Case	$a_0$	$\Delta a_0$	$a_1$	$\Delta a_1$	$a_2$	$\Delta a_2$	$a_3$	$\Delta a_3$
B	1.992	0.025	0.4773	0.0033	0.8910	0.0026	1.444	0.047

# Large-Eddy Simulation

## Parameterisation of turbulence length scales

Integral length scale

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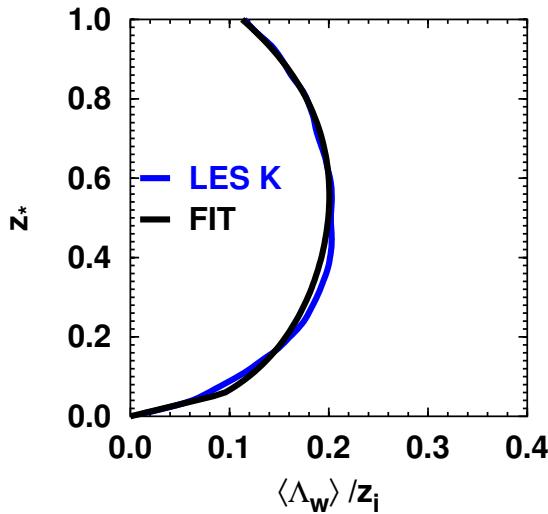
Case	$a_0$	$\Delta a_0$	$a_1$	$\Delta a_1$	$a_2$	$\Delta a_2$	$a_3$	$\Delta a_3$
G	1.704	0.015	0.3930	0.0041	0.8991	0.0046	2.457	0.140

# Large-Eddy Simulation

## Parameterisation of turbulence length scales

### Integral length scale

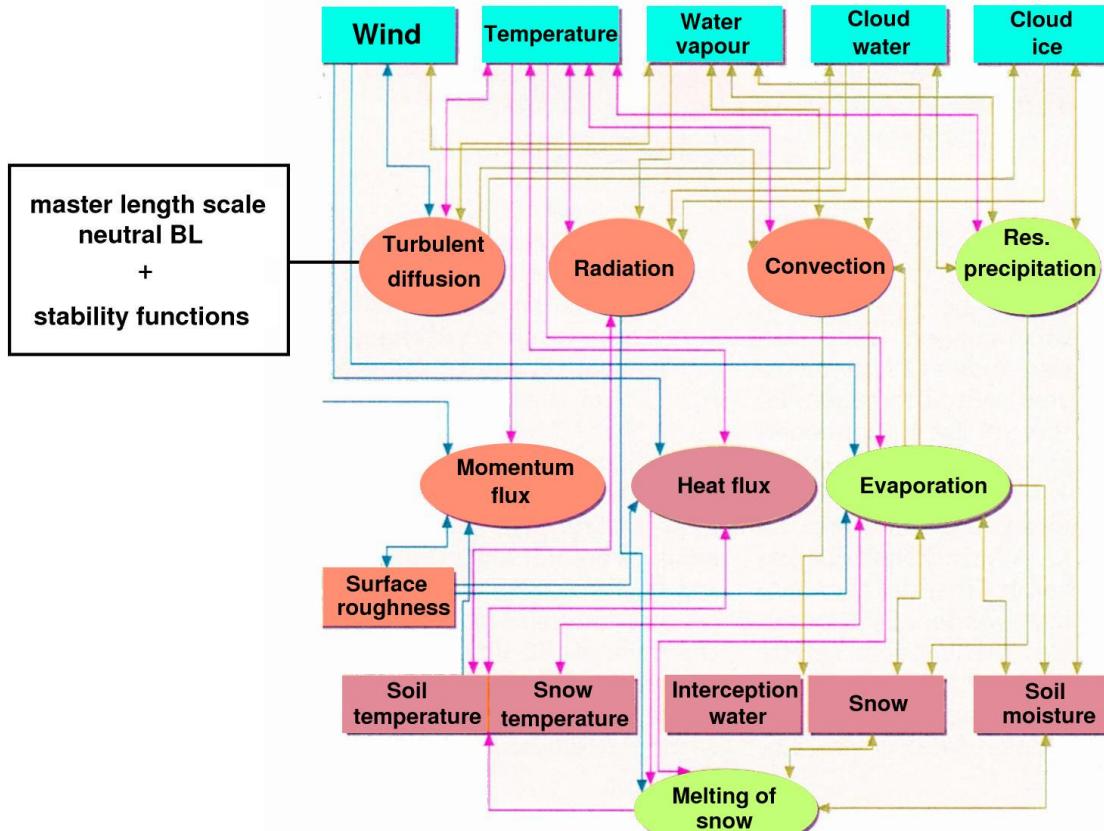
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Case	$a_0$	$\Delta a_0$	$a_1$	$\Delta a_1$	$a_2$	$\Delta a_2$	$a_3$	$\Delta a_3$
K	1.659	0.012	0.4087	0.0038	0.8452	0.0086	0.780	0.068

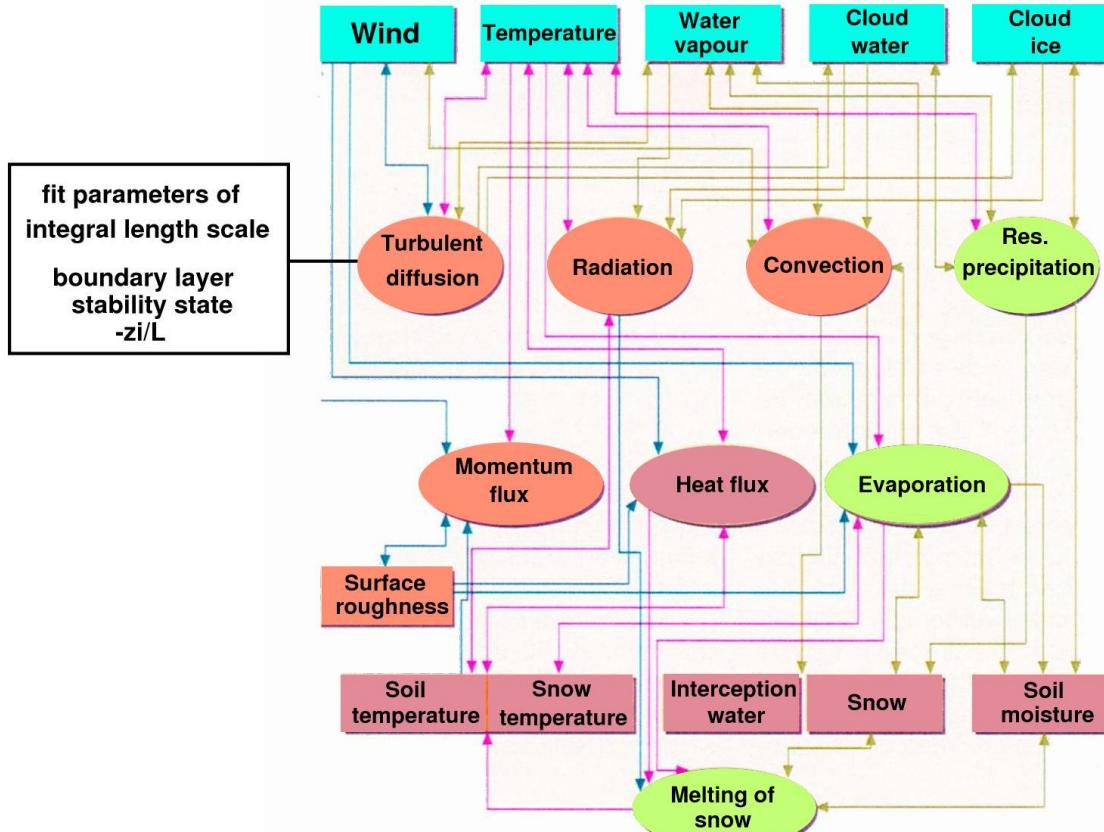
# Lokal-Modell Case Study

## Implementation of the modified mixing length



# Lokal-Modell Case Study

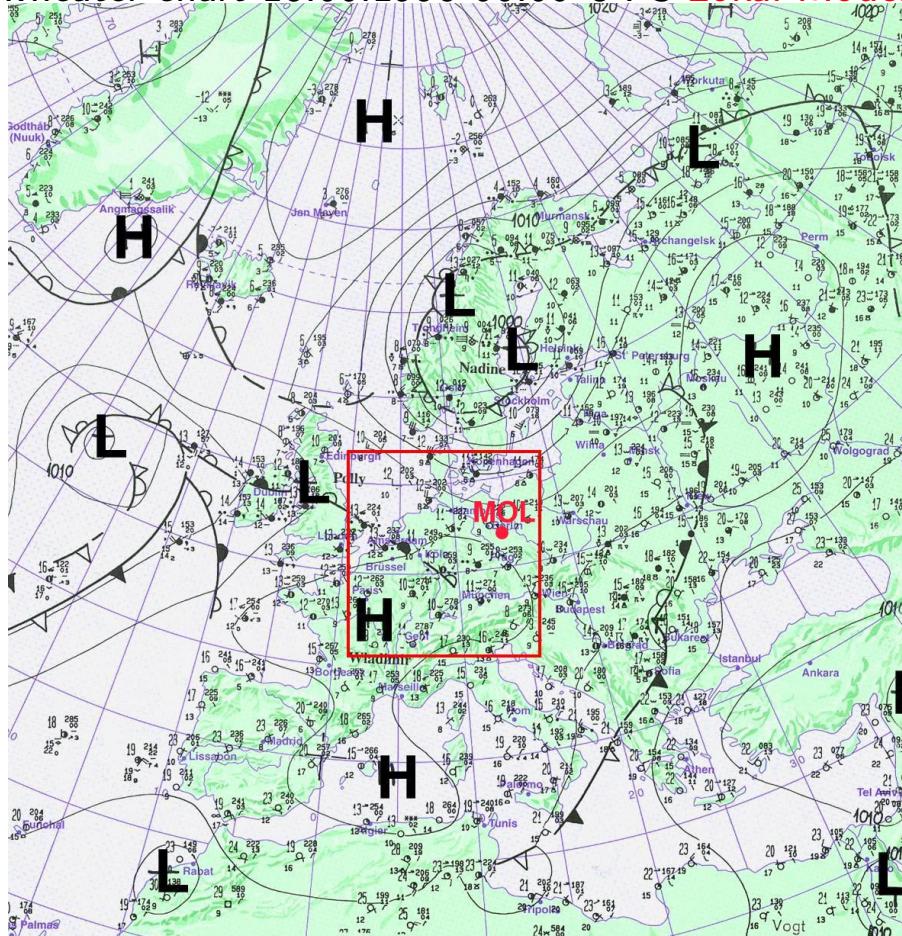
## Implementation of the modified mixing length



# Lokal-Modell Case Study

## Simulation scenario LITFASS

Surface wheater chart 18.06.1998 00:00 UTC Lokal-Modell domain



**20 hPa**

**Altitude**

**56° N**

**Latitude**

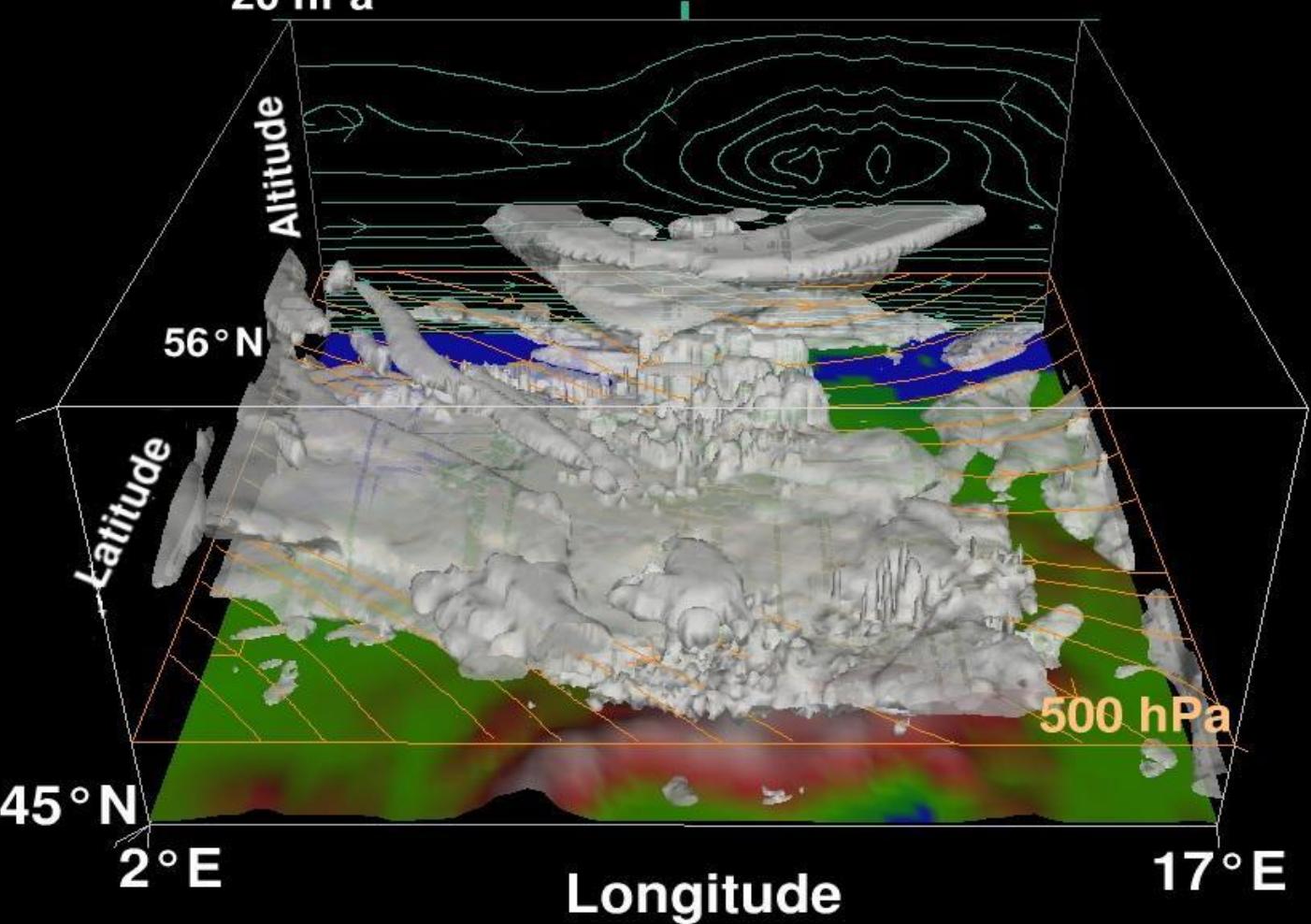
**45° N**

**2° E**

**Longitude**

**500 hPa**

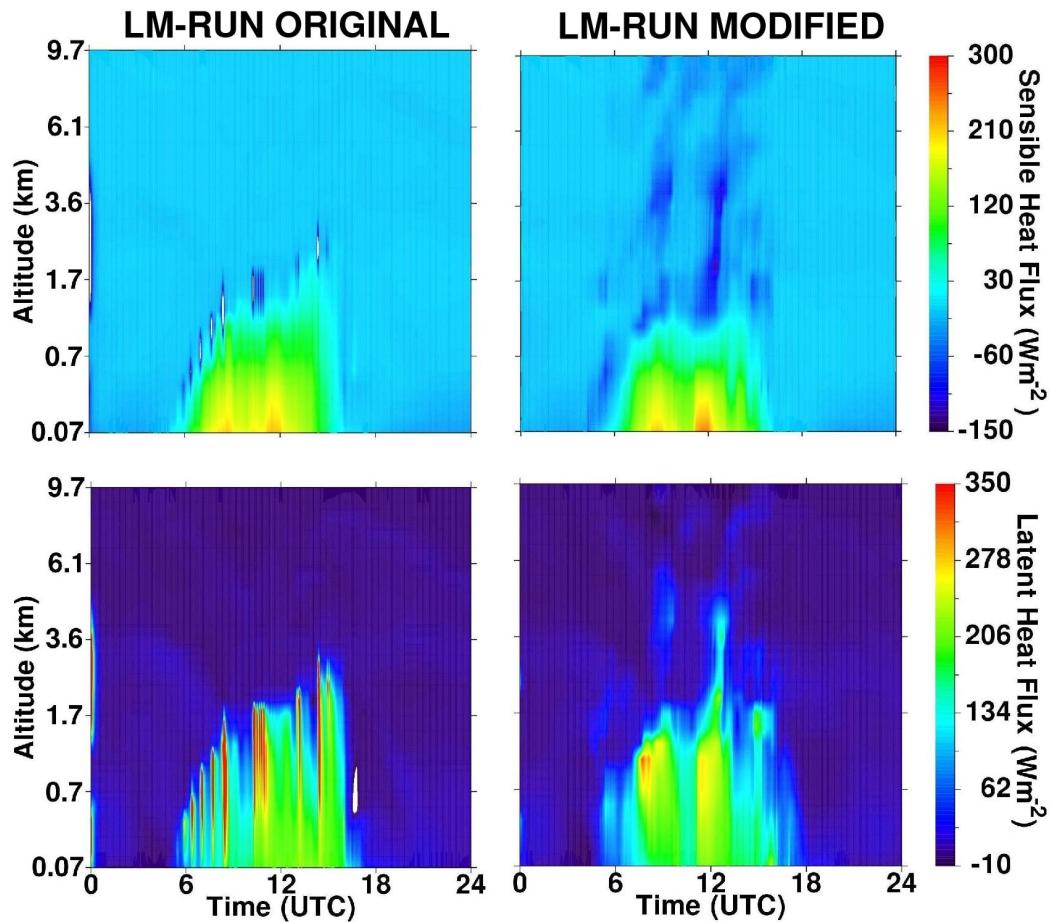
**17° E**



# Lokal-Modell Case Study

## Direct Comparison of Simulation Results

Turbulent Fluxes at grid point MOL



# Lokal-Modell Case Study

## Direct Comparison of Simulation Results

### Statistical Analysis

- Correlation coefficient

$$\rho_{A,B} = \frac{\frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (A_{i,j} - \bar{A})(B_{i,j} - \bar{B})}{\left\{ \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (A_{i,j} - \bar{A})^2 \right\}^{1/2} \left\{ \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} (B_{i,j} - \bar{B})^2 \right\}^{1/2}}$$

- Centered pattern RMS difference (CP-RMS difference)<sup>1</sup>

$$\delta_{A,B}^{\text{CP}} = \left[ \frac{1}{N_x N_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \{(A_{i,j} - \bar{A}) - (B_{i,j} - \bar{B})\}^2 \right]^{1/2}$$

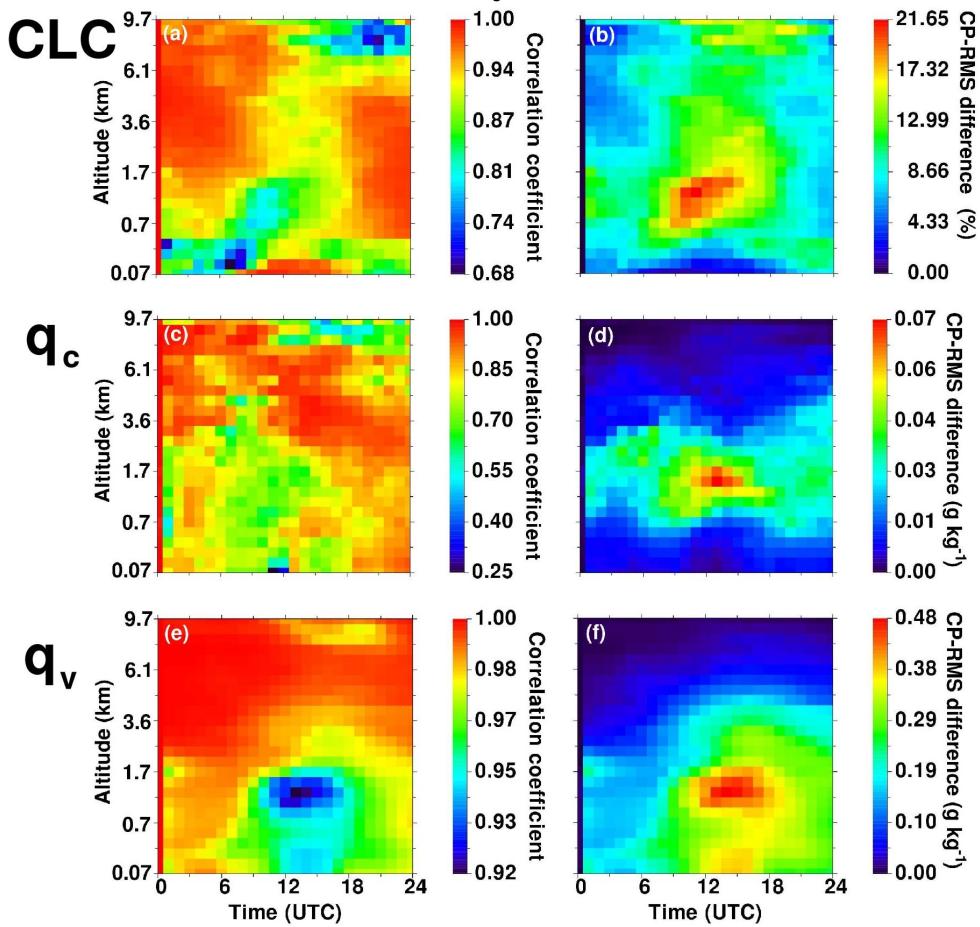
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<sup>1</sup> Taylor, 2001

# Lokal-Modell Case Study

# Direct Comparison of Simulation Results

## Statistical Analysis – 3D fields

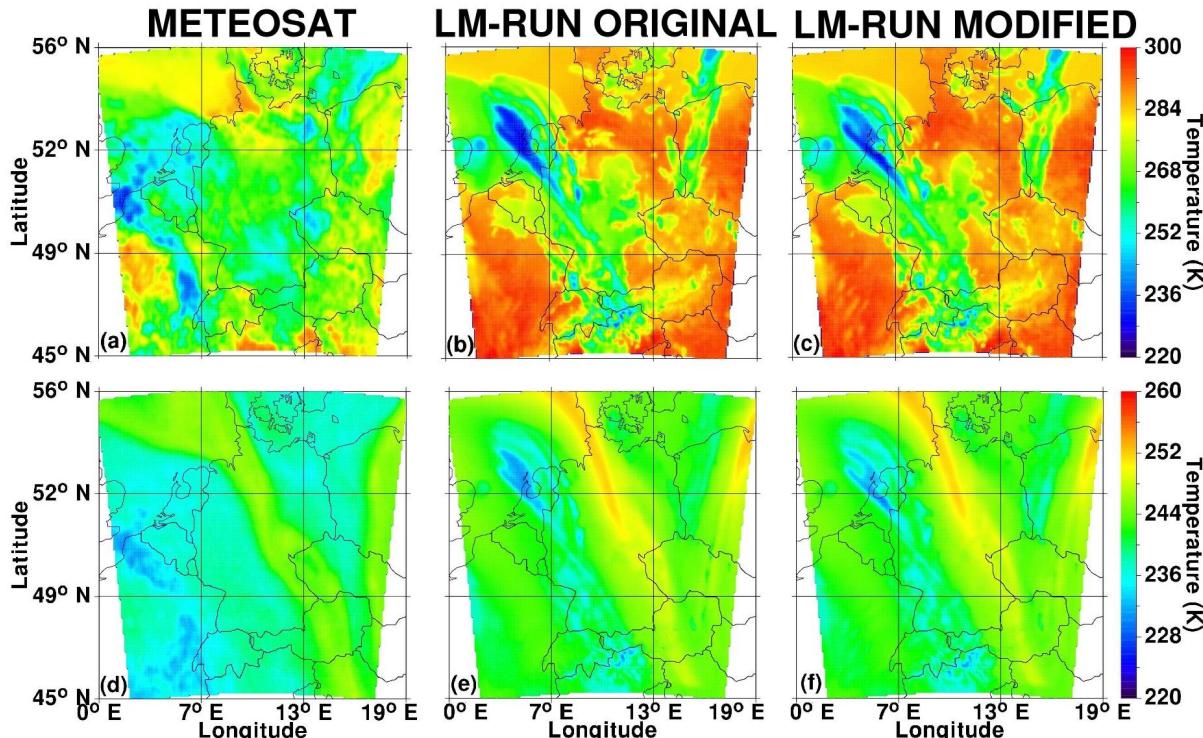


# Lokal-Modell Case Study

## Verification of Simulation Results

Brightness temperatures from METEOSAT satellite observations<sup>1</sup>

2 channels:  $T_{IR}$  10.5  $\mu\text{m}$ -12.5  $\mu\text{m}$  and  $T_{WV}$  5.7  $\mu\text{m}$ -7.1  $\mu\text{m}$



<sup>1</sup> Morcrette, 1991

# Lokal-Modell Case Study

## Verification of Simulation Results

Comparison with 4dVar-analysis data – statistical analysis

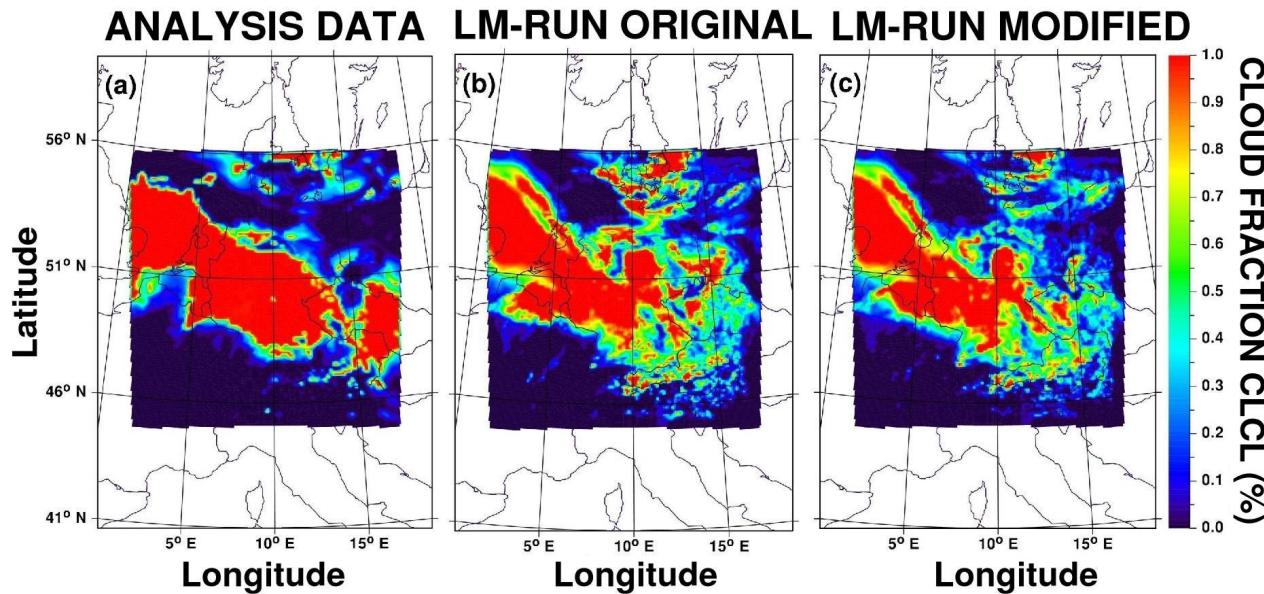
Parameter	Time [UTC]	$\varrho_{A,I}$	$\varrho_{A,II}$	$\frac{\Delta\varrho}{\varrho_{A,I}}$ [-100 %]	$\delta_{A,I}^{CP}$ [Unit of the parameter]	$\delta_{A,II}^{CP}$	$\frac{\Delta\delta^{CP}}{\delta_{A,I}^{CP}}$ [-100 %]
$T_{2\text{ m}}$	06:00	0.95	0.951	0.1	0.84	0.835	-0.616
	12:00	0.913	0.91	-0.343	1.637	1.714	4.7
	18:00	0.955	0.954	-0.158	1.088	1.106	1.646
$U_{10\text{ m}}$	06:00	0.955	0.944	-1.127	0.864	0.921	6.598
	12:00	0.857	0.829	-3.363	1.284	1.376	7.175
	18:00	0.781	0.774	-0.883	1.367	1.352	-1.068
CLCH	06:00	0.595	0.601	1.006	0.372	0.37	-0.6
	12:00	0.443	0.423	-4.479	0.446	0.455	2.012
	18:00	0.333	0.353	6.12	0.418	0.41	-1.89
CLCM	06:00	0.822	0.823	0.148	0.245	0.245	-0.32
	12:00	0.705	0.7	-0.82	0.318	0.319	0.392
	18:00	0.653	0.651	-0.348	0.342	0.341	-0.235
CLCL	06:00	0.749	0.746	-0.42	0.289	0.291	0.491
	12:00	0.629	0.698	10.88	0.358	0.319	-10.959
	18:00	0.711	0.707	-0.592	0.315	0.314	-0.455

# Lokal-Modell Case Study

## Verification of Simulation Results

Comparison with 4dVar-analysis data – 2D fields

Example: cloud fraction of low clouds 18.06.98 12:00 UTC



# Conclusions

- Problems in parameterisation of turbulent transports
- Characteristic turbulence length scales as turbulent mixing length in turbulence closure models
- Three-dimensional large-eddy simulations of convective boundary layers for a wide range of stability states
- Development of an approximation for the vertical profile of integral length scale
- Case study with the Lokal-Modell of the DWD
- Sensitivity of cloud fraction of low clouds on turbulent mixing length

# Outlook

- Large-eddy simulations
  - Further investigations of the moisture – and cloud impact on characteristic turbulence length scales
  - Two-dimensional spectra and auto-covariances → reduced sampling complexity
- Mesoscale Simulations
  - Larger number of synoptic situations
  - Use of further assimilated data in analysis data for verification purposes

# Acknowledgment

Olaf Hellmuth

Andreas Chlond

Hans-Joachim Herzog

Gerd Vogel

Barbara Heide

# **Examination of the impact of mixing-length formulation on mesoscale simulation results**

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